Noise, Error and Bandwidth in Polarimeters

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Summary and Conclusions

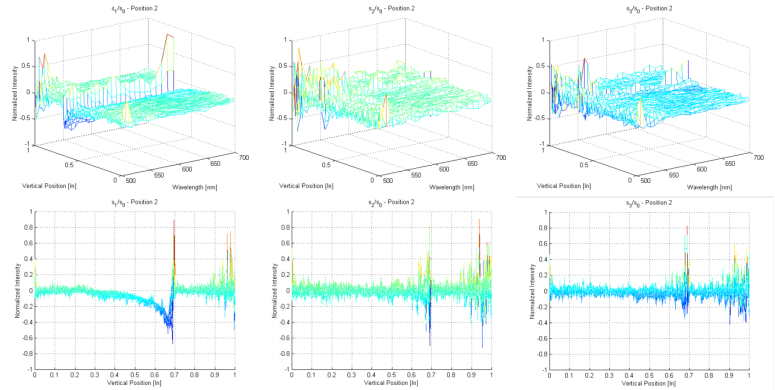
$$\underline{\mathbf{S}}\left(\vec{\Theta}\right) = \underline{\mathbf{S}}\left(x, y, t, \sigma, \theta, \phi, \ldots\right) \qquad \underline{\underline{\mathbf{M}}}\left(\vec{\Theta}\right) = \underline{\underline{\mathbf{M}}}\left(x, y, t, \sigma, \theta, \phi, \ldots\right)$$

- The polarimetric object that you are trying to reconstruct has some bandwidth in any number of potential modulation dimensions:
  - Scene can be spatially modulated using microgrid or polarimetric interference
  - Scene can be spectrally modulated using a channeled system
  - Scene can be temporally modulated using a variable optic
  - Scene can be angle-modulated using a high-NA beam and angle-varying optics
- Your system has some specified space/time/spectral/angular resolution (bandwidth)
  - The bandwidth of the object in the various dimesnions of sensitivity impact your choice of modulation strategies
  - Interplay between object and system bandwidth places modified crosstalk (channels) and sampling requirements
  - Each modulation strategy comes with particular set of image artifacts
- Your choices of how your modulation strategy uses the bandwidth of the system affects the quality of the reconstruction
  - Well-conditioned systems will be least sensitive to noise
  - Well-designed systems will have minimal instrumental variation in the dimensions of modulation
  - The reconstruction should recognize that the object is varying in the modulation dimensions and take this into account - use your bandwidth wisely

Outline

Early Observations

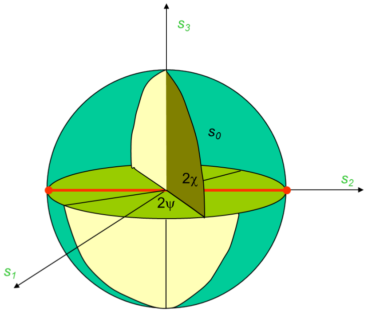
s_1 / s_0 s_2 / s_0 s_3 / s_0



This variable retardance spectropolarimeter data[1] troubled us for several months. We did not understand why we were seeing different SNR in the different polarimeter channels.

[1] J. S. Tyo and T. S. Turner, *Appl. Opt.* **40**:1450 (2001)

System Conditioning[2,3,4,5]



The now well-known result that the measurements should be maximally separated on the Poincaré sphere is related to system condition. Minimization of redundancy in the measurements maximizes noise performance. Similar effects are apparent for calibration.

Several groups discovered and re-discovered this concept. This is a form of bandwidth maximization – attempting to measure as much information as possible with each successive analysis state. We will revisit this time and time again in the context of Stokes and Mueller polarimeters.

[2] R. M. A Azzam, *et al.*, *JOSA A* **5**:681 (1998)

[3] A. Ambirajan and D. C. Look, *Opt. Eng* **34**:1651 (1995)

[4] D. S. Sabatke, *et al.*, *Opt. Lett.* **25**:802 (2000)

[5] J. S. Tyo, *Opt. Lett.* **25**:1198 (2000)

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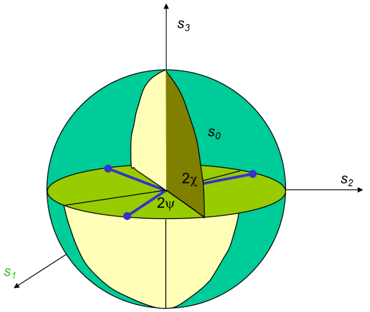

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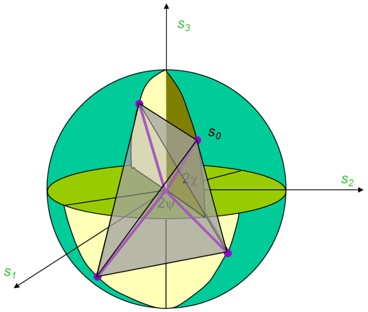

