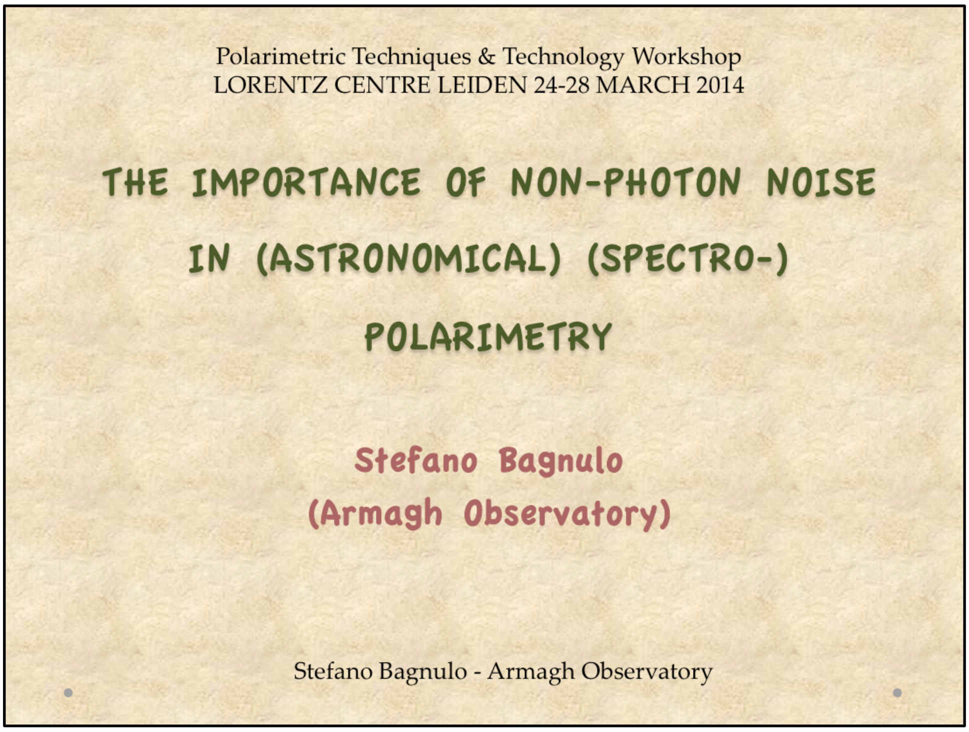
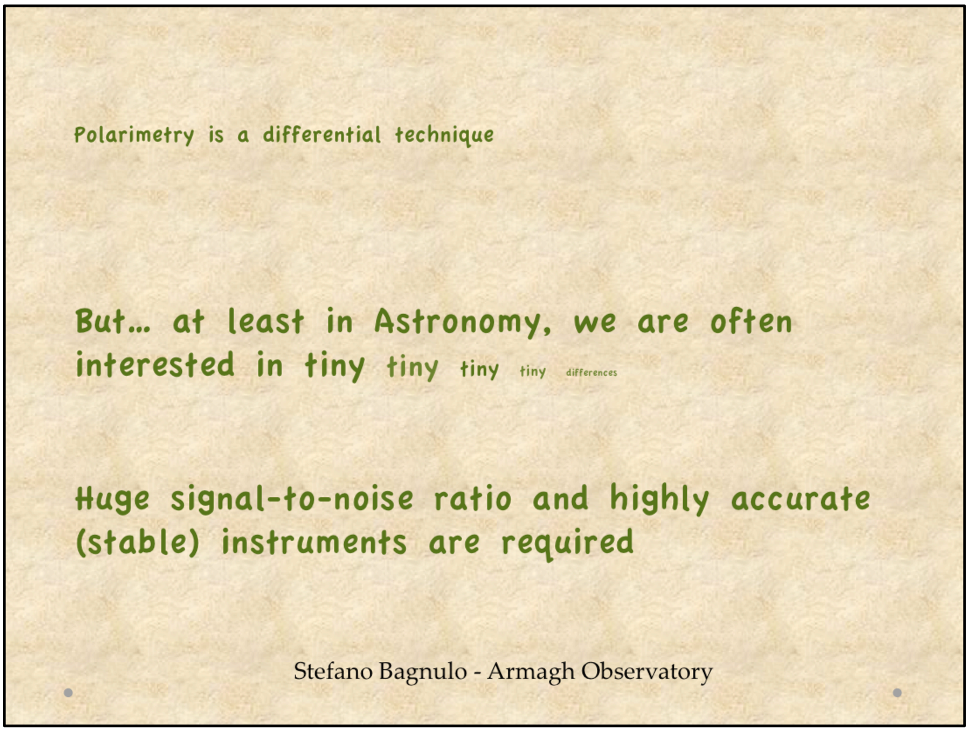


Polarimetric Techniques & Technology Workshop
LORENTZ CENTRE LEIDEN 24-28 MARCH 2014

THE IMPORTANCE OF NON-PHOTON NOISE IN (ASTRONOMICAL) (SPECTRO-) POLARIMETRY

Stefano Bagnulo
(Armagh Observatory)



Polarimetry is a differential technique

But… at least in Astronomy, we are often interested in tiny tiny tiny tiny differences

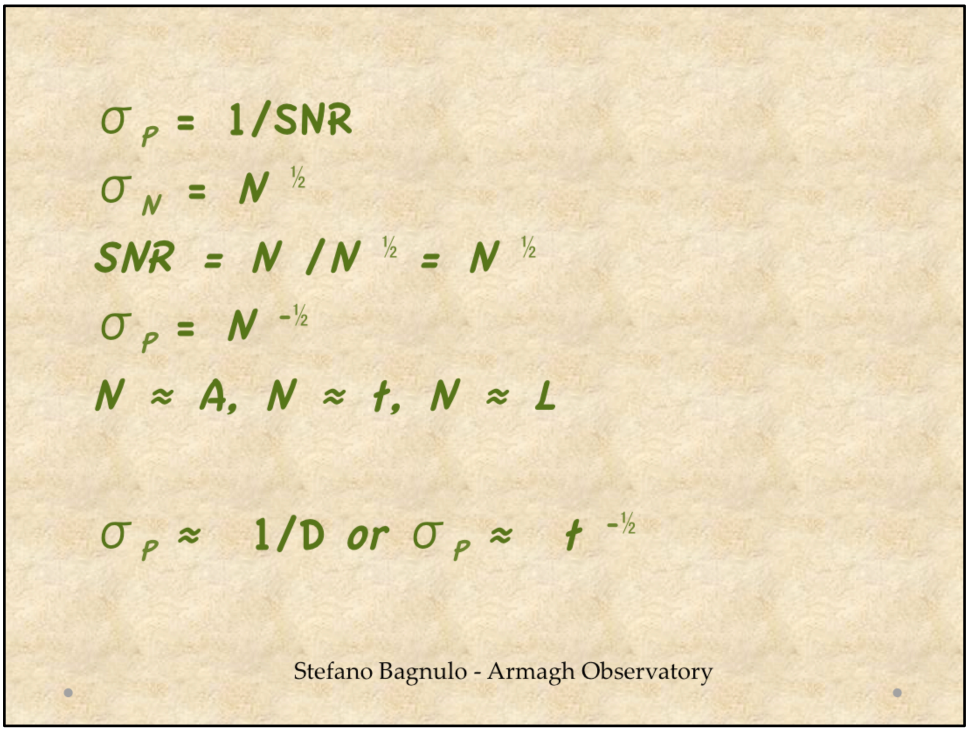
Huge signal-to-noise ratio and highly accurate (stable) instruments are required

Stefano Bagnulo - Armagh Observatory

One the one hand, astronomical polarimetry is a differential technique, hence conceptually more robust than other techniques.

On the other hand, at least in astronomy, we are interested in measuring tiny signals. In fact, astronomers often base their expectations on results that are at the limit of what can be achieved with the available instrumentation, sometimes beyond.

To measure the signals in which we are interested, we often need to obtain measurements characterised by huge signal-to-noise ratios.
Reaching such a high signal-to-noise ratio in term of simple number of photons is already a challenging task, what often happens is that we forget that photon-noise is not the entire business.



