Signal detection
A Bayesian perspective

A. Asensio Ramos
Instituto de Astrofísica de Canarias

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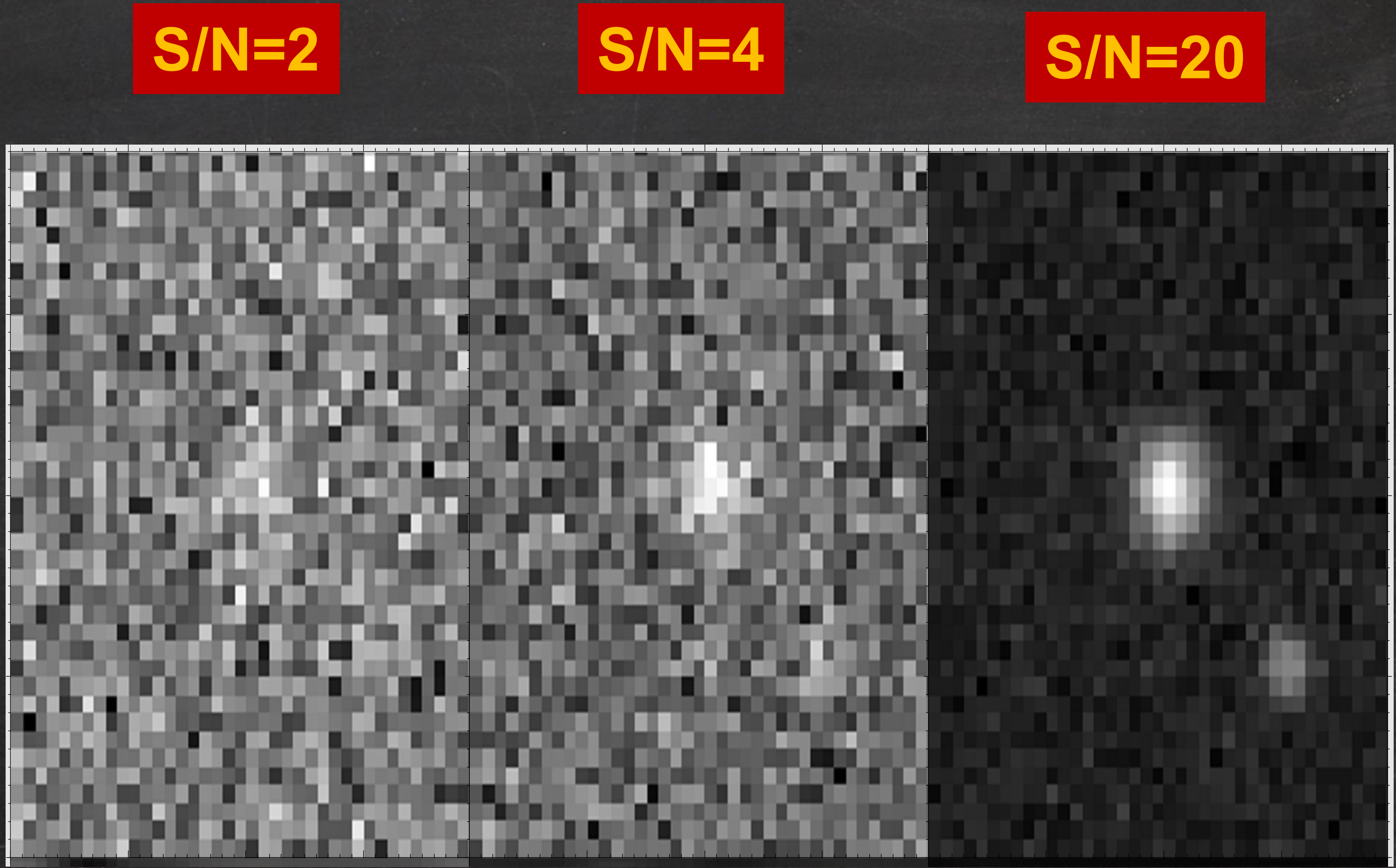
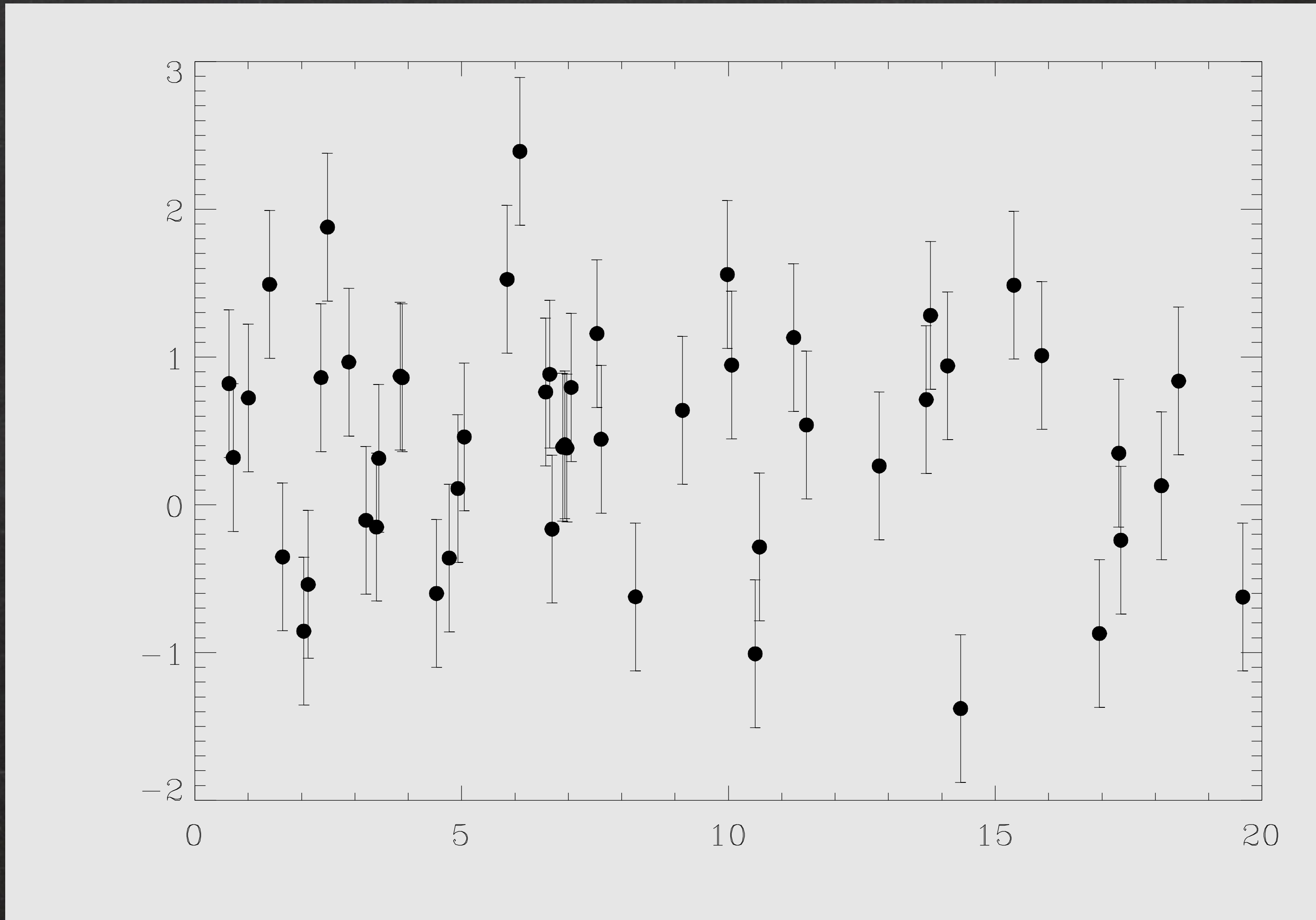
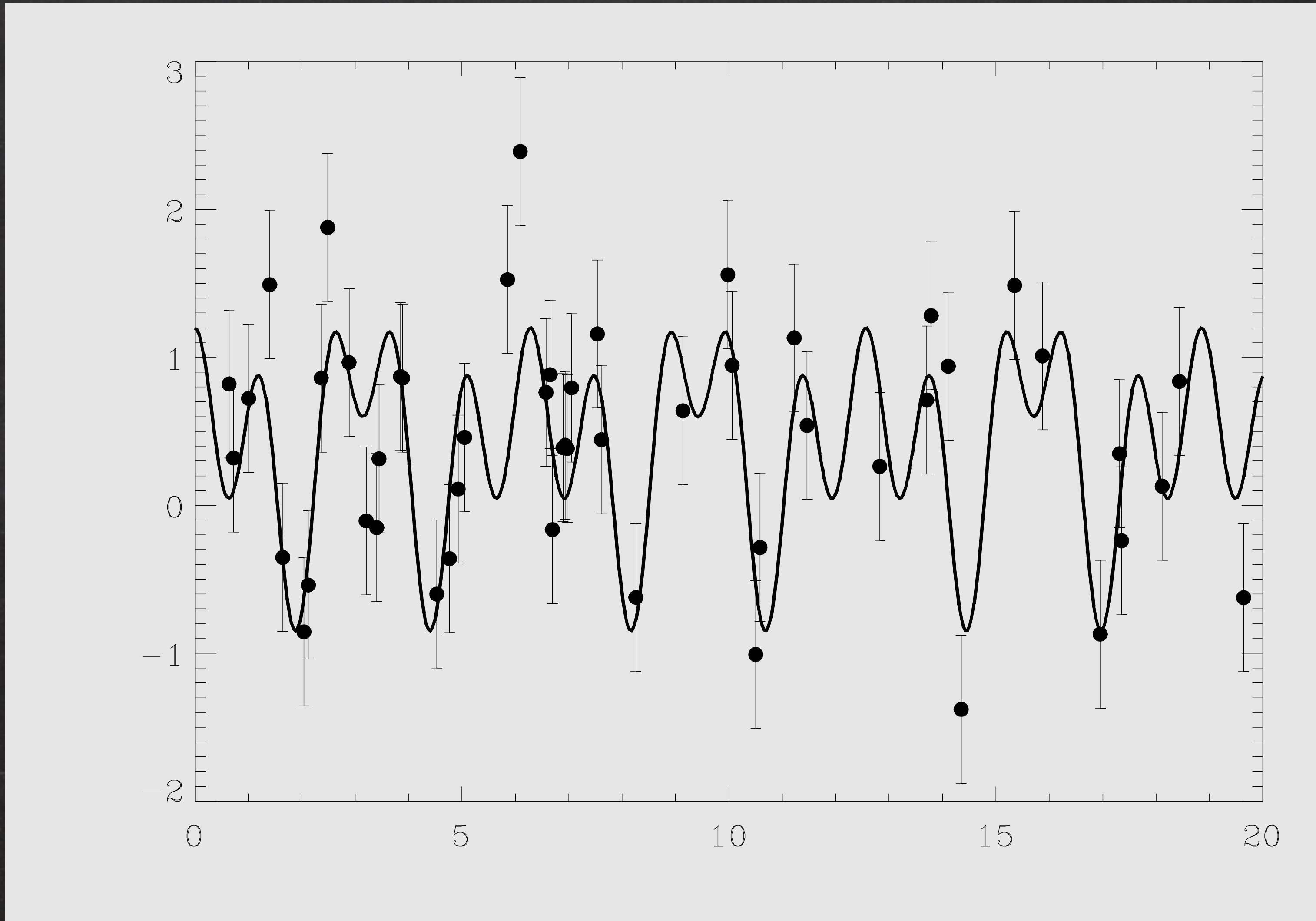
Signal detection