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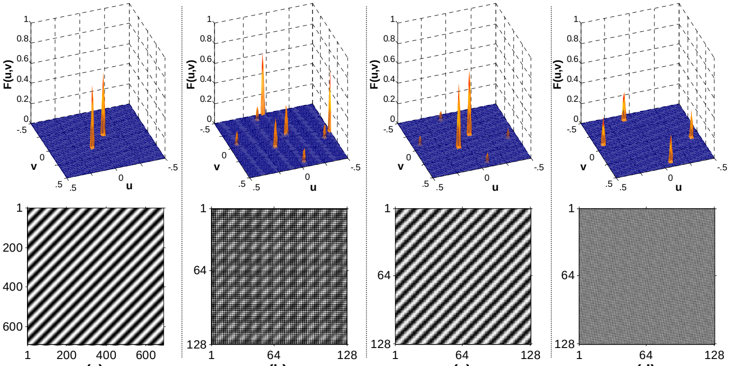
[5] J. S. Tyo, *Opt. Lett.* **25**:1198 (2000)

Frequency Cross-Modulation

Later we were working with microgrid polarimeters and determined that the spatial frequencies in the reconstructed images were not the same as those in the input[6].

This implied that the system was not LSI, but we did not understand the implications immediately.



(a) (b) (c) (d)

[6] B. M. Ratliff, *et al.*, *Opt. Exp.* **17**:9112 (2009)

Instrument Bandwidth

Fig. 4. Intensity pattern with mesh-like fringes.

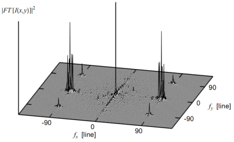
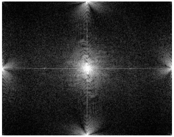


Fig. 5. Power spectrum of the intensity pattern.

The use of spatial channels from a prismatic polarimeter[a] motivated us to apply a similar strategy to microgrid images.



[a]M. W. Kudenov, *et al.*, *Opt. Exp.* **16**:13720 (2008)

This strategy can be generally extended to modulation in any domain (space, time, wavenumber, angle of incidence) or even multiple domains.

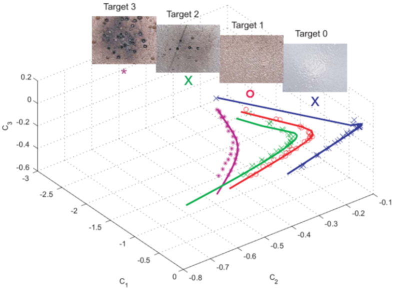
A single general formalism encompasses all of them[a].

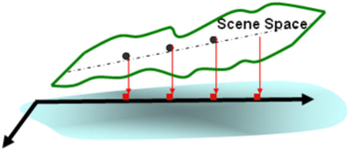
Choice of filter depends on the data being examined, and there is a tradeoff between the channel bandwidth, system condition, noise performance.

- The solution of LaCasse is the maximum bandwidth solution that attempts to reconstruct all frequencies within channels.
- The standard DRM formalism is the opposite extreme. The Null space of the operator is all frequencies other than the modulation frequencies - maximum noise rejection.

[a]C. F. Lacasse, *et al.*, *Opt. Exp.* **19**:14976 (2011)

Partial Mueller Polarimeters





Good case- the reconstruction of data lies in a direction of sensor space with higher SNR

We developed active polarimeters that could perform detection tasks using subsets of Mueller elements and combinations thereof[a].

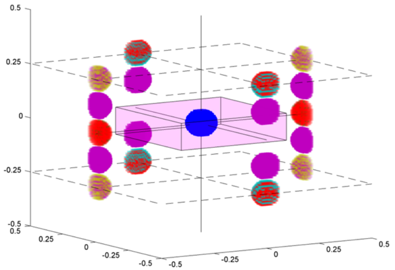
[a]Hoover & Tyo, *Appl. Opt.* **46**:8364 (2007)

Other work with tunable multispectral sensors had taught us to consider the relation between the spaces spanned by the sensor measurements and the scene elements[a].

[a]Z. Wang, *et al.*, *JOSA A* **25**:2528 (2008)

Multidomain Modulation/Beam Swapping

- Channeled polarimeters are essentially all single-domain instruments
- Strategies for multiple domain modulation do exist
  - Rotating Retarder + Microgrid [a]
  - Time Modulated + Channeled Spectropolarimeter
  - Complicated space-time modulations using pixellated, addressible devices
- Ultimately the question arises about how to best use the available space/time/spectral bandwidth and sensitivities of the detector in order to maximize SNR (and accuracy!) in the reconstructed polarimetric data
- Concepts are all easily generalized to active Mueller polarimeters



[a] LaCasse, *et al.*, *Proc. SPIE* **8160** (2011)

Motivating Problems

Polarimeters are inherently information extraction systems, and data-space-based methods of analyzing their performance are highly appropriate.