$\sigma_P = 1/SNR$

$\sigma_N = N^{1/2}$

$SNR = N / N^{1/2} = N^{1/2}$

$\sigma_P = N^{-1/2}$

$N \approx A,\ N \approx t,\ N \approx L$

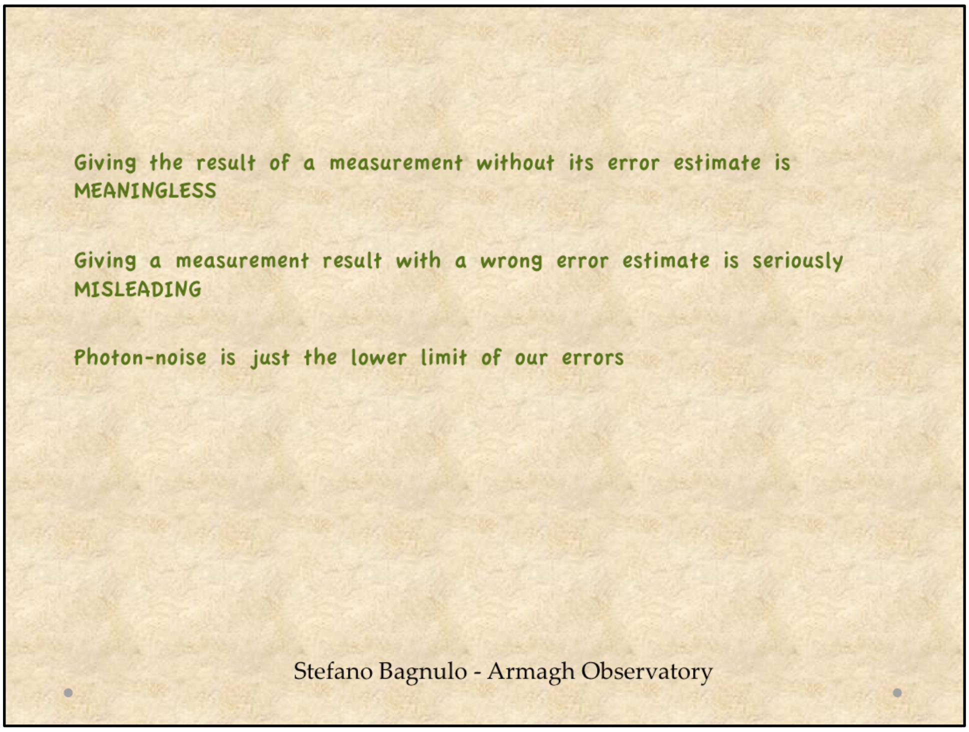
$\sigma_P \approx 1/D$ or $\sigma_P \approx t^{-1/2}$

Stefano Bagnulo - Armagh Observatory

One very basic thing is that polarimetric error bars are proportional to one over the signal-to-noise ratio of your photon measurement. For instance, if we want to measure Q/I with an error bar of 1%, we need to count your photons with a signal-to-noise ratio of 100. If we want to achieve an accuracy of 0.1% you need a SNR of 1000 and so on.

Poisson statistics tells us that when we count N photons, our error bar is the square root of N, therefore the signal-to-noise ratio increases only as the square root of the number of photons, and the error bar on the reduced Stokes parameters decreases only as fast as one over the square root of the photons that we are counting.

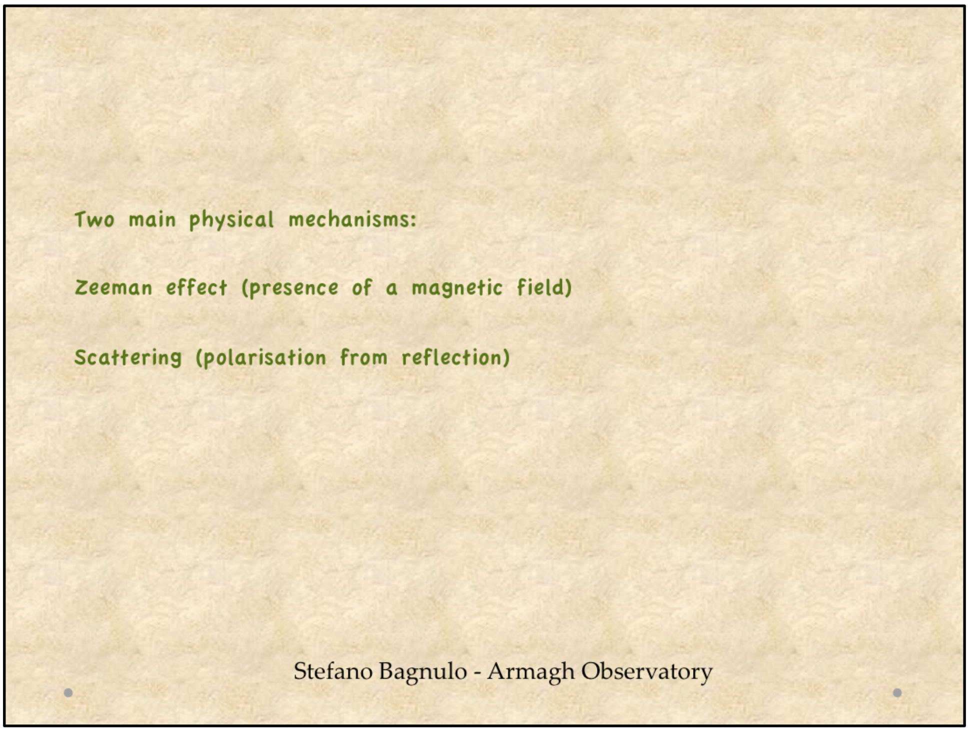
The number of photons that we measure is linearly porportional to the collecting area of our telescopes, or the to the time of our exposures or the luminosity of our targets, therefore our error bars decreases as 1 over the diameter of our telescopes, or 1 over the square root of the time we wait with the shutter open.
If we need 1h telescope time to get an error bar of 1%, then we need 4 hours telescope time to get a 0.5% error bar!



Giving the result of a measurement without its error estimate is MEANINGLESS

Giving a measurement result with a wrong error estimate is seriously MISLEADING

Photon-noise is just the lower limit of our errors



Two main physical mechanisms:

Zeeman effect (presence of a magnetic field)

Scattering (polarisation from reflection)

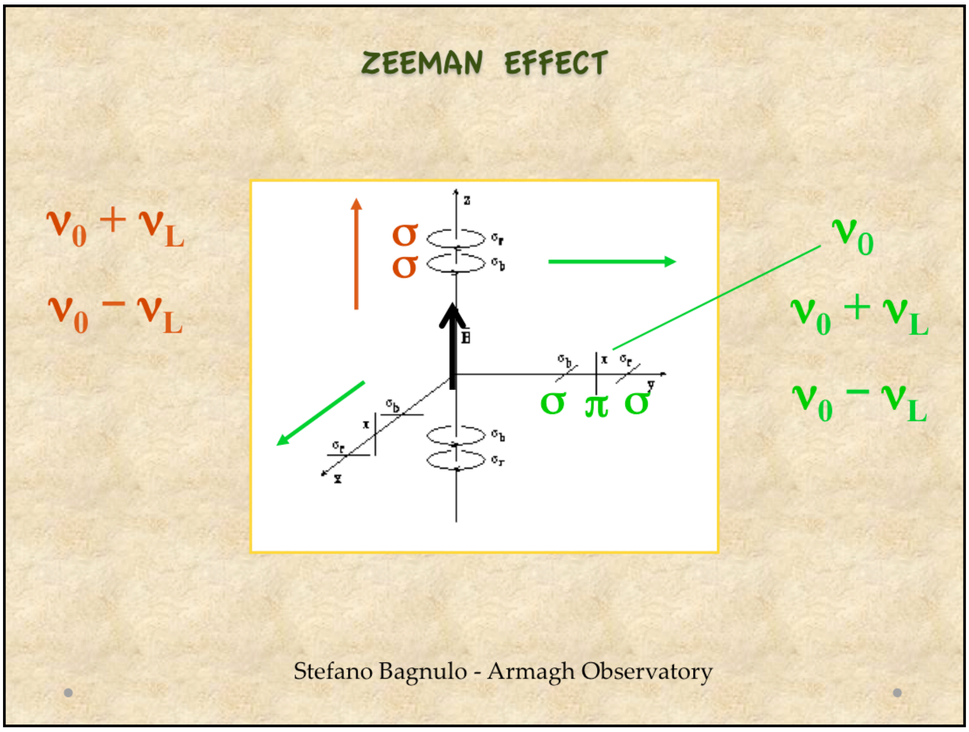
Stefano Bagnulo - Armagh Observatory

There are two physical mechanisms that generate polarisation and that are of astronomical interest: the Zeeman effect, due to the presence of a magnetic field in the Sun or in a star, and light scattering, for instance the reflection of the light from the surface of an asteroid.

Stefano Bagnulo - Armagh Observatory

Perhaps the most important tool in astronomy is the spectral analysis. In particular, we know that when the intensity is decomposed in the various contributions at various wavelengths, we see drops in intensity at specific wavelengths, which corresponds to the presence of a certain element, like Hydrogen, Helium, Calcium, Iron etc.

The main diagnostic tool for detection of stellar magnetic fields is the analysis of the Zeeman effect on the Stokes profiles of spectral lines.