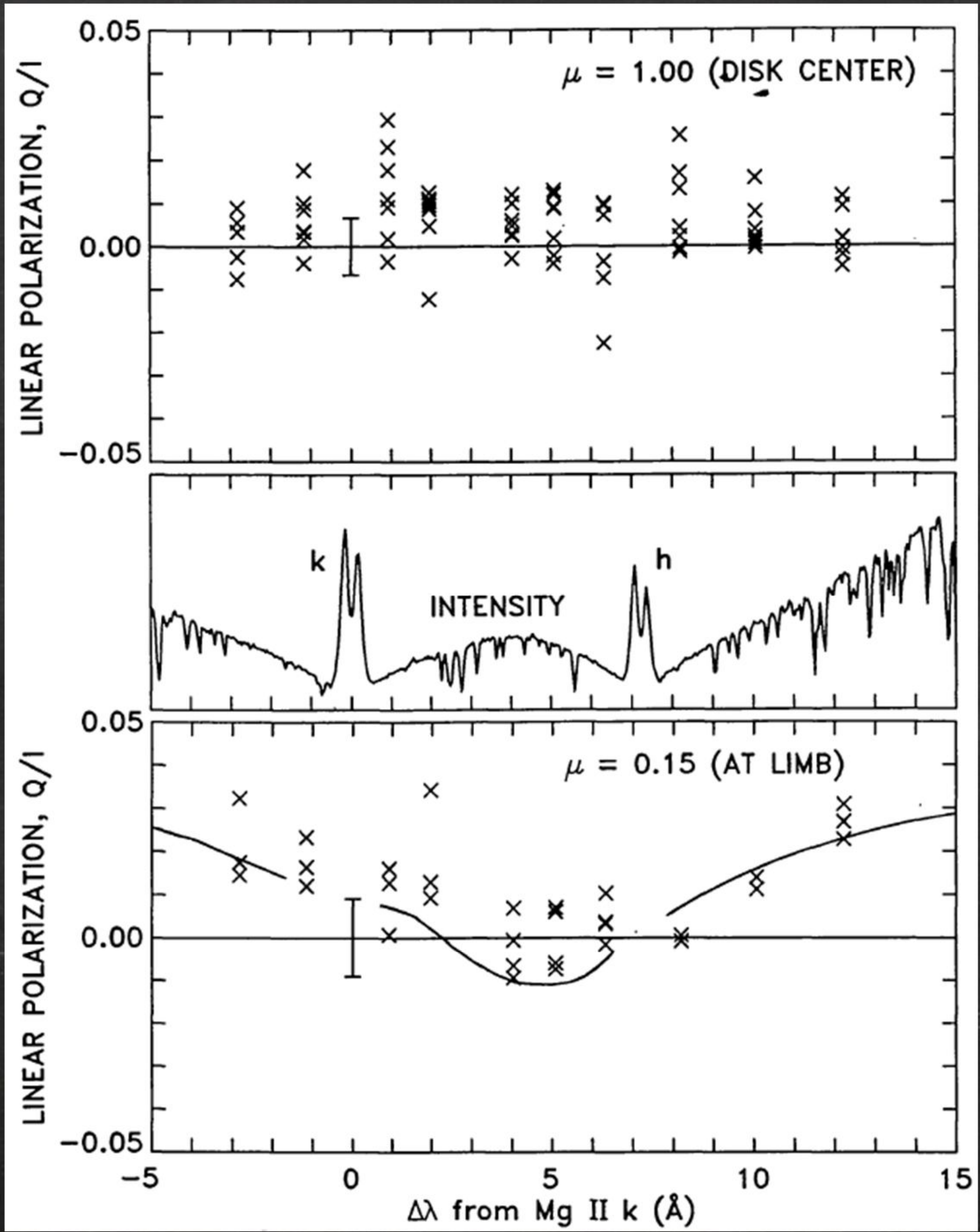
Signal detection



Signal detection



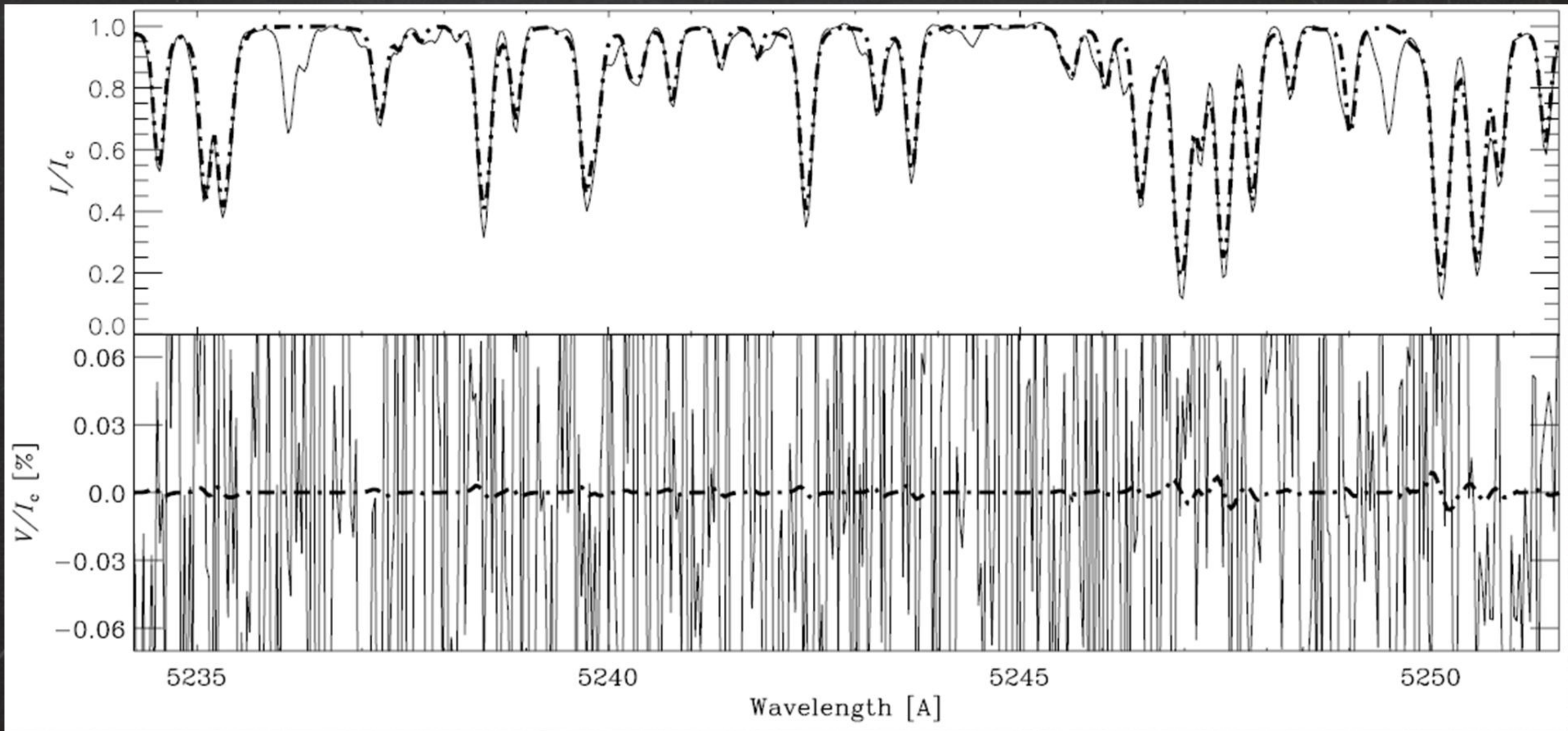
Signal detection in spectroscopy/spectropolarimetry



Henze & Stenflo (1987)

Signal detection in spectroscopy/spectropolarimetry

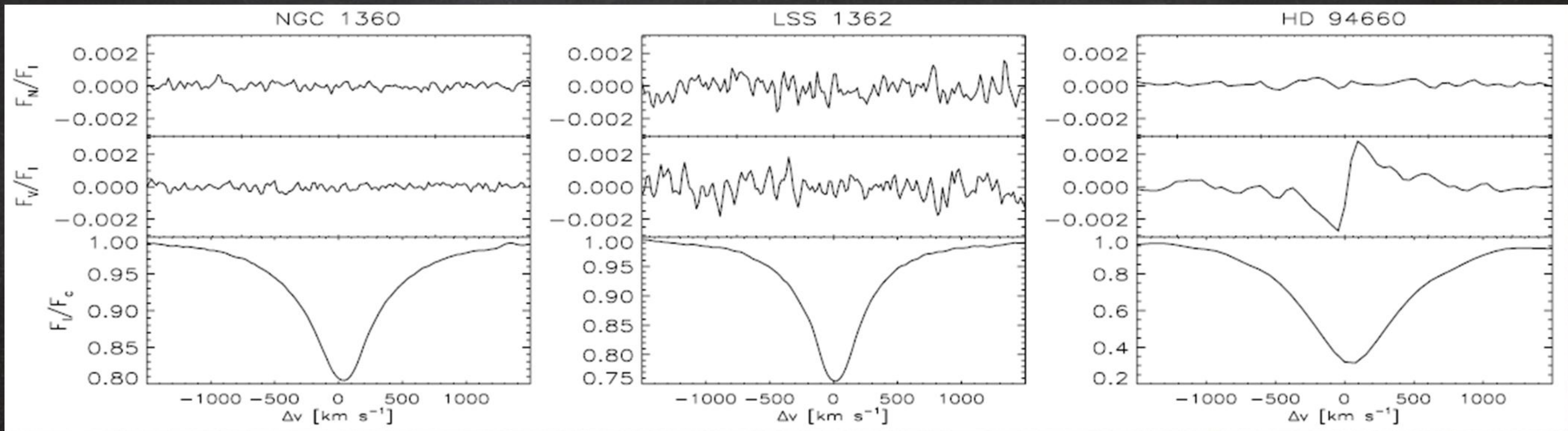
Detection of magnetic field in Arcturus?