Data-space based methods impact the performance of the sytem in terms of its

- Noise – System conditioning inherently relates the detector SNR to the reconstructed SNR in the final images or measurements [7]
- Uniformity – Uniform distributions in measurement space lead to overall better performance in terms of both noise and error [8]
- Calibration & Error – The choice of calibration targets affects the accuracy of the reconstruction due to effects of instrumental error [9]
- System Bandwidth – A polarimeter necessarily uses some of the available detector bandwidth (space, time, spectrum) to make measurements that can be used to recover the invisible polarization signals [10]

[7] J. S. Tyo and T. S. Turner, *Appl. Opt.* **40**:1450 (2001)

[8] J. S. Tyo, *JOSA A* **15**:359 (1998)

[9] J. S. Tyo, *Appl. Opt* **41**:619 (2002)

[10] J. S. Tyo, *et al.*, *Opt. Lett.* **34**:3187 (2009)

Generalized Channeled Polarimetry[11]

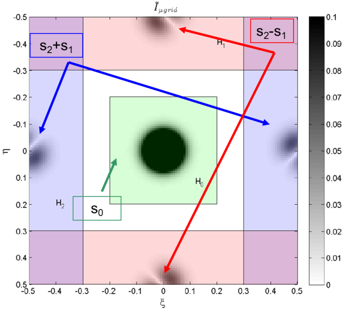
Oka, Dereniak, Kudenov, and their collaborators have popularized the class of "snapshot" polarimeters based on channeled concepts.

Tyo's group has shown that Division of Time and Division of Focal Plane (microgrid) polarimeters are really just channeled devices.

Brady's group has developed related strategies (primarily for spectrometers) that use "random codes," which are just another form of channel

Common modulation schemes include:

- Rotating retarder or PEM-based (time modulated)
- Microgrids (space modulated)
- Birefringent prisms (space modulated)
- High-order retarders (spectrally modulated)
- Combinations thereof



[11] A. S. Alenin & J. S. Tyo, *JOSA A* (2014, In Press)

Introduction

Problem:

- Channeled polarimetry papers often describe modulation/demodulation strategies using manually arithmetic
- Splitting is tracked by hand and some channels are ignored to simplify reconstruction

Solution:

- Automate channel construction directly from the analytical modulations present
- Generate a matrix describing the mapping between the elements and the channels
- Rely on linear algebra to invert the process rather than arithmetic operations

Design Automation Process

Every modulation will perform a convolution in the corresponding frequency domain

- With arbitrary modulation dimensions, we might need a N-dimensional convolution
- If successive modulations are within the same dimension, they can be combined

The entire channeled modulation can be represented by combining individual dimensions

$$\underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{ij}} = \underline{\mathbf{q}}_{\{\tau\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\omega\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\xi\};m_{ij}} \otimes \underline{\mathbf{q}}_{\{\eta\};m_{ij}}$$

If modulations are interleaved, it may be easier to deal with convolutions directly

$$\underline{\mathbf{q}}_{\xi_{e_1}/\eta_{e_2}/\xi_{e_3}/\eta_{e_4};m_{ij}} = \mathrm{vec}\left(\underline{\mathbf{q}}_{\xi_{e_1};m_{ij}} * \underline{\mathbf{q}}_{\eta_{e_2};m_{ij}} * \underline{\mathbf{q}}_{\xi_{e_3};m_{ij}} * \underline{\mathbf{q}}_{\eta_{e_4};m_{ij}}\right)$$

Similarly, we can express the construction of those channels in terms of PSA and PSG modulations:

$$\underline{\mathbf{q}}_{m_{ij}} = \mathrm{vec}\left(\underline{\mathbf{q}}_{g_i} * \underline{\mathbf{q}}_{a_j}\right) \rightarrow \underline{\underline{\mathbf{D}}} = \underline{\mathbf{A}}\,\underline{\mathbf{G}}^T \rightarrow \mathcal{F}\left\{\underline{\underline{\mathbf{D}}}\right\} = \mathcal{F}\left\{\underline{\mathbf{A}}\right\} * \mathcal{F}\left\{\underline{\mathbf{G}}\right\}^T$$

Combining sixteen of those vectors will produce the corresponding $\underline{\underline{\mathbf{Q}}}$ matrix,

$$\underline{\underline{\mathbf{Q}}} = \left(\ \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{00}} \ \cdots \ \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{03}} \ \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{10}} \ \cdots \ \underline{\mathbf{q}}_{\{\tau,\omega,\xi,\eta\};m_{33}}\ \right)$$