ZEEMAN EFFECT

$\nu_0 + \nu_L$

$\nu_0 - \nu_L$

σ

σ

σ π σ

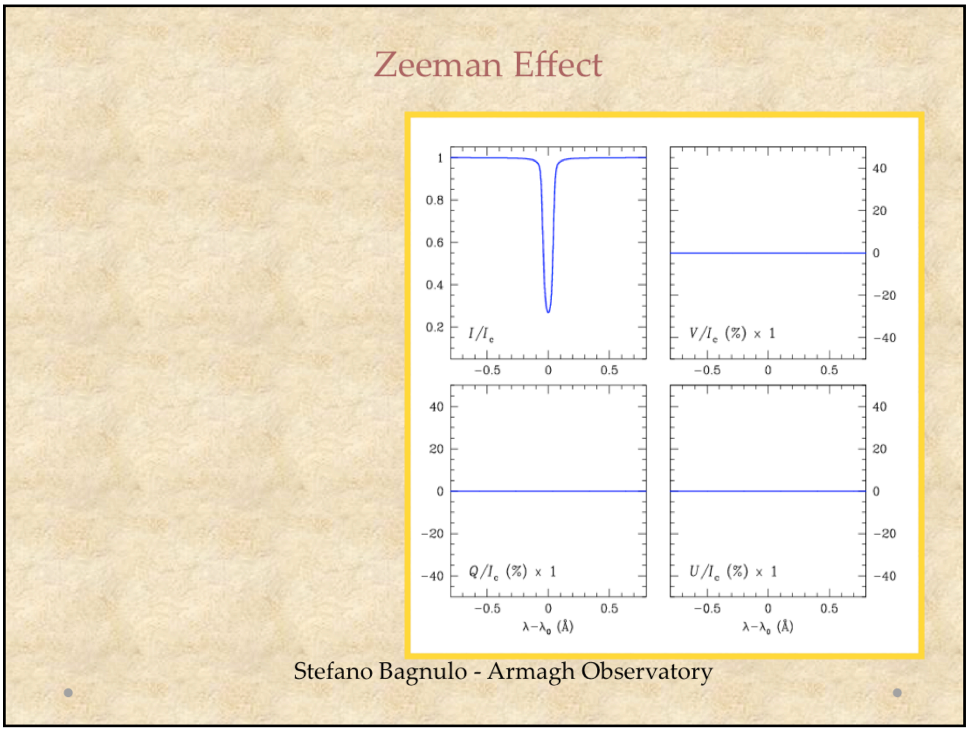
ν_0

$\nu_0 + \nu_L$

$\nu_0 - \nu_L$

Stefano Bagnulo - Armagh Observatory

Let us consider an atomic oscillator in the presence of a magnetic field. Observing in the direction perpendicular to the magnetic field vector, we see three oscillations, one equal to the natural frequency of the oscillator, which is called pi component, one equal to the natural frequency of the oscillator increased by an amount equal to the Larmor frequency, and one equal to the frequency of the oscillator decreased by an amount equal to the Larmoor frequency, these two latter components are called sigma components. The oscillation appear linear, the pi component oscillation is parallel to the magnetic field, and the sigma component oscillates perpendicularly to the magnetic field vector. Observing in the direction of the magnetic field vector, the oscillator will appear split in two components, one equal to the natural frequency of the oscillator increased by an amount equal to the Larmor frequency, and one equal to the frequency of the oscillator decreased by an amount equal to the Larmoor frequency. These two oscillations are called the sigma components. The oscillation appears circular to the observer. This has some consequences in the process of line formation.



Zeeman Effect

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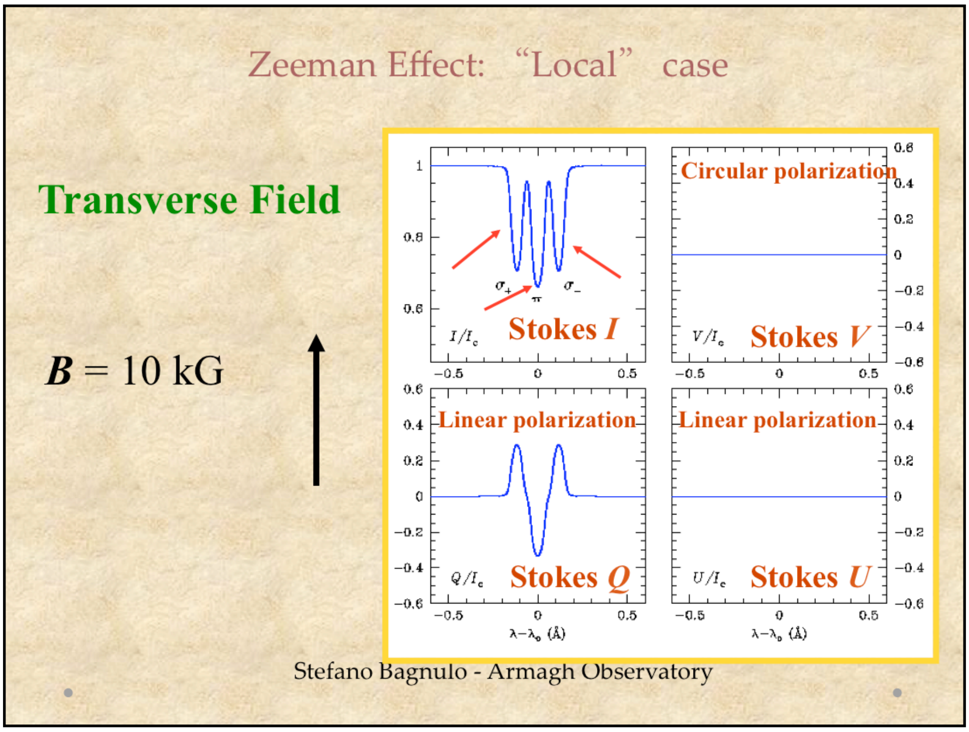
Here we se a deep spectral line formed in an element of the stellar surface, with no magnetic field



Zeeman Effect: "Local" case

Transverse Field

$\boldsymbol{B} = 10$ kG

Stefano Bagnulo - Armagh Observatory

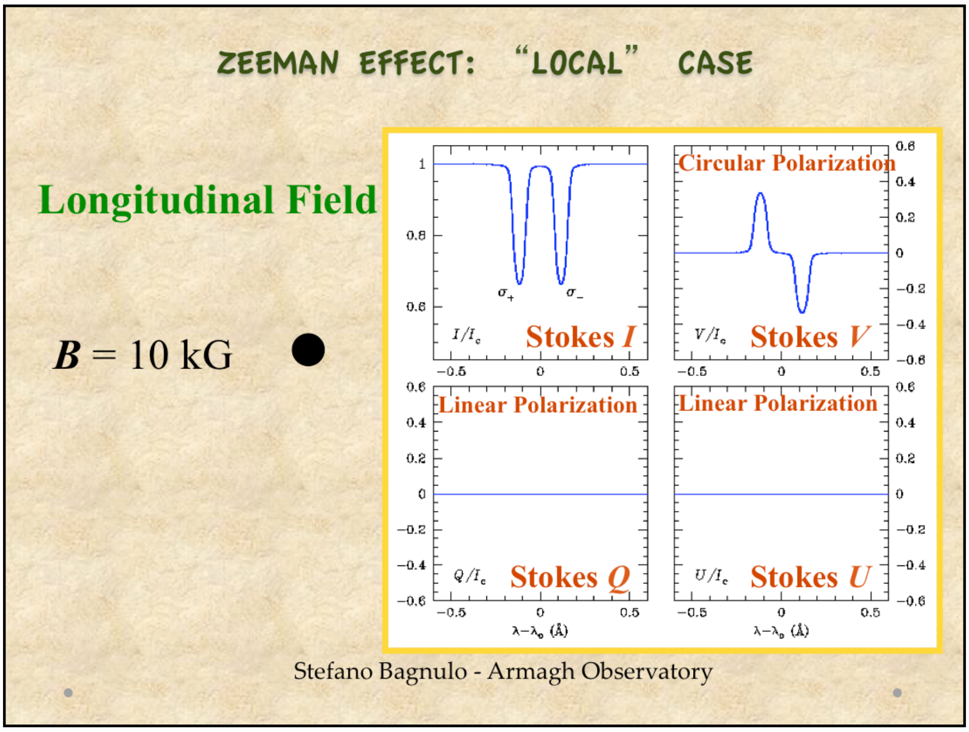
Let' s suppose that we are observing a single element of the stellar surface characterised by a magnetic field oriented perpendicularly to the line of sight. Stokes *I*, the`usual' intensity, will be **split** in three components:

A π component, at the rest frequency, is polarized parallel to the direction of the magnetic field;

A red and a blue **sigma** components are shifted by a quantity equal to the Larmor frequency, and polarized in the direction perpendicular to the magnetic field.