Sennhauser & Berdyugina (2011)

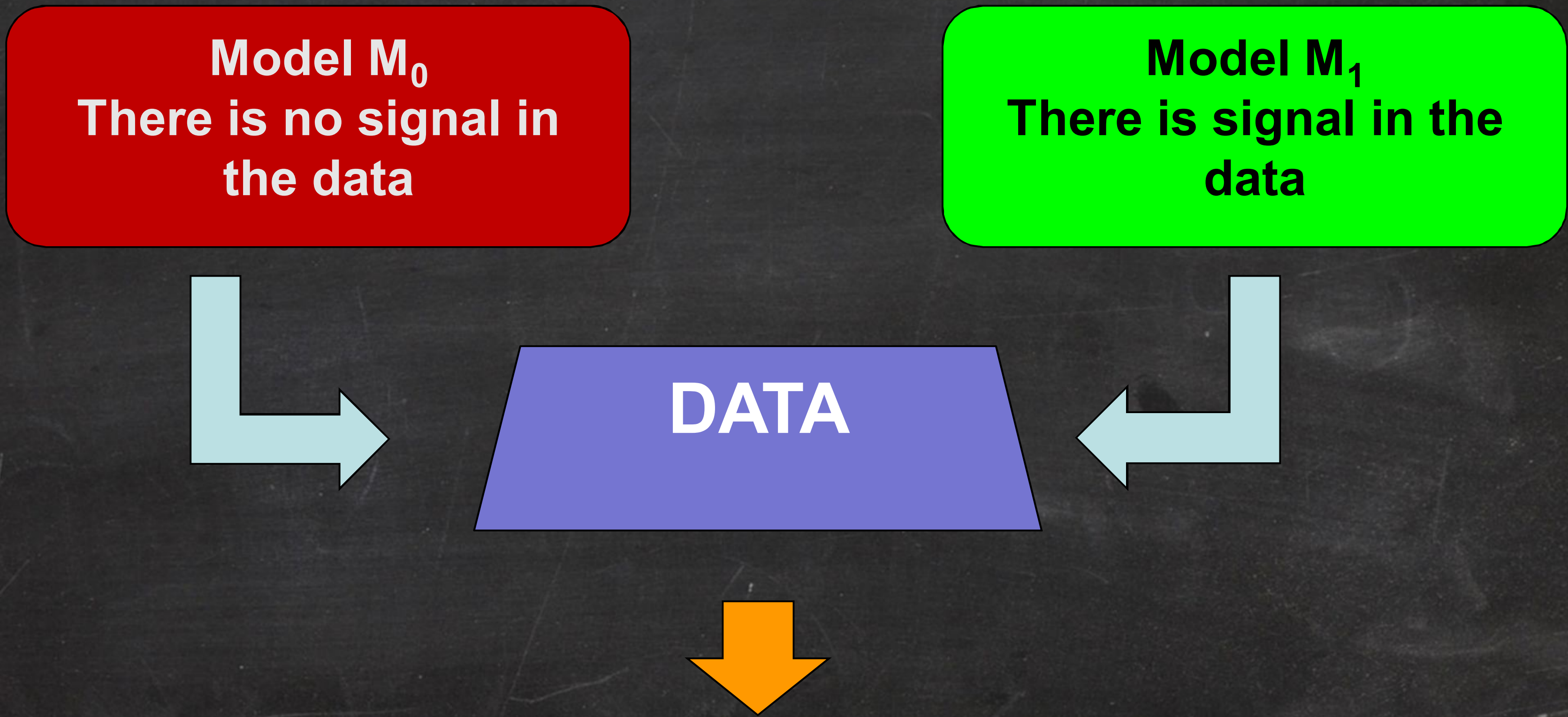
Signal detection in spectroscopy/spectropolarimetry

Non-detection of magnetic field in central stars of PNe



Leone et al. (2011)

Bayesian signal detection as an instance of model selection



$$R = \frac{p(\mathbf{d}|M_1)}{p(\mathbf{d}|M_0)}$$

Bayesian signal detection (BSD) in parametric models
How-to

Model depends on the set of parameters θ

$$p(\mathbf{d}|M) = \int d\boldsymbol{\theta} p(\mathbf{d}|\boldsymbol{\theta}, M) p(\boldsymbol{\theta}|M)$$

Likelihood

Prior

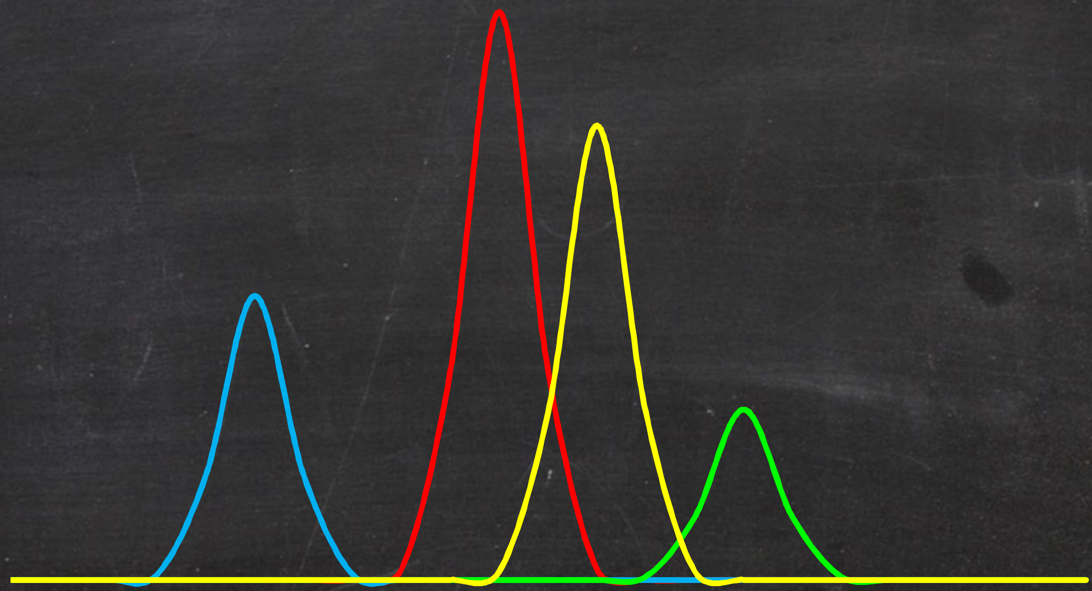
- The integral is usually difficult to compute (rely on advanced Montecarlo techniques)
- Model comparison

BSD – Non-parametric model

- We propose a sufficiently general model
  - It might potentially have infinite number of parameters
  - We let data decide which contributions are important
- Ideal for the detection of ‘surprises’
- With appropriate priors, its complexity adapts to the data