Multiple Function Modulation

The following function is a representative modulation encountered in channeled systems

$$f_M(x) = \prod_{m=1}^{M} {}^{\cos}_{\sin}(2\pi\xi_m x)$$

For which we can consider every possible modulation function/frequency combination

$$\underline{\underline{\mathbf{F}}}_M = \left[\begin{array}{cccc} \underline{\mathbf{f}}_1 & \underline{\mathbf{f}}_2 & \cdots & \underline{\mathbf{f}}_M \end{array} \right], \text{ where } f_{m,k} = \begin{cases} 0 & \text{if } \cos \\ 1 & \text{if } \sin \end{cases}$$

$$\underline{\underline{\mathbf{O}}}_M = \left[\begin{array}{cccc} \underline{\mathbf{o}}_1 & \underline{\mathbf{o}}_2 & \cdots & \underline{\mathbf{o}}_M \end{array} \right], \text{ where } o_{m,\ell} = \begin{cases} -1 & \text{if } -\xi_i \\ +1 & \text{if } +\xi_i \end{cases}$$

from which we can calculate the Frequency Phase Matrix as

$$\underline{\underline{\mathbf{P}}}_M = \frac{1}{2^M} \exp\left[\frac{-\mathrm{j}\pi}{2}\left(\underline{\underline{\mathbf{F}}}_M \underline{\underline{\mathbf{O}}}_M^T\right)\right]$$

Frequency Phase Matrix

The result of the multiplication can be shown with the phase of the coefficients encoded into hue

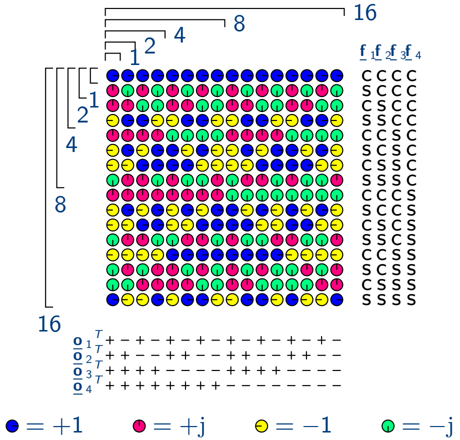


Figure: Frequency phase matrix for modulations containing up to four sinusoids

Spectrally Channeled Polarimeter[12] - I

$$\underline{\underline{\mathbf{X}}}(\sigma) = \underline{\underline{\mathbf{LP}}}(0)\,\underline{\underline{\mathbf{LR}}}(c_4\sigma, 45^\circ)\,\underline{\underline{\mathbf{LR}}}(c_3\sigma, 0^\circ)\,\underline{\underline{\mathbf{M}}}(\sigma)\,\underline{\underline{\mathbf{LR}}}(c_2\sigma, 0^\circ)\,\underline{\underline{\mathbf{LR}}}(c_1\sigma, 45^\circ)\,\underline{\underline{\mathbf{LP}}}(0)$$

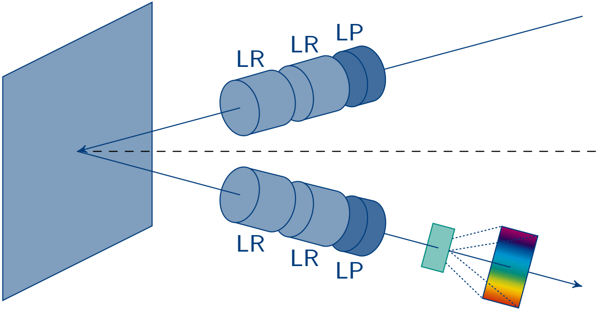


Figure: System Layout

[12] N. Hagen, *et al.*, *Opt. Lett.* **32**:2100 (2007)

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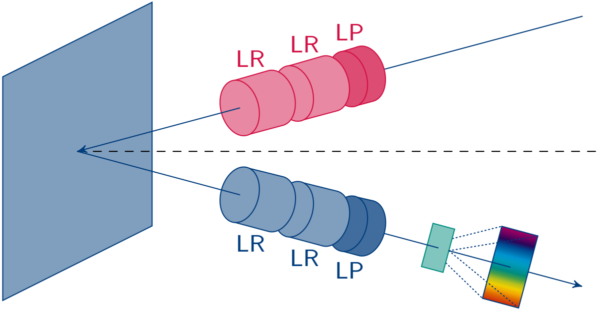

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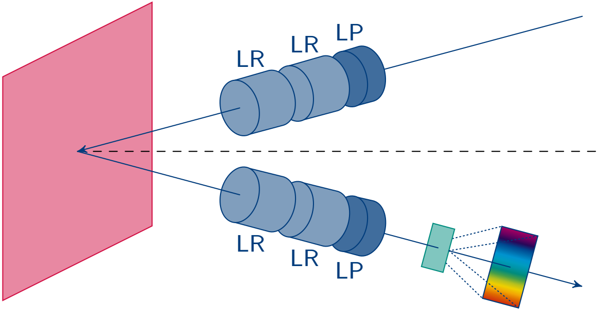

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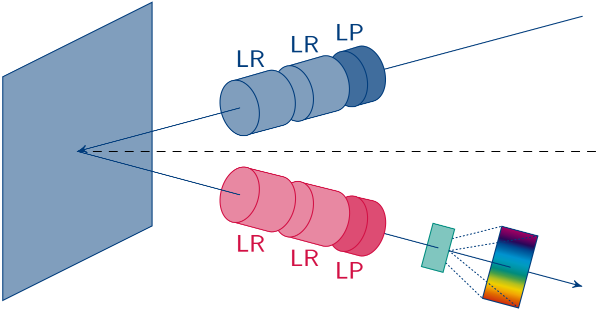