The **linear polarization** is measured by the Stokes parameters *Q* and *U*. The line appears linearly polarized, and the fact that Stokes *U* is zero depends uniquely on the reference system that we adopted for defining the Stokes parameters

The **circular polarization**, measured bythe Stokes parameter *V*, is zero



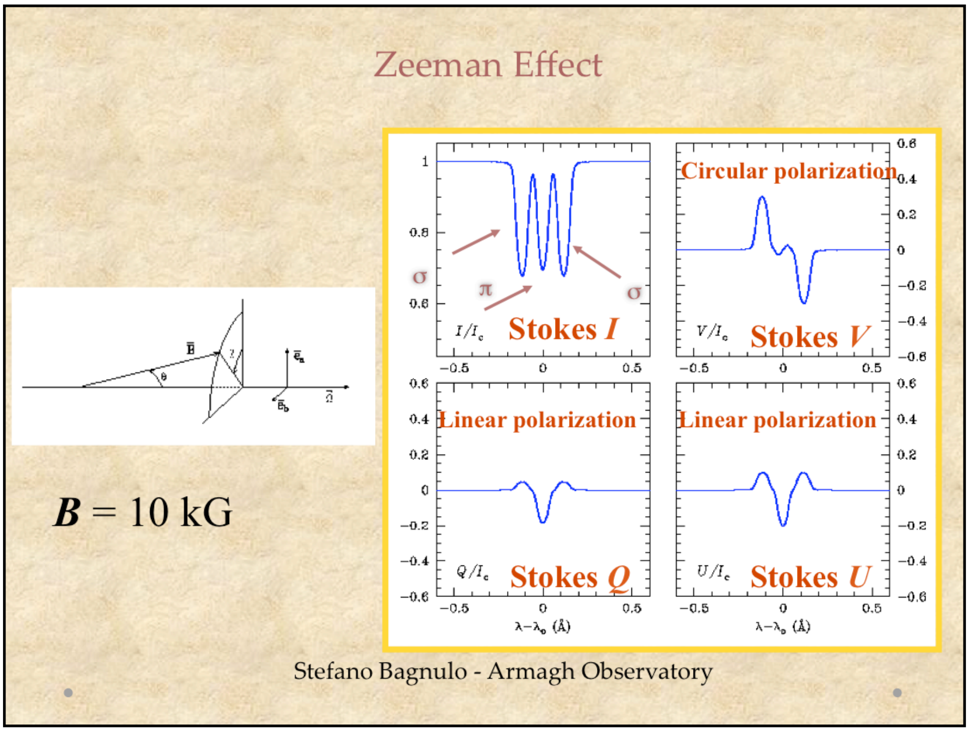
ZEEMAN EFFECT: "LOCAL" CASE

Longitudinal Field

B = 10 kG ●

Stefano Bagnulo - Armagh Observatory

In the case of a longitudinal field, the spectral line is split in two sigma component *circularly polarized.* The linear polarization is zero.



Zeeman Effect

$\boldsymbol{B}$ = 10 kG

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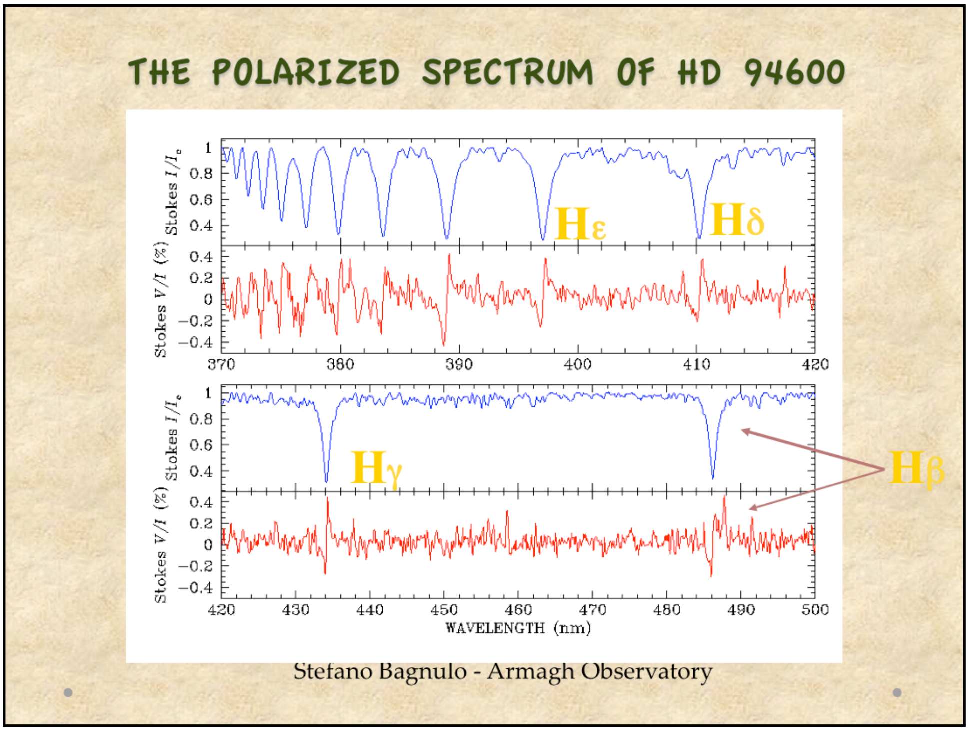
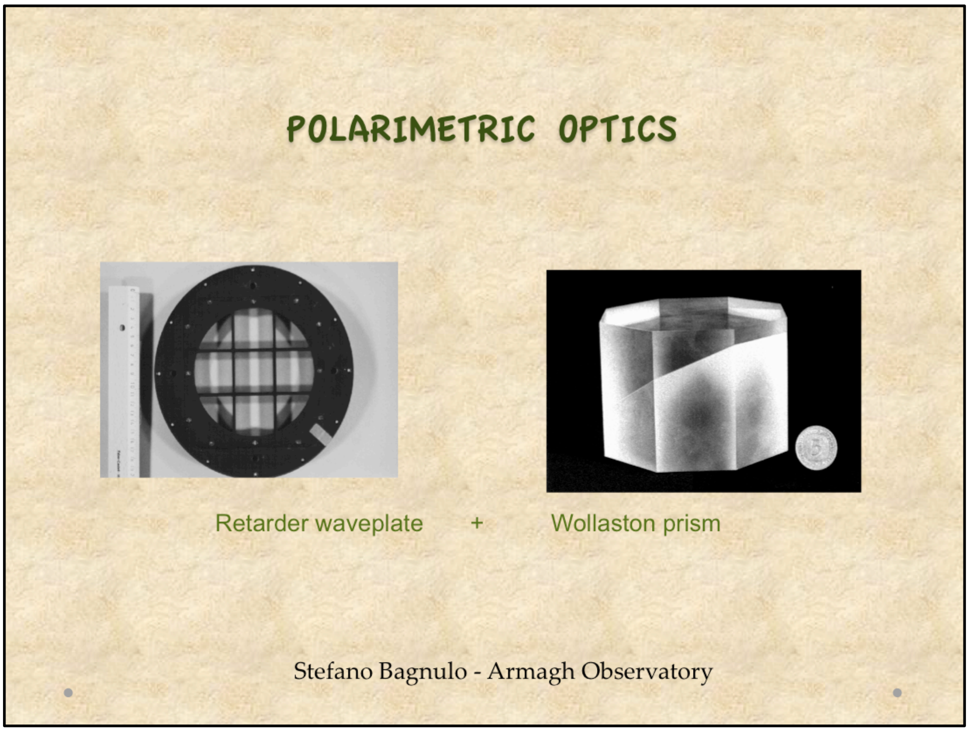
In the general case of a magnetic field tilted at an arbitrary direction, all Stokes profiles will be non zero

THE POLARIZED SPECTRUM OF HD 94600



Stefano Bagnulo - Armagh Observatory

This is the example of the polarised spectrum of a magnetic star, observed with relatively low resolution… you can see the typical S profile of Stokes V associated to each Hydrogen Balmer line. We can see that if we want to measure a 0.4% signal in a 1 A spectral bin, we need to have an error bar not higher than 0.1% hence a signal-to-noise ratio of at least 1000 over 1 A spectral bin!