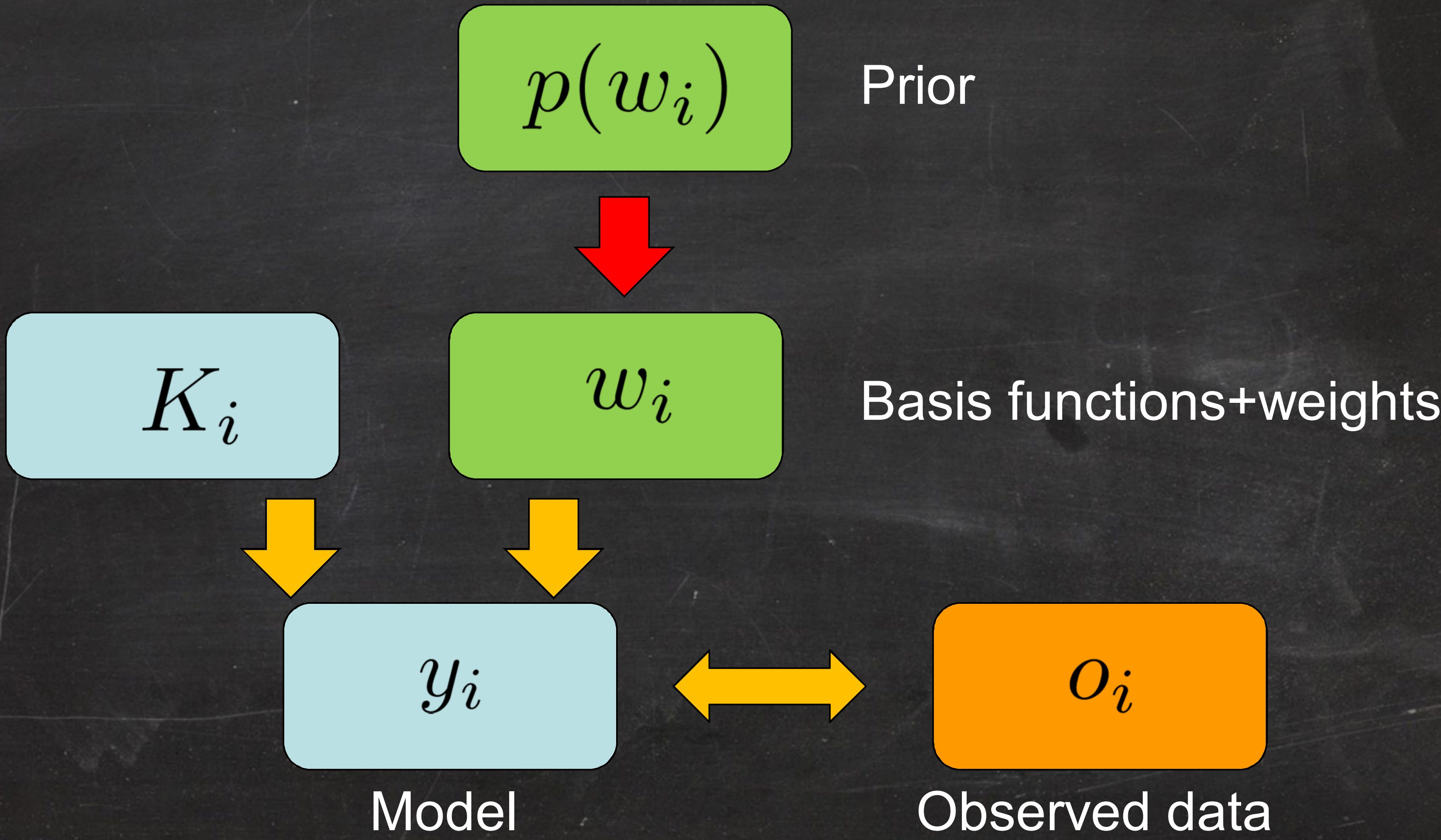
BSD – General linear regression

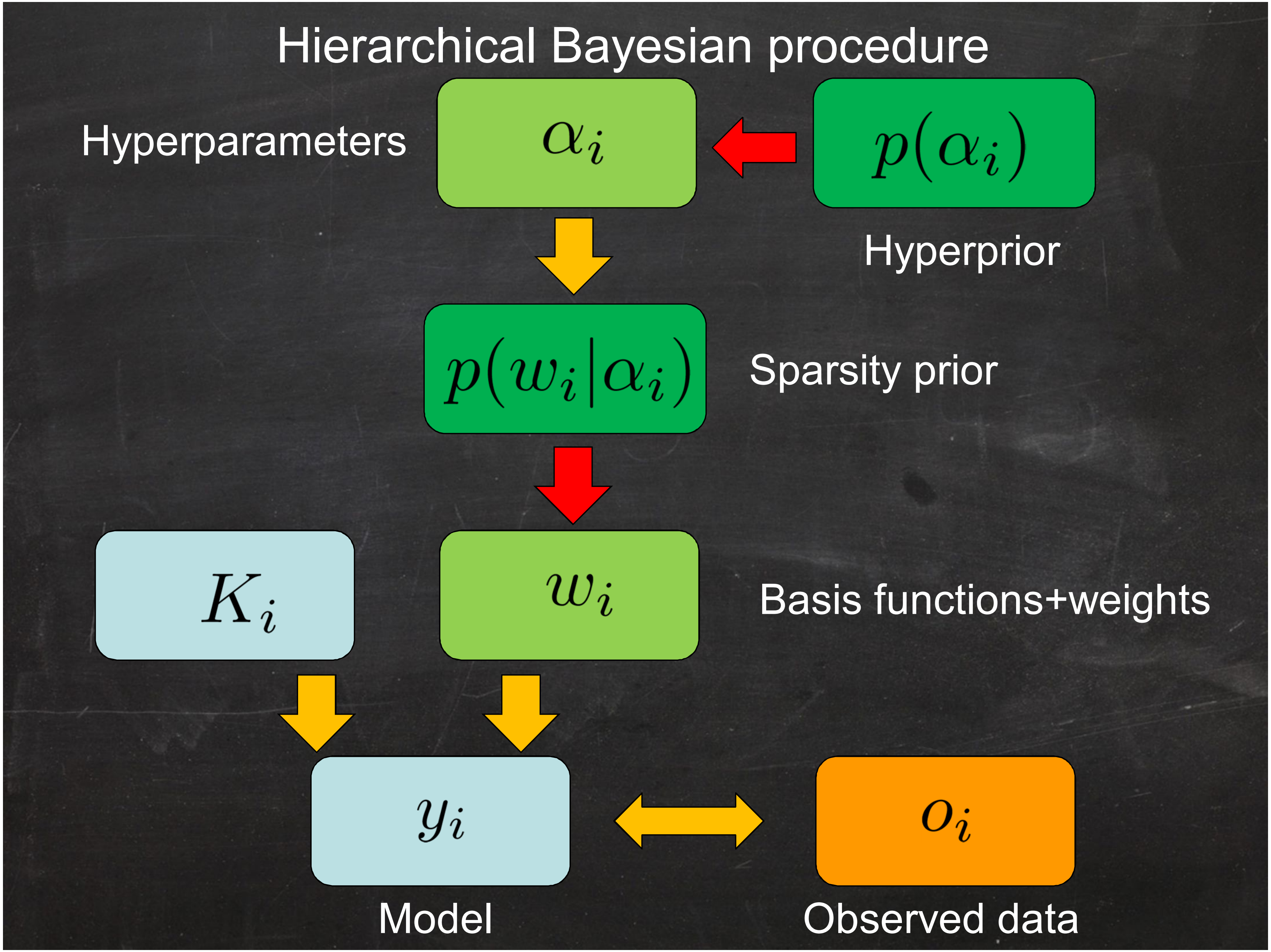
$$y(\mathbf{x};\mathbf{w}) = \sum_{j=1}^{M} w_j K_j(\mathbf{x})$$



We use kernel functions centered at observed points
Shapes: Gaussian, DoG, dispersion profile,
periodic function, polynomial, etc.)

Usual Bayesian procedure

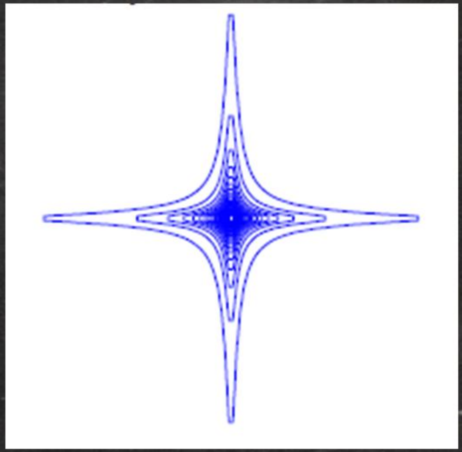



Hierarchical Bayesian procedure
Hyperparameters
α_i
$p(\alpha_i)$
Hyperprior
$p(w_i|\alpha_i)$
Sparsity prior
K_i
w_i
Basis functions+weights
y_i
o_i
Model
Observed data

Non-parametric models – sparsifying prior

$$p(\mathbf{w}|\boldsymbol{\alpha}) = \prod_{i=1}^{M} \mathcal{N}(w_i|0, \alpha_i^{-1})$$

$$p(\mathbf{w}) = \int d\mathbf{w} p(\mathbf{w}|\boldsymbol{\alpha}) p(\boldsymbol{\alpha})$$



This prior forces to use as few $K_j(x)$ functions as possible

Bayesian signal detection – Non-parametric models

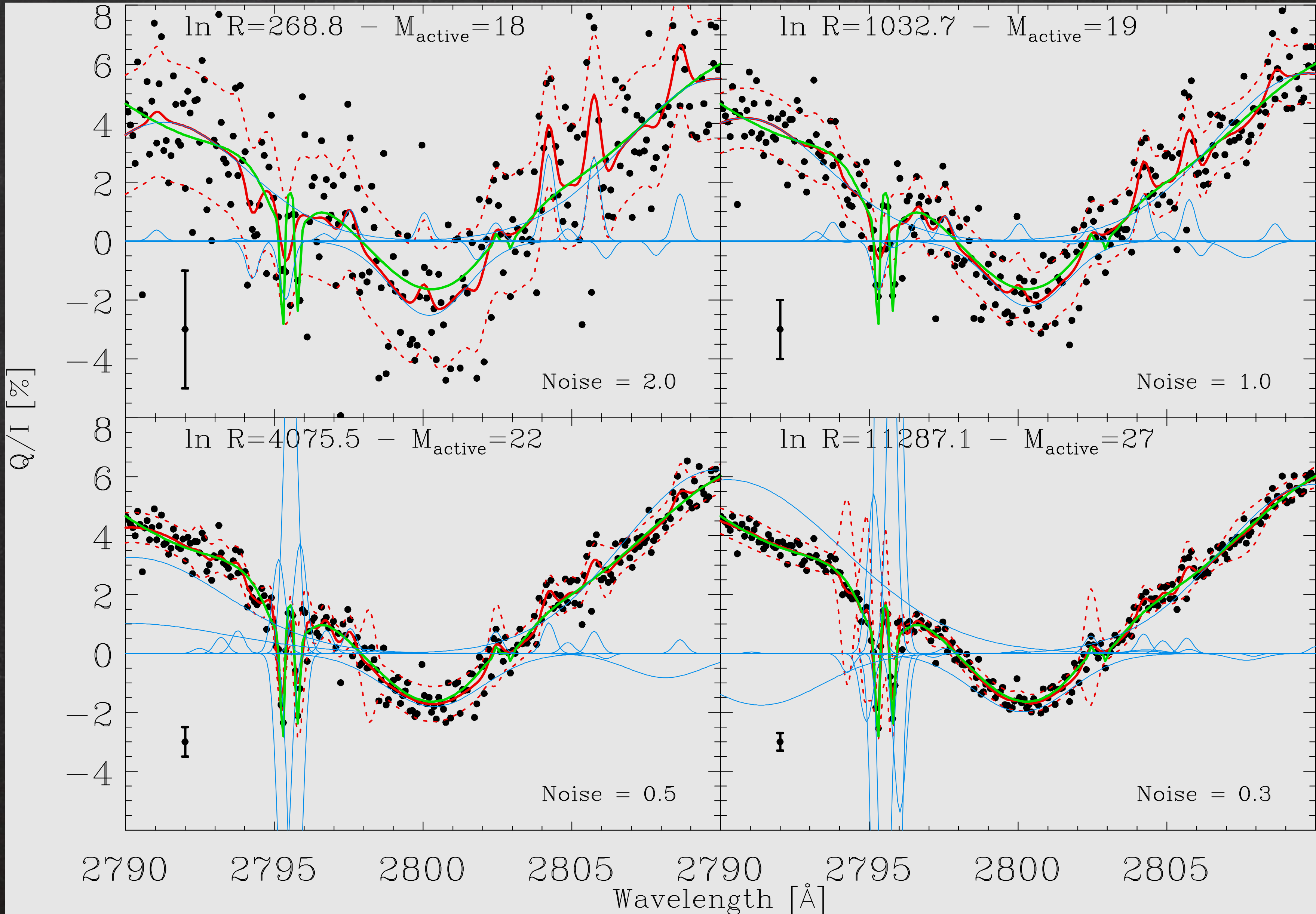
Model with signal

$$p(\mathbf{d}|M_1) = \int \mathrm{d}\mathbf{w}\mathrm{d}\boldsymbol{\alpha}\mathrm{d}\sigma^2 \, p(\mathbf{d}|\mathbf{w},\sigma^2)p(\mathbf{w}|\boldsymbol{\alpha})p(\boldsymbol{\alpha})p(\sigma^2)$$