Figure: System Layout

[12] N. Hagen, *et al.*, *Opt. Lett.* **32**:2100 (2007)

Spectrally Channeled Polarimeter - II

$$\underline{\underline{\mathbf{X}}} = \begin{pmatrix} x_{00} & x_{01} & x_{02} & x_{03} \\ x_{10} & x_{11} & x_{12} & x_{13} \\ x_{20} & x_{21} & x_{22} & x_{23} \\ x_{30} & x_{31} & x_{32} & x_{33} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} I & I & 0 & 0 \\ I & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where

$$\begin{aligned} I = m_{00} & + m_{01}\cos(c_1\sigma) \\ + m_{02}\sin(c_1\sigma)\sin(c_2\sigma) & + m_{03}\sin(c_1\sigma)\cos(c_2\sigma) \\ + m_{10}\cos(c_4\sigma) & + m_{11}\cos(c_1\sigma)\cos(c_4\sigma) \\ + m_{12}\sin(c_1\sigma)\sin(c_2\sigma)\cos(c_4\sigma) & + m_{13}\sin(c_1\sigma)\cos(c_2\sigma)\cos(c_4\sigma) \\ + m_{20}\sin(c_3\sigma)\sin(c_4\sigma) & + m_{21}\cos(c_1\sigma)\sin(c_3\sigma)\sin(c_4\sigma) \\ + m_{22}\sin(c_1\sigma)\sin(c_2\sigma)\sin(c_3\sigma)\sin(c_4\sigma) & + m_{23}\sin(c_1\sigma)\cos(c_2\sigma)\sin(c_3\sigma)\sin(c_4\sigma) \\ - m_{30}\cos(c_3\sigma)\sin(c_4\sigma) & - m_{31}\cos(c_1\sigma)\cos(c_3\sigma)\sin(c_4\sigma) \\ - m_{32}\sin(c_1\sigma)\sin(c_2\sigma)\cos(c_3\sigma)\sin(c_4\sigma) & - m_{33}\sin(c_1\sigma)\cos(c_2\sigma)\cos(c_3\sigma)\sin(c_4\sigma) \end{aligned}$$

with the argument

$$c_i\sigma = 2\pi\tau_i\sigma = 2\pi d_o d_i \lambda_o B\sigma \longrightarrow \tau_i = d_o d_i \lambda_o B$$

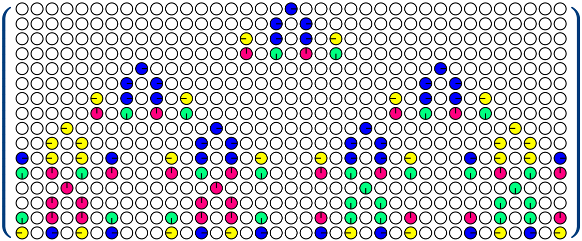
Spectrally Channeled Polarimeter - III

C_n	Channel Content $\times(64/S_{in,0})$
0	$16m_{00}$
± 1	$8m_{01} + 4m_{02} \pm 4im_{03}$
± 2	$-m_{22} \pm im_{23} \mp im_{32} - m_{33}$
± 3	$-4m_{02} \mp 4im_{03}$
± 4	$2m_{21} + m_{22} \mp im_{23} \pm 2im_{31} \pm im_{32} + m_{33}$
± 5	$4m_{20} \pm 4im_{30}$
± 6	$2m_{21} + m_{22} \pm 1m_{23} \pm 2im_{31} \pm im_{32} - m_{33}$
± 7	$-2m_{12} \pm 2im_{13}$
± 8	$-m_{22} \mp im_{23} \mp im_{32} + m_{33}$
± 9	$4m_{11} + 2m_{12} \mp 2im_{13}$
± 10	$8m_{10}$
± 11	$4m_{11} + 2m_{12} \pm 2im_{13}$
± 12	$m_{22} \mp im_{23} \mp im_{32} - m_{33}$
± 13	$-2m_{12} \mp 2im_{13}$
± 14	$-2m_{21} - m_{22} \pm im_{23} \pm 2im_{31} + im_{32} + m_{33}$
± 15	$-4m_{20} \pm 4im_{30}$
± 16	$-2m_{21} - m_{22} \mp im_23 \pm 2im_{31} \pm im_{32} - m_{33}$
± 18	$m_{22} \pm im_{23} \mp im_{32} + m_{33}$

$$c_n(\sigma) = \frac{1}{s_{in,0}(\sigma)} \mathcal{F}^{-1}\left\{w(\tau)C_n(\tau)\right\}$$