POLARIMETRIC OPTICS

Retarder waveplate + Wollaston prism

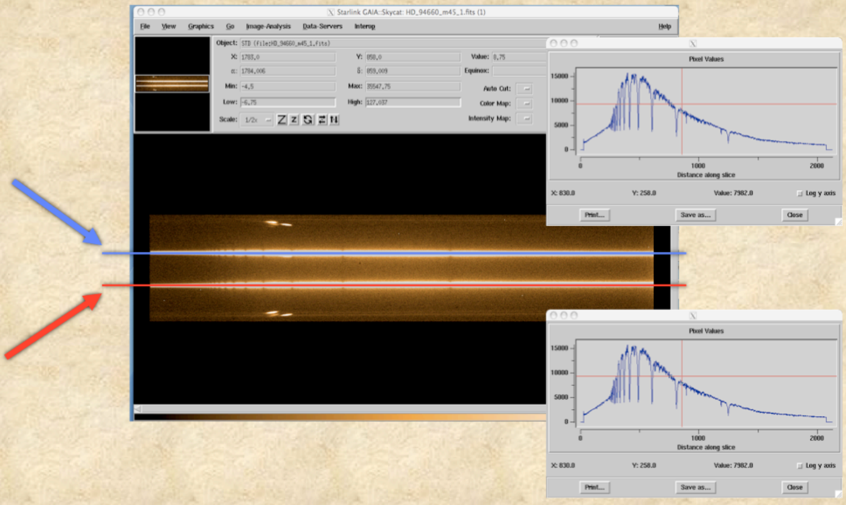
Stefano Bagnulo - Armagh Observatory

Most of astronomical polarimeters include a retarder waveplate and a beam splitter device, for instance a Wollaston prism, that will split the incoming radiation in two beams of opposite polarisation

STOKES V FROM RCP AND LCP

I+V
(RCP)

I-V
(LCP)

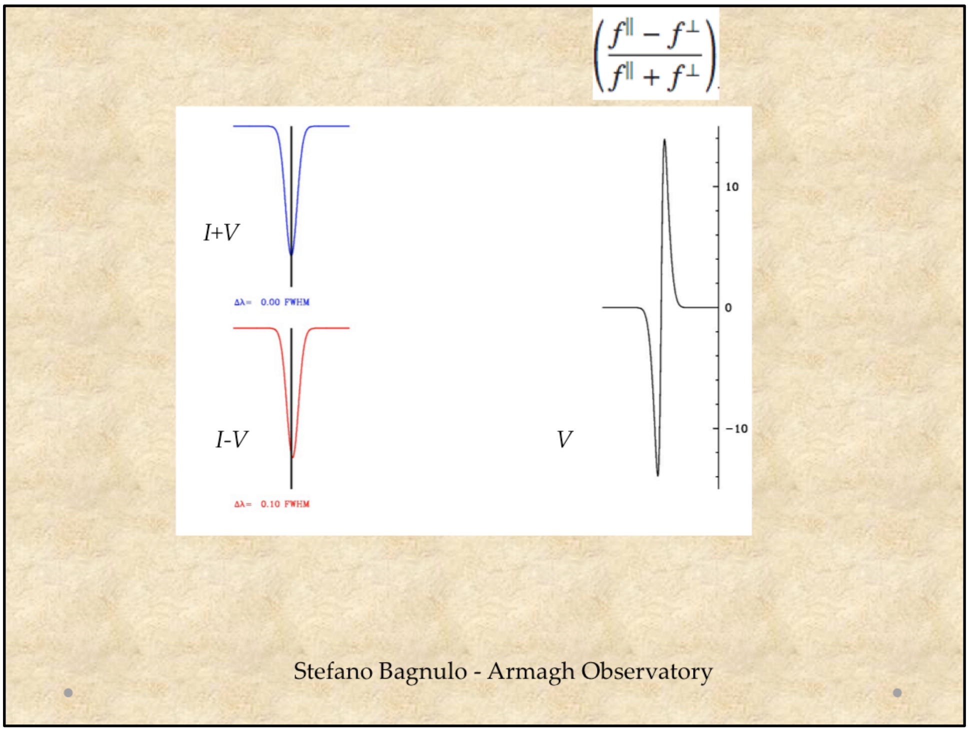


Object: STD (File:HD_94660_m45_1.fits)
X: 1783,0
Y: 858,0
Value: 8,75
α: 1784,006
δ: 859,009
Equinox:
Min: -4,5
Max: 35547,75
Low: -6,75
High: 127,037
Scale: 1/2x
Auto Cut:
Color Map:
Intensity Map:

X: 830.0
Y: 258.0
Value: 7982.0

Stefano Bagnulo - Armagh Observatory
The Open University

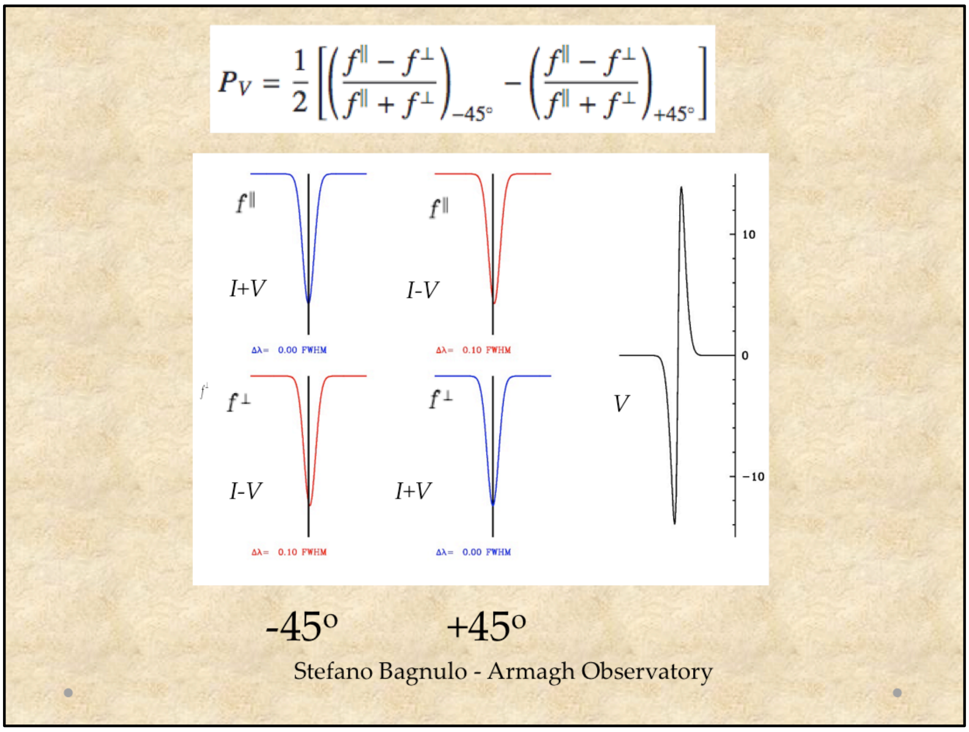
This is an example of raw spectropolarimetric data.
The upper beam is proportional to the right circular polarization, and the beam at the bottom is proportional to the left circular polarization. Stokes V is obtained from their difference, which is VERY SMALL.



$$\left(\frac{f^{\parallel} - f^{\perp}}{f^{\parallel} + f^{\perp}}\right).$$

Stefano Bagnulo - Armagh Observatory

If there is a shift between the positions of two spectral lines, their difference will give us a non zero Stokes profile. And even very small shifts may produce large signals. In this slide we see that an offset of one tenth the full width half maximum of a spectral line gives a Stokes profile with a nearly 30% amplitude.

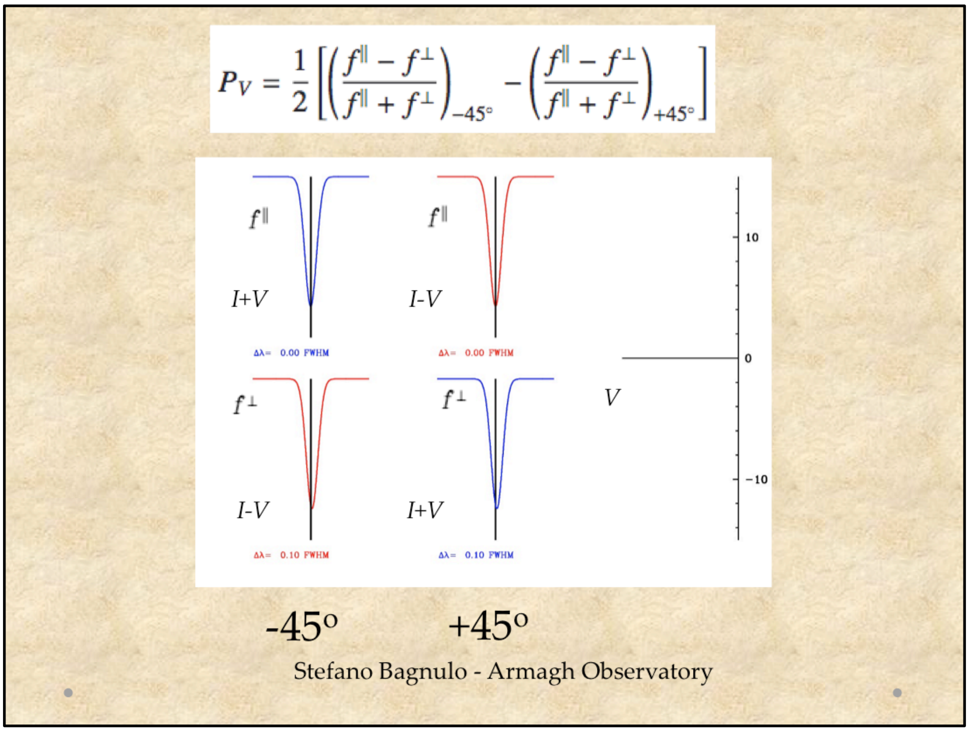


$$P_V = \frac{1}{2}\left[\left(\frac{f^{\parallel}-f^{\perp}}{f^{\parallel}+f^{\perp}}\right)_{-45^\circ} - \left(\frac{f^{\parallel}-f^{\perp}}{f^{\parallel}+f^{\perp}}\right)_{+45^\circ}\right]$$