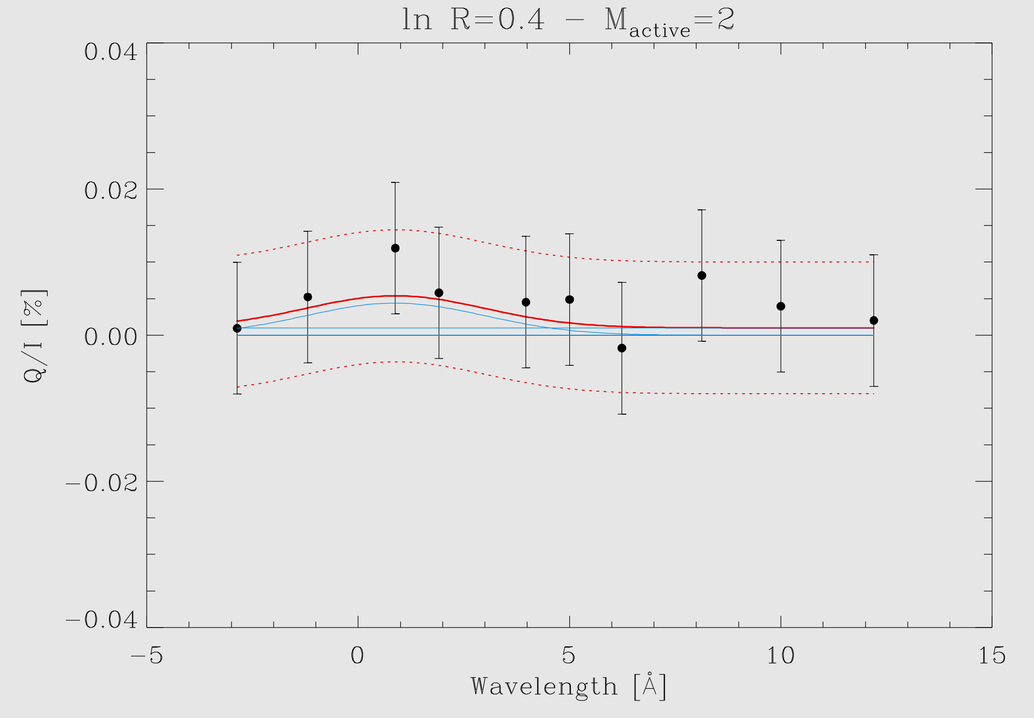
Model without signal

$$p(\mathbf{d}|M_0) = \prod_{n=1}^{N} (2\pi\sigma^2)^{-1/2} \exp\left[-\frac{y_n^2}{2\sigma^2}\right]$$