$m_{ij}(\sigma)$	Hagen's Algebraic Reconstruction
$m_{00}(\sigma) =$	$4c_0$
$m_{01}(\sigma) =$	$8(c_1 + c_3)$
$m_{02}(\sigma) =$	$-16\Re[c_3]$
$m_{03}(\sigma) =$	$16\Re[c_1]$
$m_{10}(\sigma) =$	$8c_{10}$
$m_{11}(\sigma) =$	$16(c_7 + c_9)$
$m_{12}(\sigma) =$	$-32\Re[c_7]$
$m_{13}(\sigma) =$	$32\Im[c_7]$
$m_{20}(\sigma) =$	$16\Re[c_5]$
$m_{21}(\sigma) =$	$32\Re[c_2 + c_4]$
$m_{22}(\sigma) =$	$-32\Re[c_2 + c_8]$
$m_{23}(\sigma) =$	$32\Im[c_2 - c_8]$
$m_{30}(\sigma) =$	$16\Re[c_5]$
$m_{31}(\sigma) =$	$16\Im[c_2 + c_4 + c_6 + c_8]$
$m_{32}(\sigma) =$	$-32\Im[c_2 + c_8]$
$m_{33}(\sigma) =$	$32\Re[c_8 - c_2]$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$  ^{T}

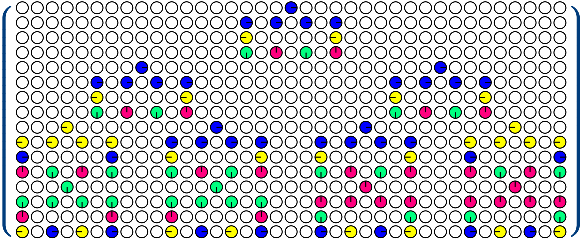
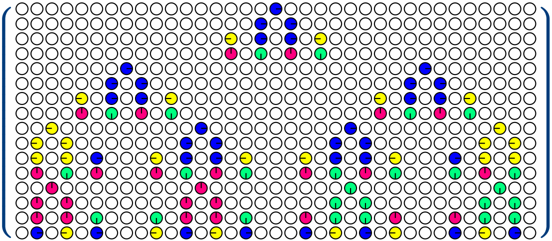
$\underline{\underline{\mathbf{Q}}}^{+} =$ 

Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 0 channels ignored, $\mathrm{EWV} = 187.000$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$



T

$\underline{\underline{\mathbf{Q}}}^{+} =$

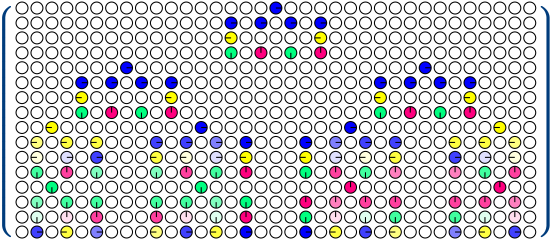
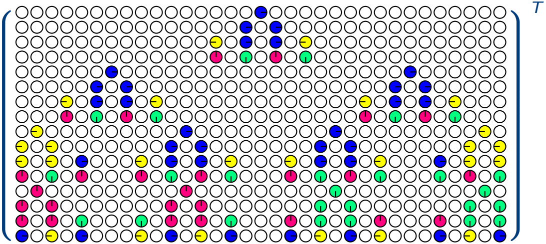


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 2 channels ignored, $\mathrm{EWV} = 225.400$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$



$\underline{\underline{\mathbf{Q}}}^{+} =$

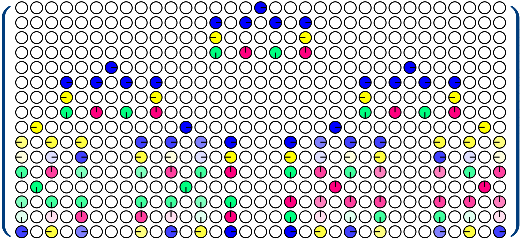


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 4 channels ignored, $\mathrm{EWV} = 225.400$

Spectrally Channeled Polarimeter - IV

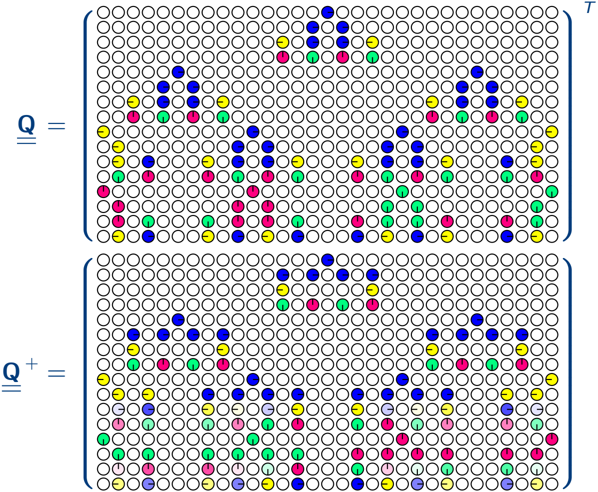
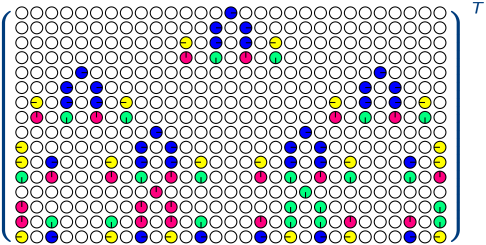