-45°   +45°

Stefano Bagnulo - Armagh Observatory

Therefore in astronomical polarimetry we often adopt the so-called beam-swapping technique. If we rotate the retarder waveplate, then the polarisation in the two beams split by the Wollaston reverses, and Stokes V can be measured with the method of the double difference. If the offset between the two beams is due to Zeeman effect, then the offset will reverse its signs



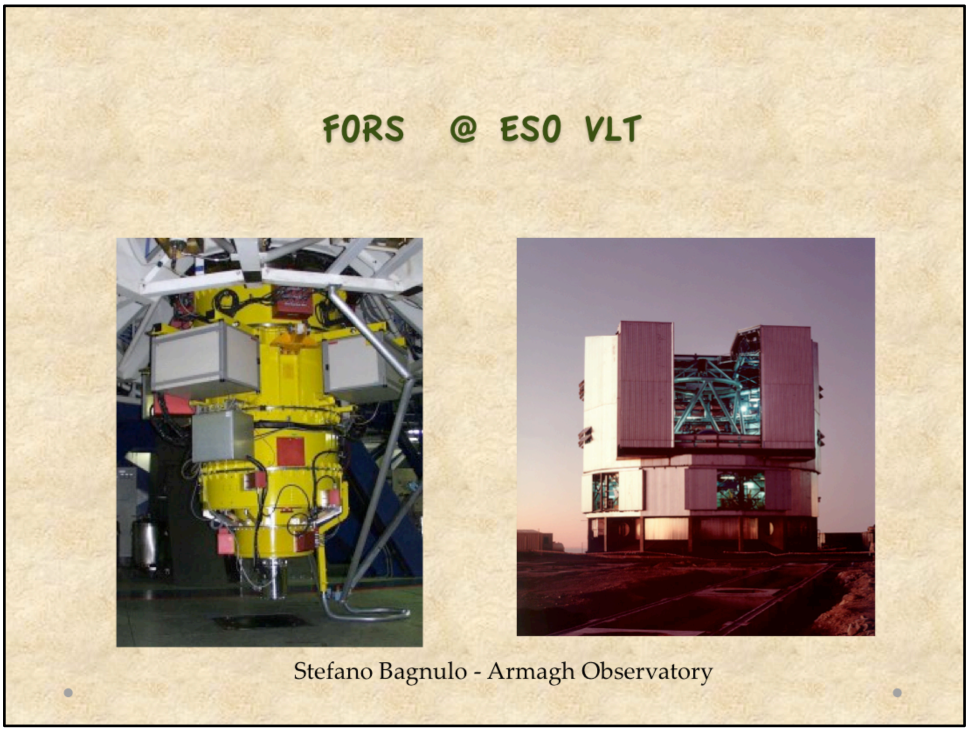
$$P_V = \frac{1}{2}\left[\left(\frac{f^{\parallel}-f^{\perp}}{f^{\parallel}+f^{\perp}}\right)_{-45^\circ} - \left(\frac{f^{\parallel}-f^{\perp}}{f^{\parallel}+f^{\perp}}\right)_{+45^\circ}\right]$$

-45° +45°

Stefano Bagnulo - Armagh Observatory

If the offset between the two beams is spurious, then it probably keeps the same sign when the retarder waveplate is rotated, therefore it disappears with the double difference method.

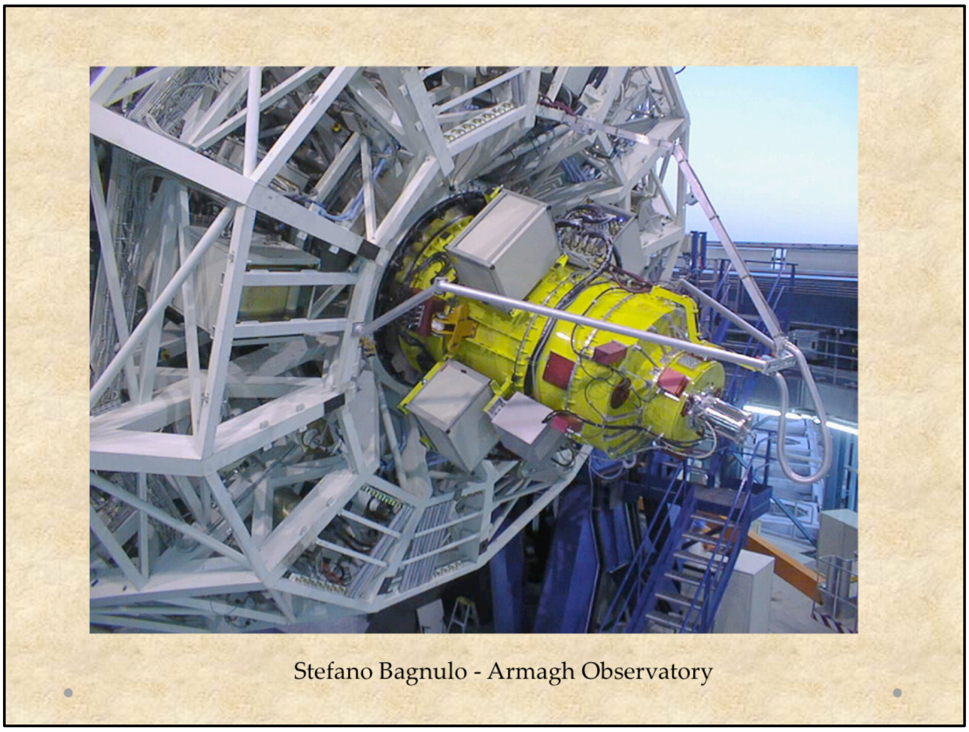
Advantages of the beam swapping techniques are thoroughly discussed in
Bagnulo et al. 2009, PASP, 121, 993 “: Stellar Spectropolarimetry with Retarder Waveplate and Beam Splitter Devices”
and in
Bagnulo et al. 2013, A&A, 559, 103: “The importance of non-photon noise in stellar spectropolarimetry. The spurious detection of a non-existing magnetic field in the A0 supergiant HD 92207”



FORS @ ESO VLT

Stefano Bagnulo - Armagh Observatory

FORS is a multipurpose instrument attached to one of the 8 m units of the ESO VLT. It is capable of doing imaging and low resolution spectroscopy and is equipped with polarimetric optics



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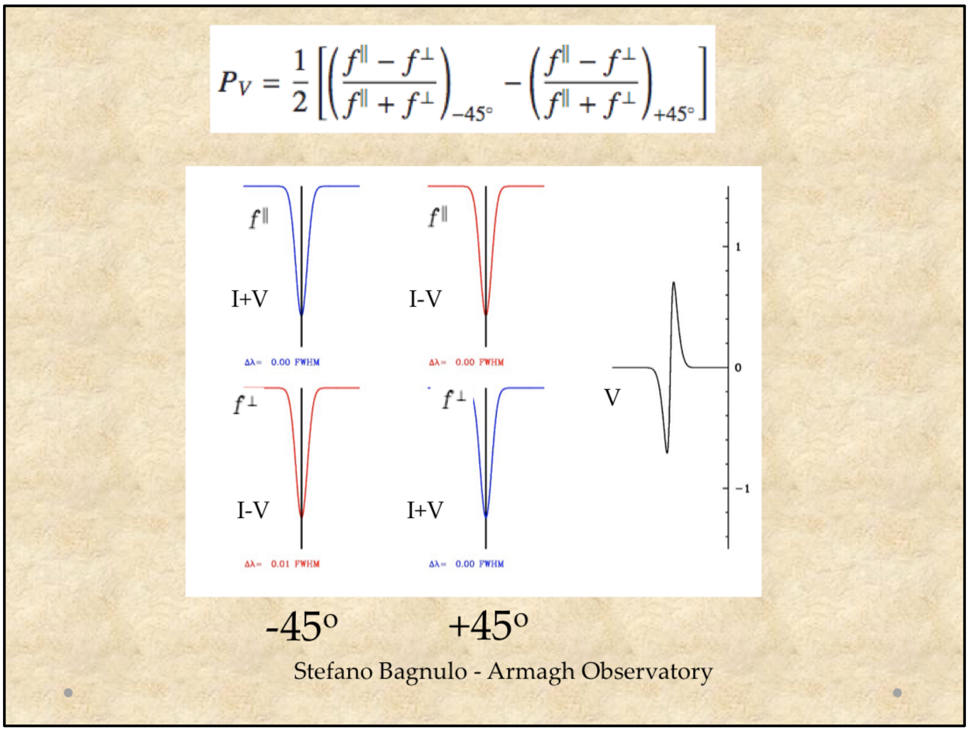
The problem is that observing conditions may be less than optimal. For instance if we attach the instrument at the Cassegrain focus of a telescope it is well possible that during the course of an exposure some differential micro-flexures intervene which cannot be compensated even by the swapping technique