Signal detection in Mg II h&k - Synthetic

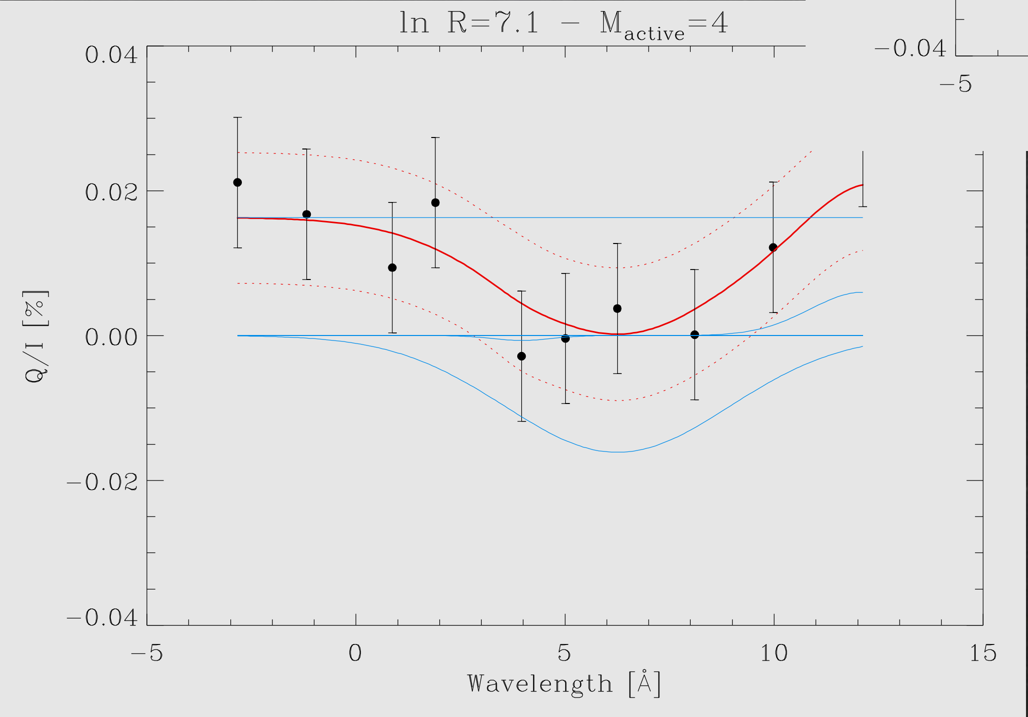


Signal detection in Mg II h&k - Observed



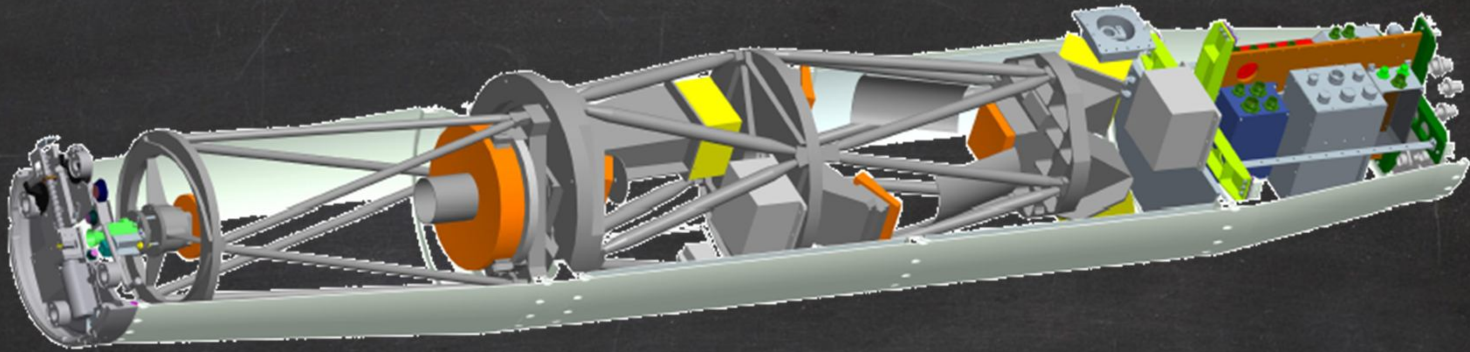
Disk center

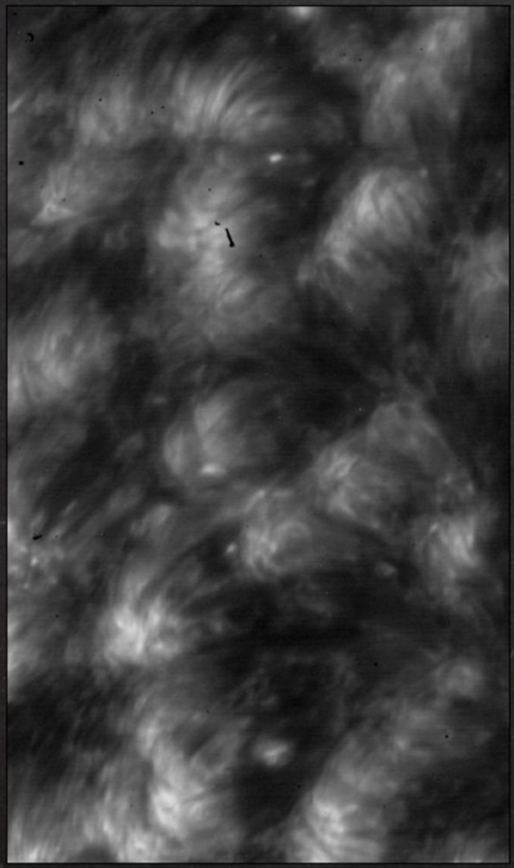
μ=0.3

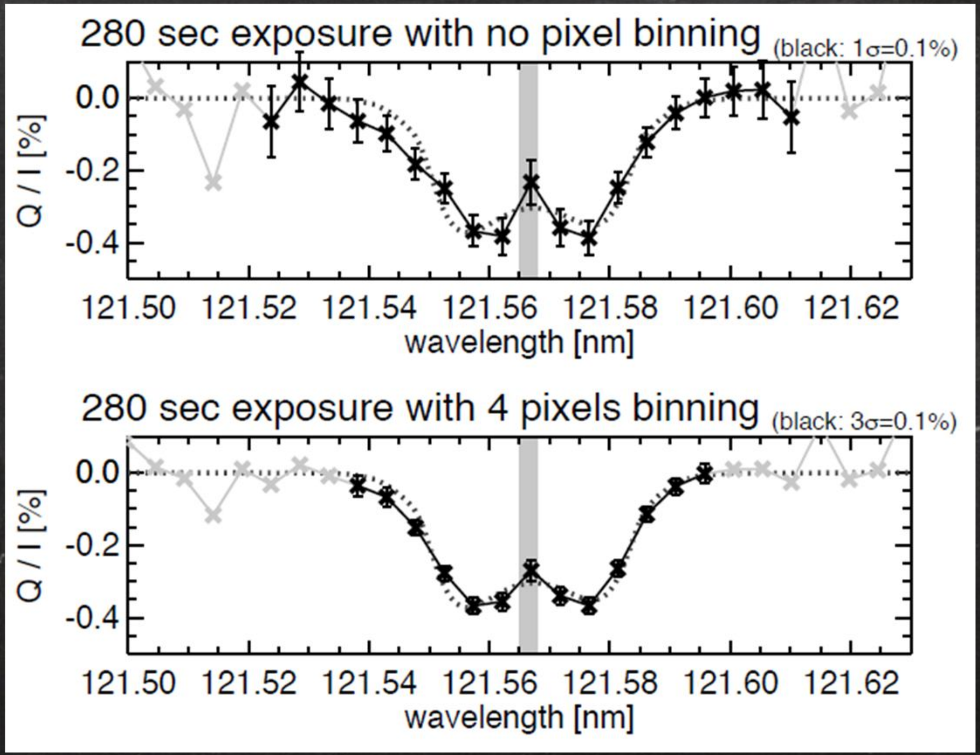


Henze & Stenflo (1987)

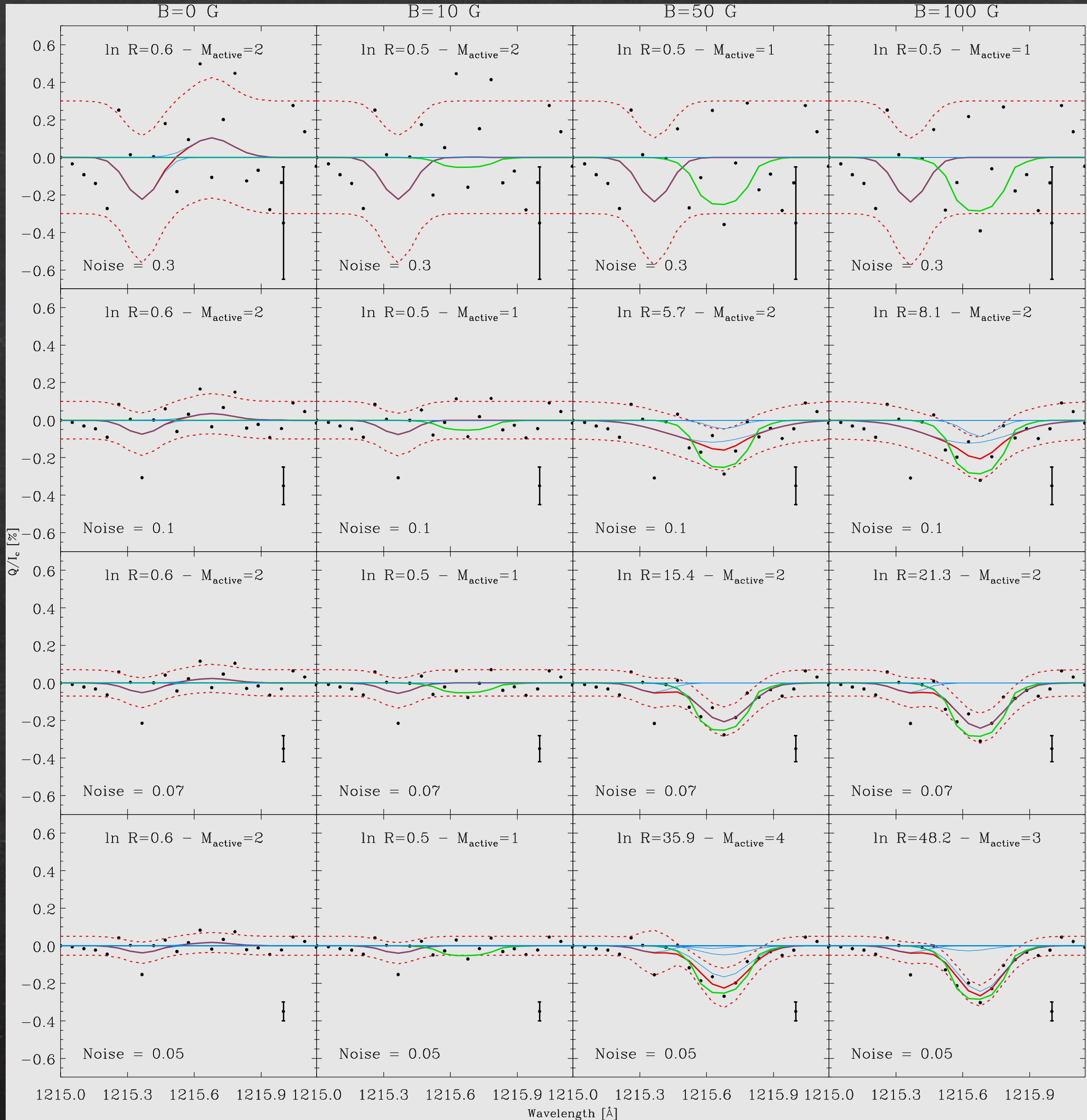
Signal detection in CLASP
Chromospheric Lyman-Alpha Spectro-Polarimeter







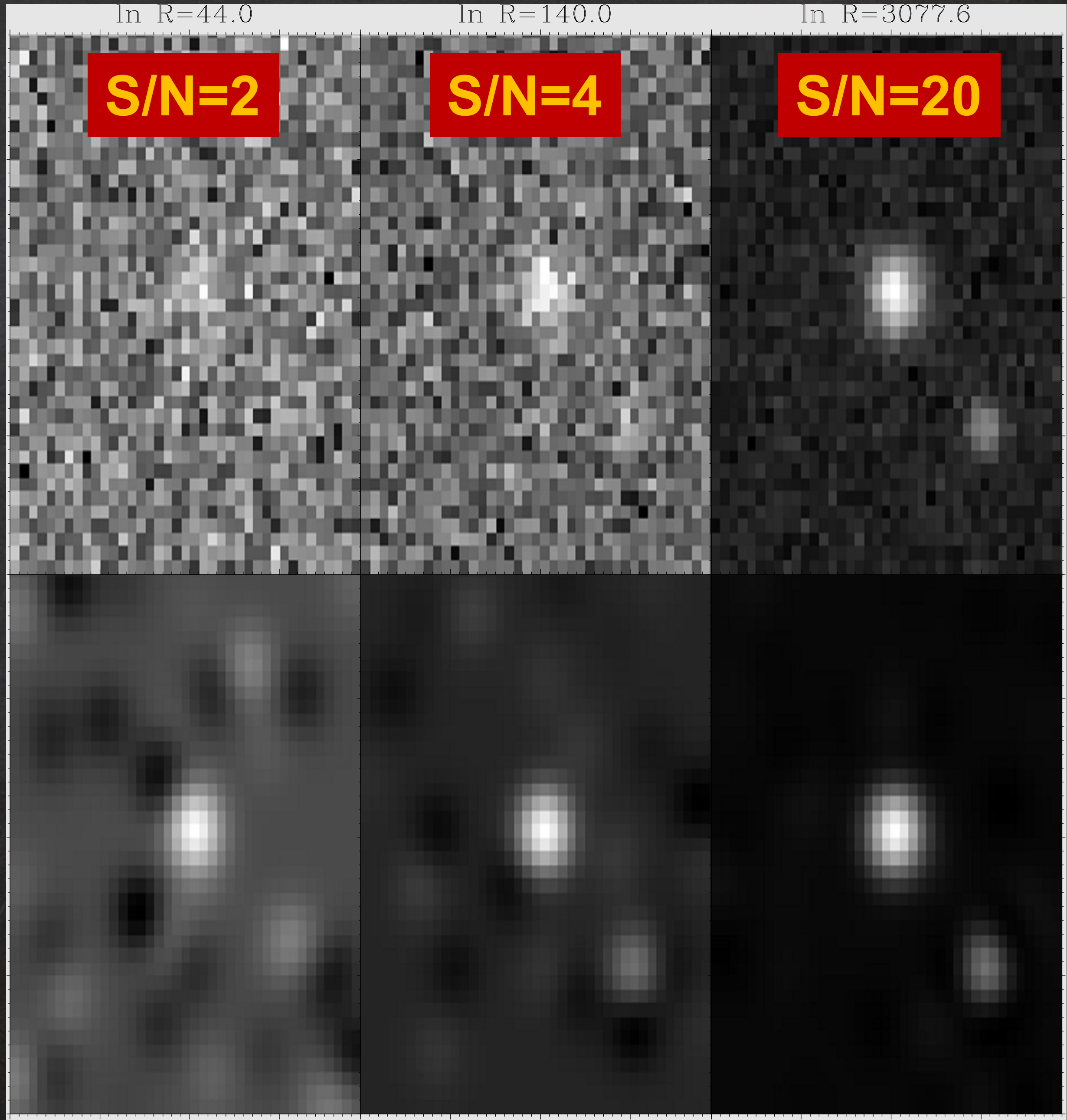
Signal detection in CLASP



Stellar object detection

Observed field

Inferred field



Conclusions

- The Bayesian framework allows to reliably detect signals
  - Giving the most probable signal
  - Giving a strength to the detection