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 6 channels ignored, $\mathrm{EWV} = 240.333$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$



$\underline{\underline{\mathbf{Q}}}^{+} =$

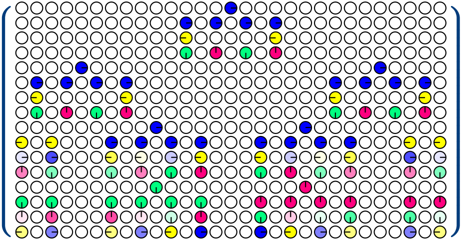


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\,1 \;\; 2 \;\; 5 \;\; 10\,)$, 8 channels ignored, $\mathrm{EWV} = 248.333$

Spectrally Channeled Polarimeter - IV

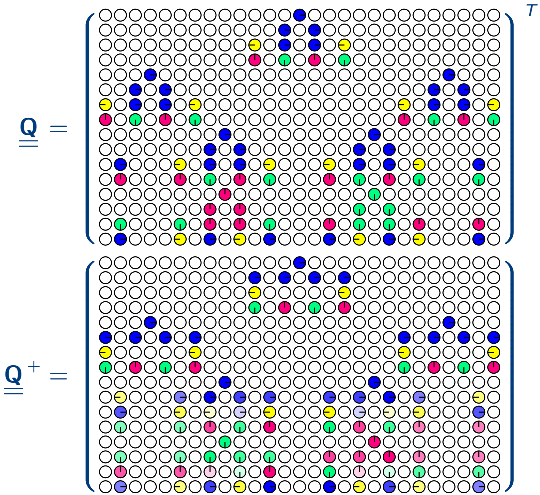
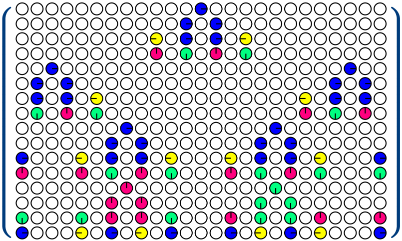


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 10 channels ignored, $\mathrm{EWV} = 268.143$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$  ^{T}

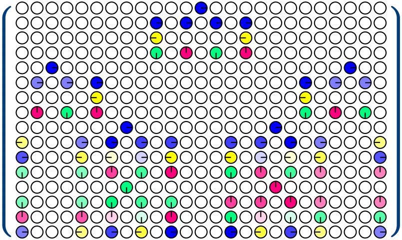
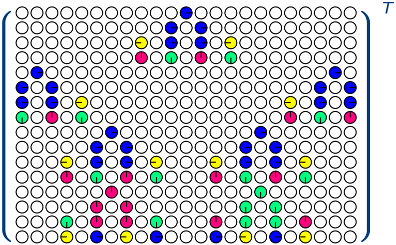
$\underline{\underline{\mathbf{Q}}}^+ =$ 

Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 12 channels ignored, $\mathrm{EWV} = 290.908$

Spectrally Channeled Polarimeter - IV

$\underline{\underline{\mathbf{Q}}} =$ 

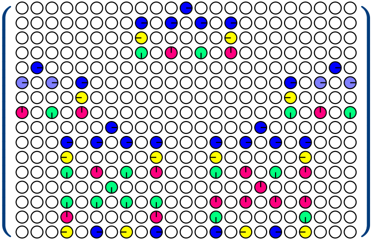
$\underline{\underline{\mathbf{Q}}}^{+} =$ 

Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 14 channels ignored, $\mathrm{EWV} = 345.667$

Spectrally Channeled Polarimeter - IV

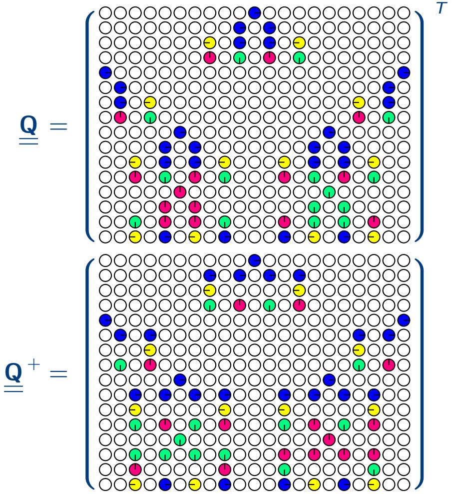


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 16 channels ignored, $\mathrm{EWV} = 355.000$