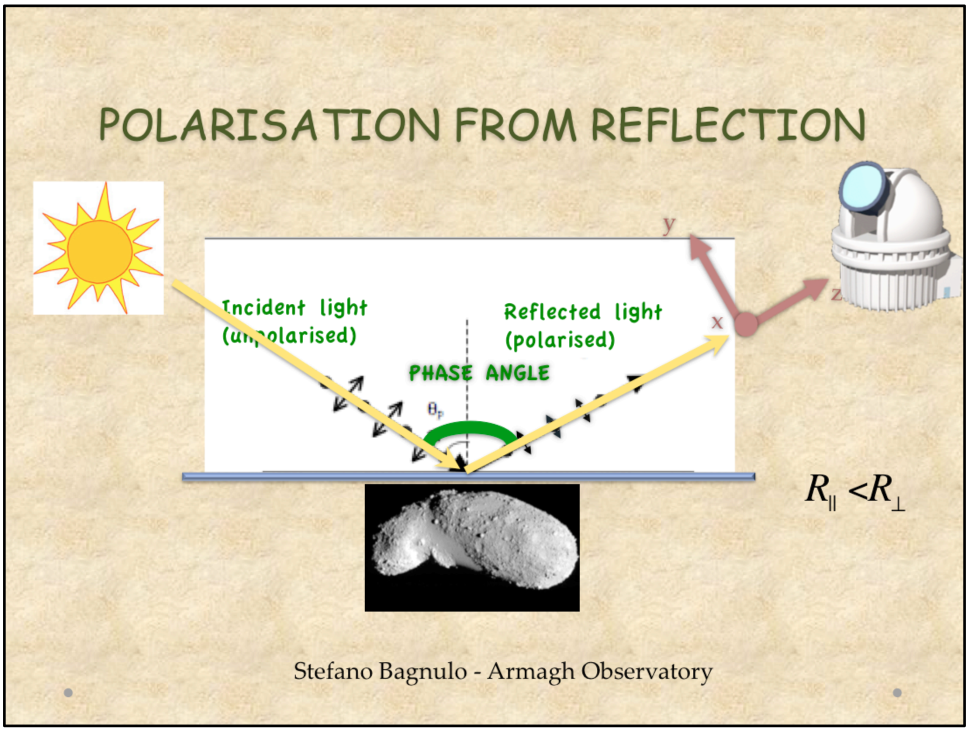


$$P_V = \frac{1}{2}\left[\left(\frac{f^{\parallel} - f^{\perp}}{f^{\parallel} + f^{\perp}}\right)_{-45^\circ} - \left(\frac{f^{\parallel} - f^{\perp}}{f^{\parallel} + f^{\perp}}\right)_{+45^\circ}\right]$$

-45°   +45°

Stefano Bagnulo - Armagh Observatory

Here we show that a differential offset introduced at only one position of the retarder waveplate by 1/100 of the FWHM is responsible for a spurious Zeeman signature of the order of 1%.



POLARISATION FROM REFLECTION

Incident light (unpolarised)

Reflected light (polarised)

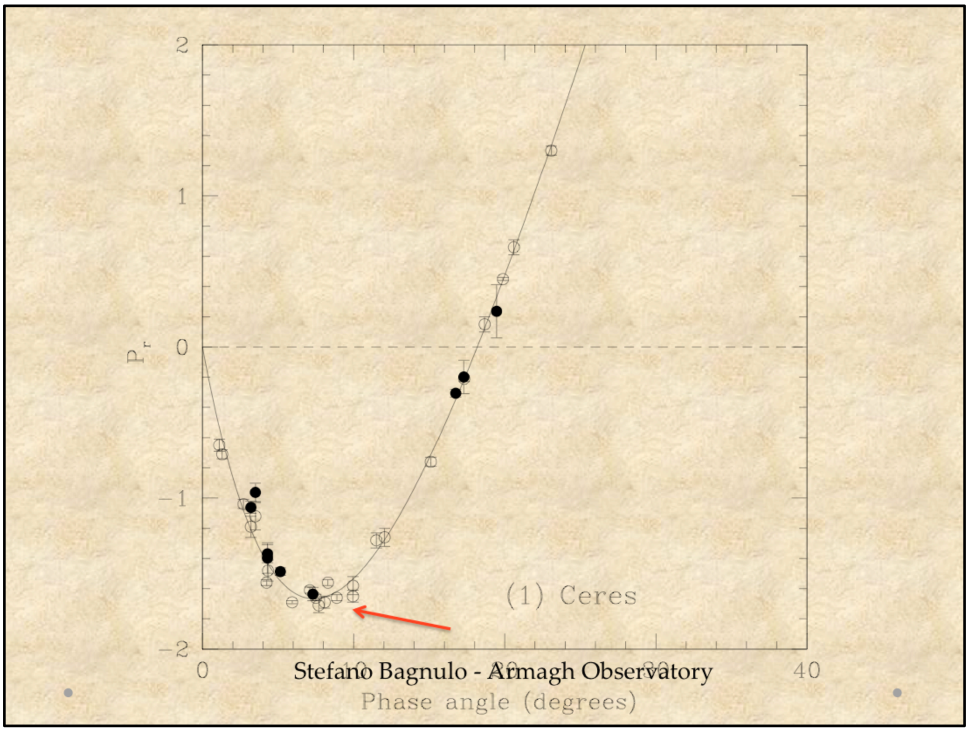
PHASE ANGLE

$R_{\parallel} < R_{\perp}$

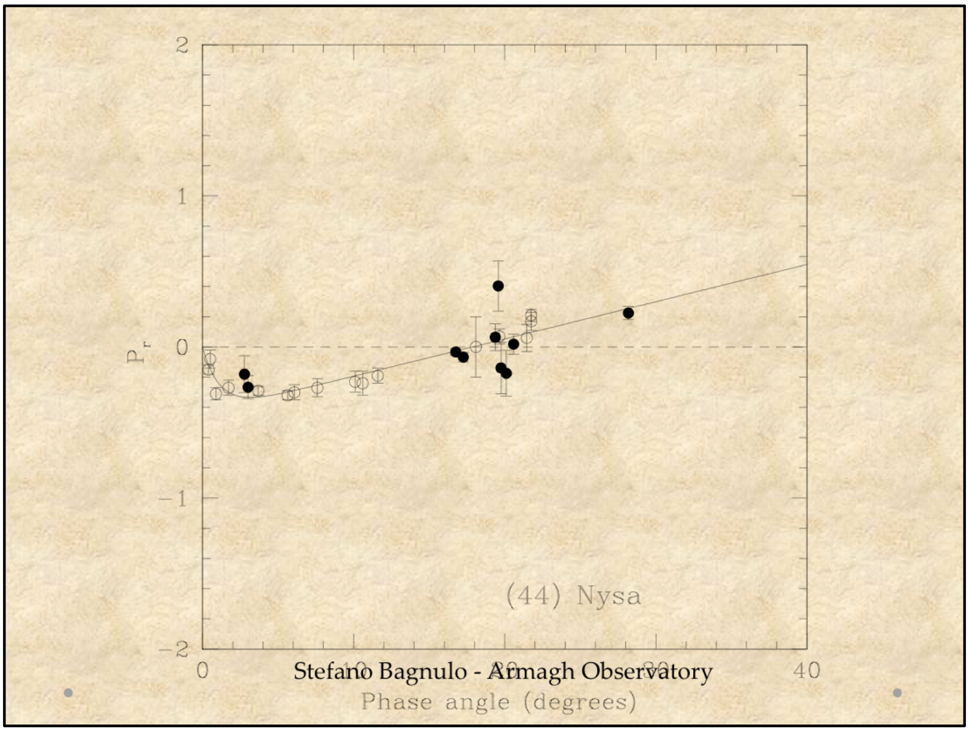
Stefano Bagnulo - Armagh Observatory

Let’s now see the case of polarisation from scattering.

From the Fresnel laws we know that the intensity and fraction of linear polarisation of the reflected light depends on the angle between the incident beam and the direction perpendicular to the surface and on the complex refractive index, a quantity that in turns depends on the surface structure and composition of the material and on wavelength.