- Non-parametric models are very general and adapt to the data quality

Bayesian signal detection (BSD) – Parametric model

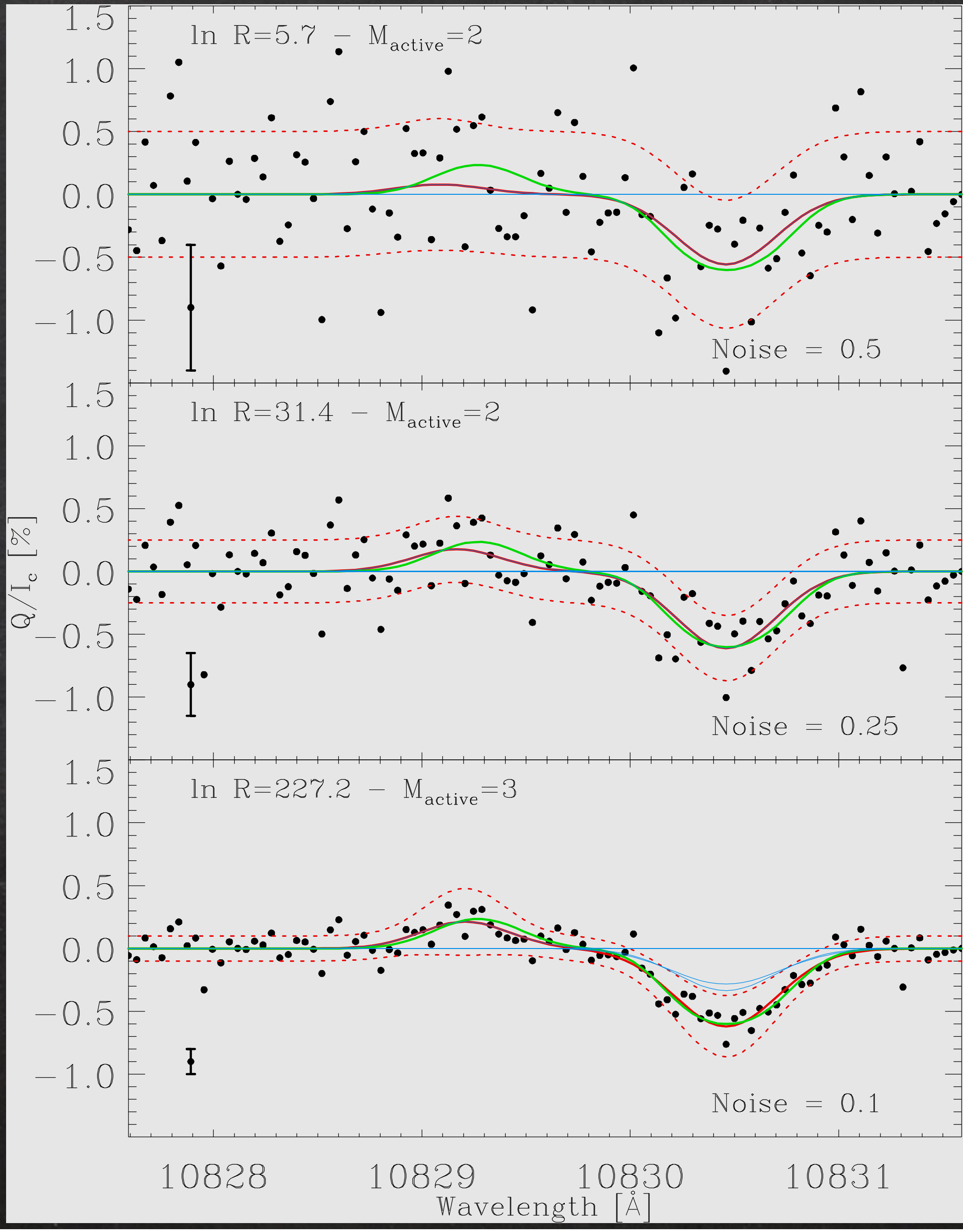
All the information is encoded in $p(\mathbf{d}|M)$

Parametric model

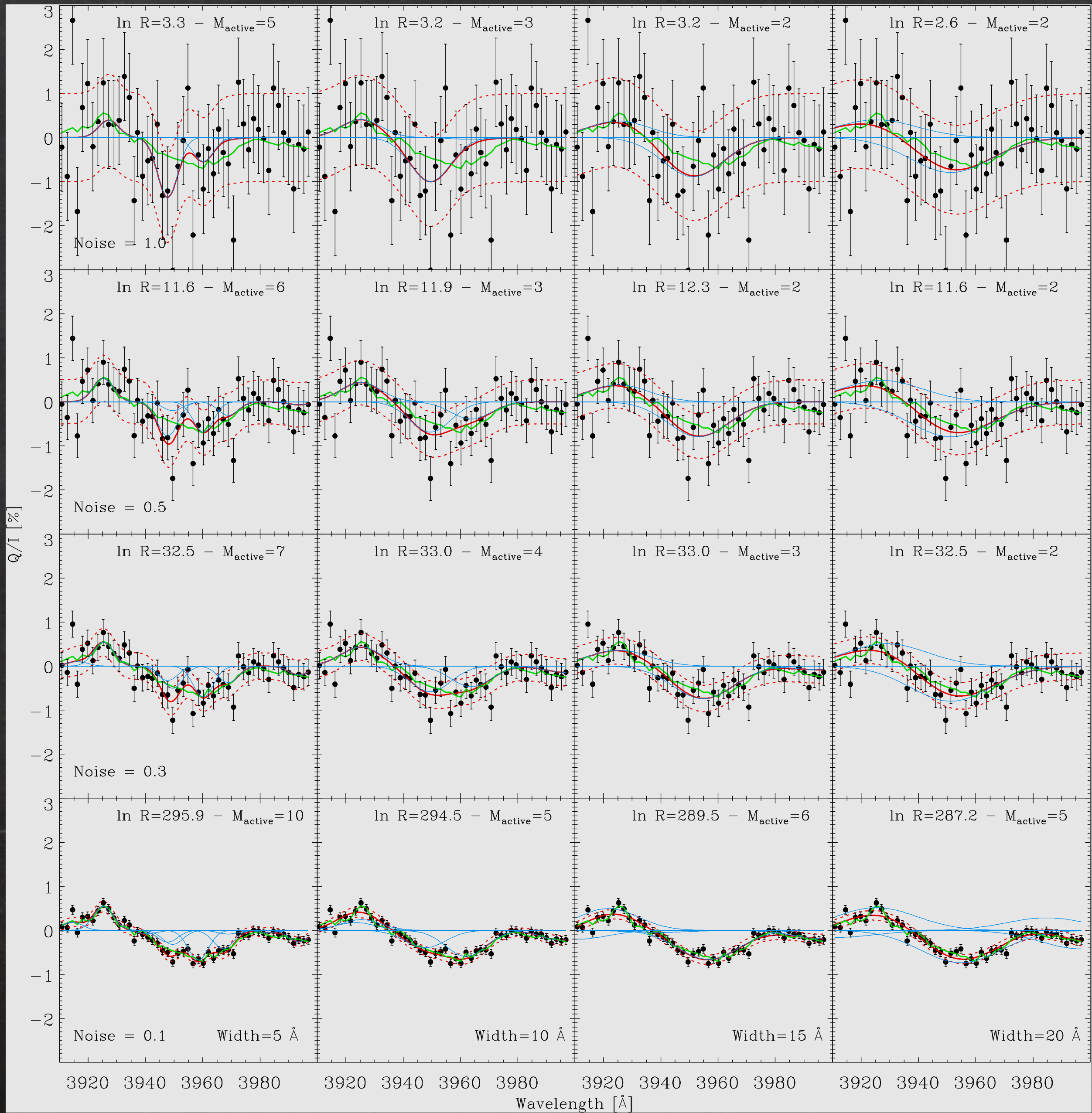
$$y(\boldsymbol{\theta}, x) = \theta_0 + \theta_1 \exp\left(-\frac{(x-\theta_2)^2}{\theta_3^2}\right)$$

- Model depends on parameters that usually have physical meaning
- A-priori knowledge of the signal
- Not adequate for discovery

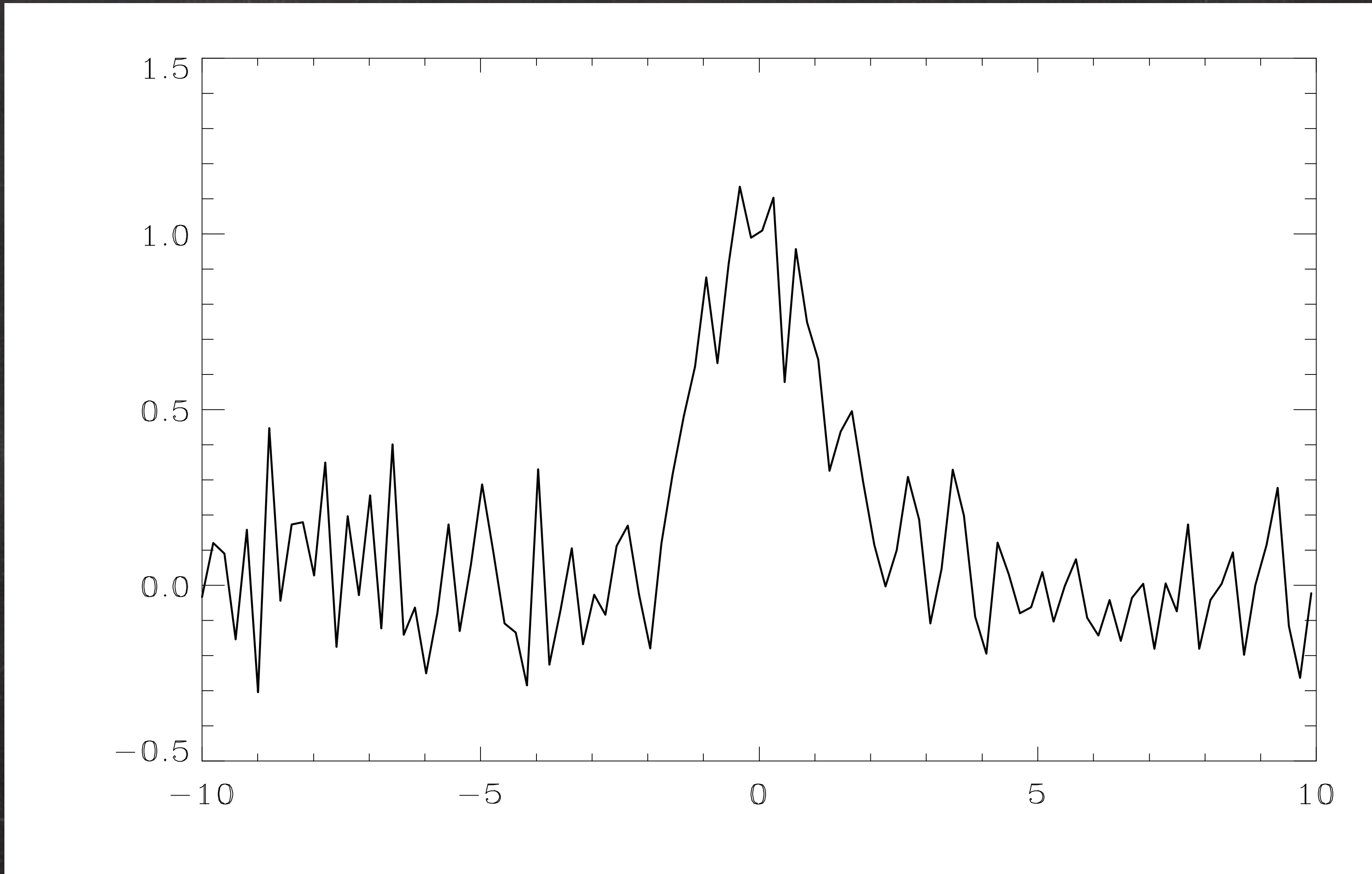
Signal detection in He I 10830 Å - Synthetic