Spectrally Channeled Polarimeter - IV

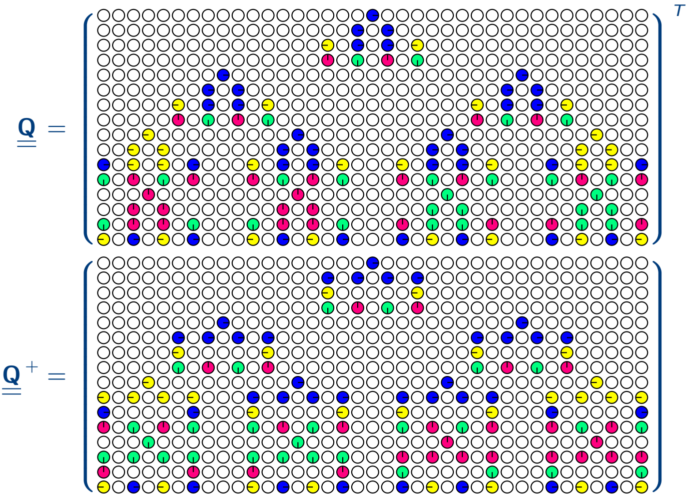


Figure: Spectrally channeled polarimeter, $\underline{\mathbf{d}} = (\ 1\ \ 2\ \ 5\ \ 10\ )$, 0 channels ignored, $\mathrm{EWV} = 187.000$

Spectrally Channeled Polarimeter - V

$\underline{\underline{\mathbf{Q}}} =$

$\underline{\underline{\mathbf{Q}}}^{+} =$

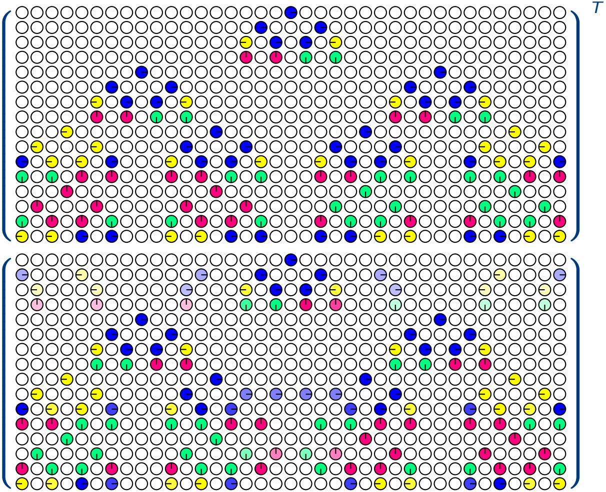


Figure: By swapping the order of the first two elements, i.e., $\underline{\mathbf{d}} = (\ 2\ \ 1\ \ 5\ \ 10\ )$, we have $\mathrm{EWV} = 130.571$

Spectrally Channeled Polarimeter - VI

$\underline{\underline{\mathbf{Q}}} =$, $\underline{\underline{\mathbf{Q}}}^{+} =$

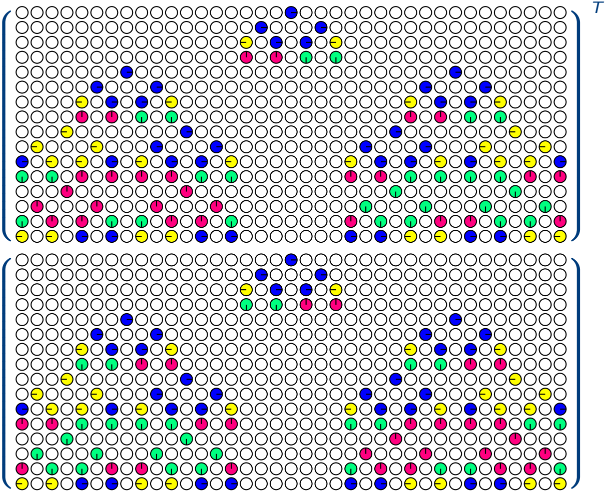


Figure: And with slight thickness modifications, $\underline{\mathbf{d}} = (\ 2\ \ 1\ \ 4\ \ 11\ )$, we can obtain $\mathrm{EWV} = 121.000$

Summary and Conclusions

$$\underline{\mathbf{S}}\left(\vec{\Theta}\right)=\underline{\mathbf{S}}(x,y,t,\sigma,\theta,\phi,\ldots) \qquad \underline{\underline{\mathbf{M}}}\left(\vec{\Theta}\right)=\underline{\underline{\mathbf{M}}}(x,y,t,\sigma,\theta,\phi,\ldots)$$

- The polarimetric object that you are trying to reconstruct has some bandwidth in any number of potential modulation dimensions:
  - Scene can be spatially modulated using microgrid or polarimetric interference
  - Scene can be spectrally modulated using a channeled system
  - Scene can be temporally modulated using a variable optic
  - Scene can be angle-modulated using a high-NA beam and angle-varying optics
- Your system has some specified space/time/spectral/angular resolution (bandwidth)
  - The bandwidth of the object in the various dimesnions of sensitivity impact your choice of modulation strategies
  - Interplay between object and system bandwidth places modified crosstalk (channels) and sampling requirements
  - Each modulation strategy comes with particular set of image artifacts
- Your choices of how your modulation strategy uses the bandwidth of the system affects the quality of the reconstruction
  - Well-conditioned systems will be least sensitive to noise
  - Well-designed systems will have minimal instrumental variation in the dimensions of modulation
  - The reconstruction should recognize that the object is varying in the modulation dimensions and take this into account - use your bandwidth wisely

Questions?