This curve shows the polarisation as a function of the phase-angle. The minimum of polarisation is strongly depended on the albedo of the object. There are other features that may be easily measured and that are unique to certain classes of asteroids. The shape of these curves may be used to classify asteroids, and in some cases to say that asteroids that orbit in completely different parts of the solar system have in fact a common origin.



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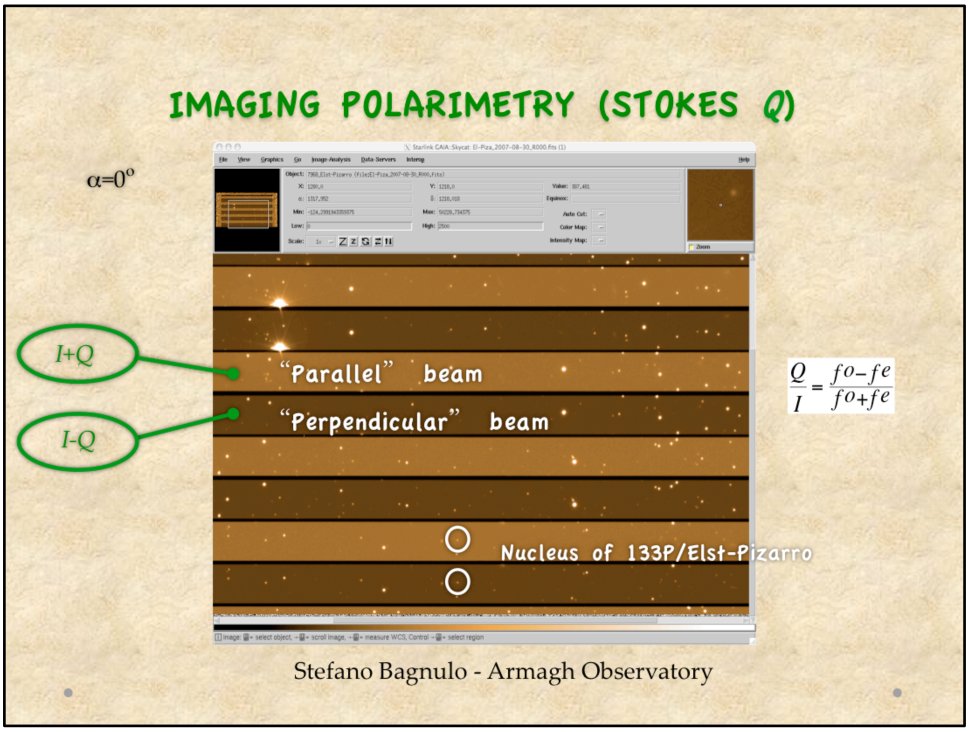
This is the example of a totally different polarimetric behaviour.



IMAGING POLARIMETRY (STOKES Q)

$\alpha=0^{o}$

$$\frac{Q}{I}=\frac{fo-fe}{fo+fe}$$

Stefano Bagnulo - Armagh Observatory

This is a field image obtained in polarimetric mode. The strips that you see are characterized by opposite polarization, and the different in intensity is due to the polarized nature of the background sky.

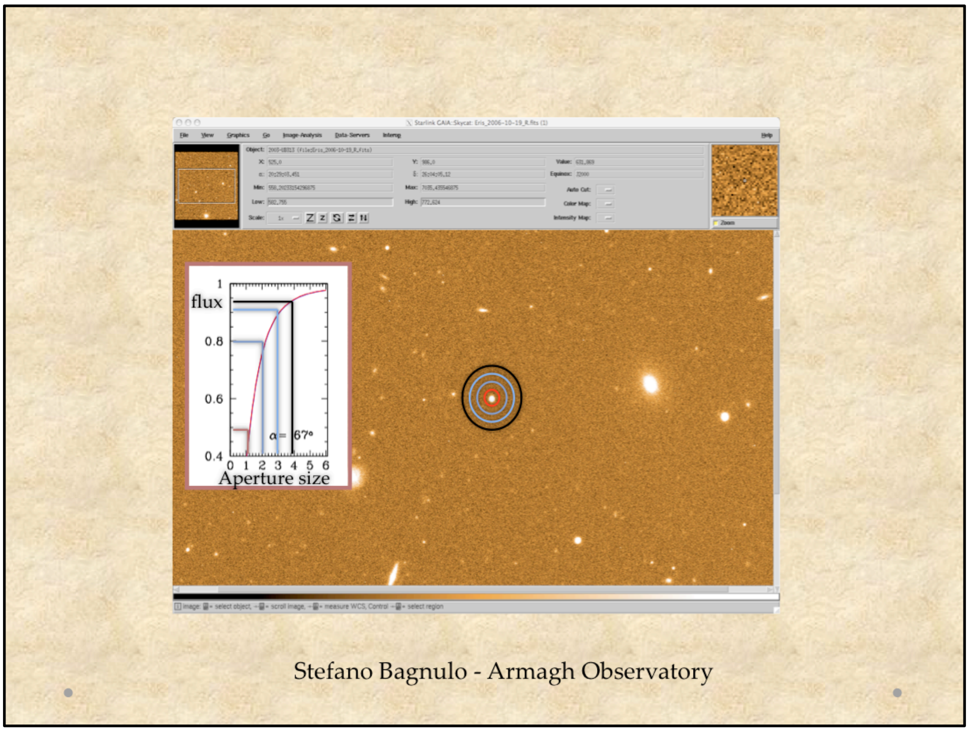
Setting the retarder waveplate at a certain position, the so-called parallel beam is proportional to I + Q, and

the perpendicular beam is proportional to I - Q.

The ratio between their difference and their sum gives us Stokes Q normalized to the intensity.

The target of these observations if the nucleus of the the main-belt object 133P/Elst-Pizarro

The beam swapping technique works also in imaging, and helps to cancel any problem due to the different transmission function in the two beams split by the Wollaston prism, but background subtraction may start to be a crucial step



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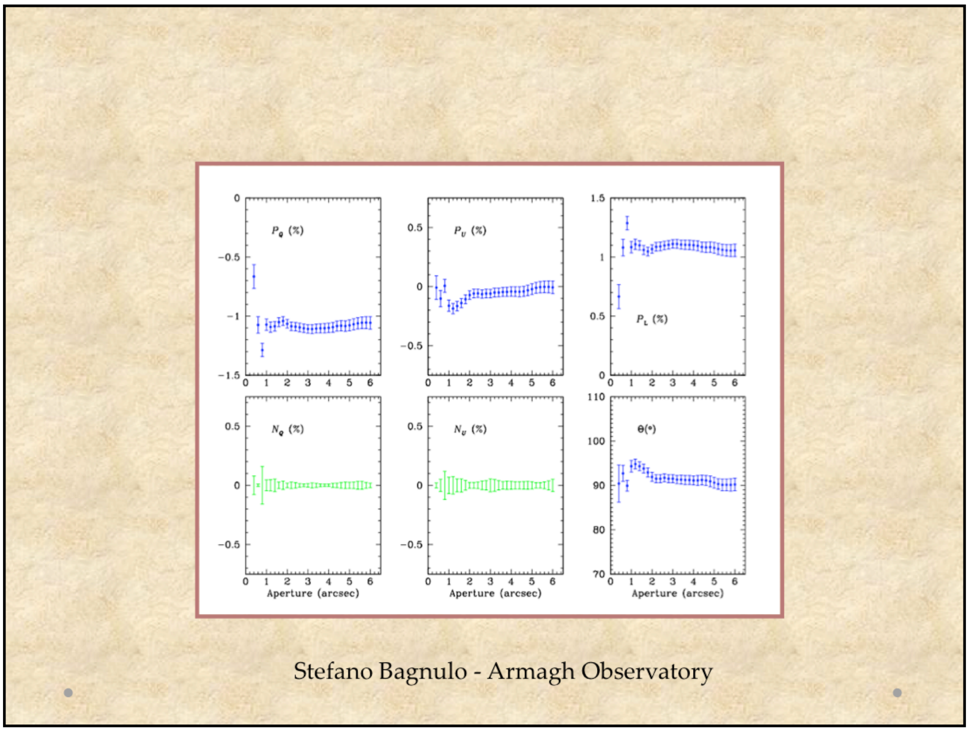
Aperture photometry consists of measuring the flux, background subtracted, for aperture of increasing size. This measured flux increases with aperture, until it reaches an asymptotic value, which is linked to be our best estimate of the object magnitude. Obviously, the larger the aperture, the larger the error introduced by background subtraction.



Stefano Bagnulo - Armagh Observatory

The aperture choice can be conveniently performed on a plot that shows the polarimetric value as a function of the aperture size. This method di explained in Bagnulo et al. 2011, JQSRT, 112, 2059: “Polarimetry of small bodies of the solar system with large telescopes”.