Signal detection in Ca II h&k - Synthetic



Model comparison – a worked example



H_0 : simple Gaussian

H_1 : two Gaussians of equal width but unknown amplitude ratio

Model comparison – a worked example

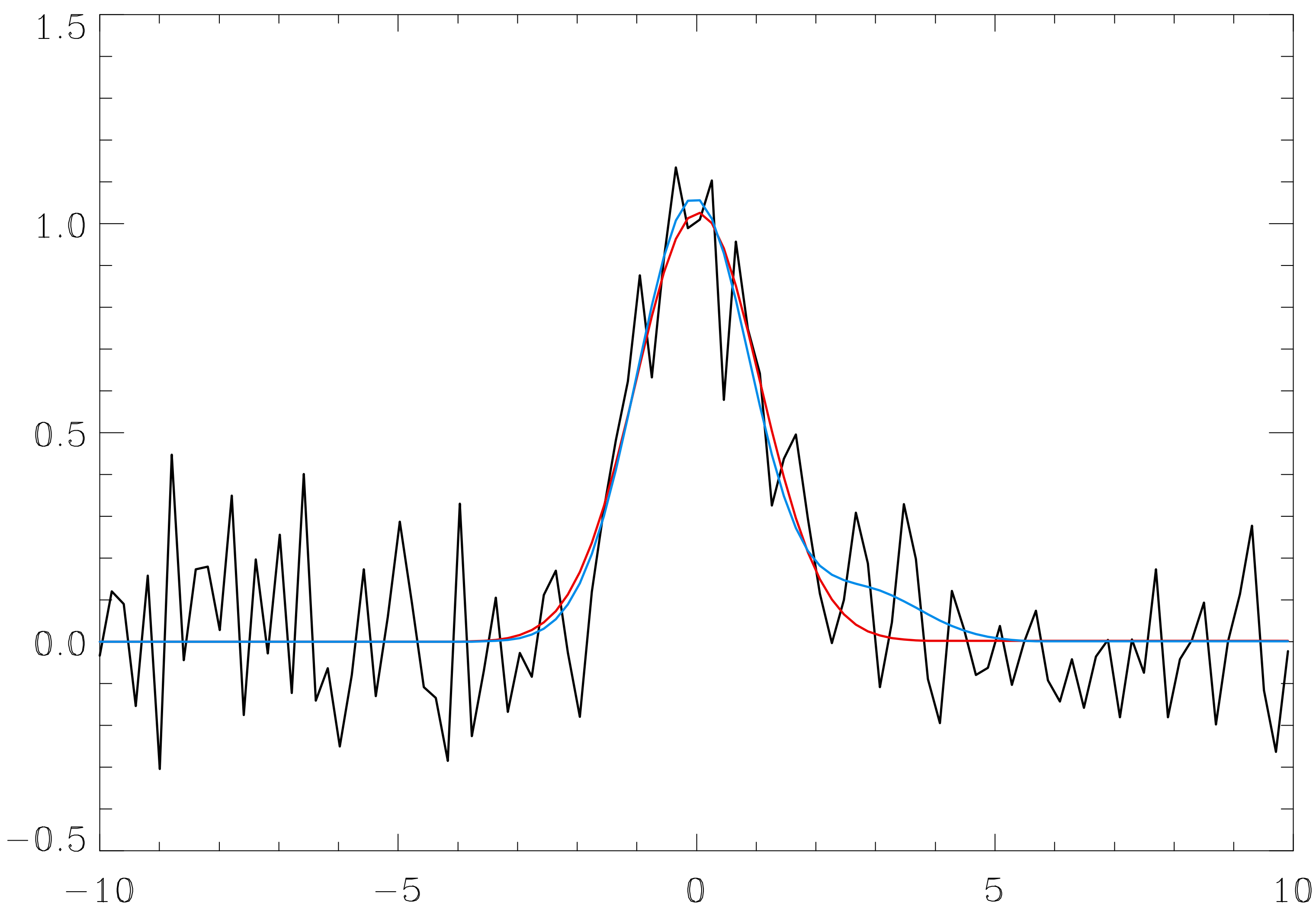
H_0 : simple Gaussian

$$y = \alpha_1 \exp\left(-\frac{(x-\alpha_2)^2}{2\alpha_3^2}\right)$$

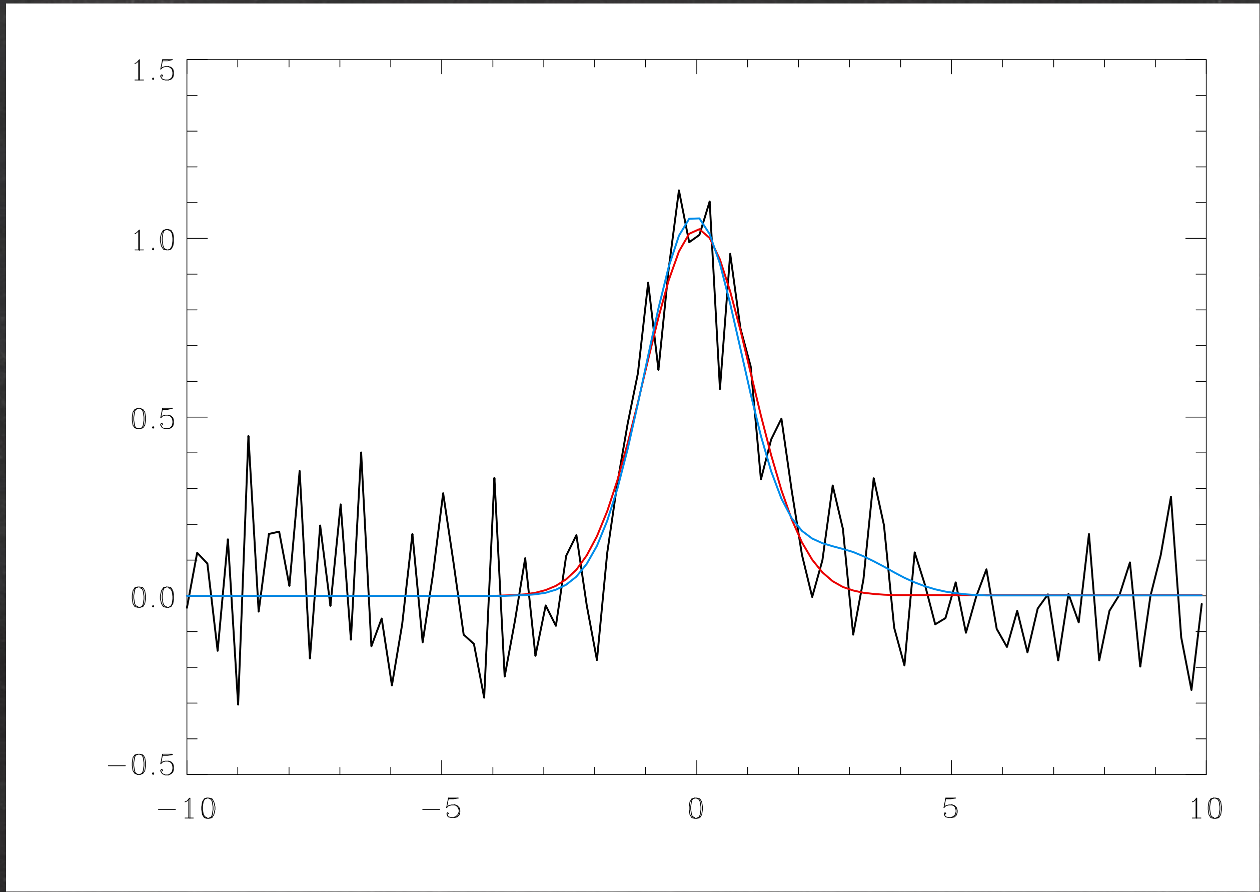
H_1 : two Gaussians of equal width but unknown amplitude ratio

$$y = \beta_1 \exp\left(-\frac{(x-\beta_2)^2}{2\beta_3^2}\right) + \beta_4 \exp\left(-\frac{(x-\beta_2-3\beta_3)^2}{2\beta_3^2}\right)$$

Model comparison – a worked example



Model comparison – a worked example



$$\mathcal{L}(H_1) = -67.44$$

$$\mathcal{L}(H_0) = -69.66$$

In R=2.22 → weak-moderate evidence in favor of model 1