Relating the Statistics of the Angle of Linear Polarization (AoLP) to Measurement Uncertainty of the Stokes Vector

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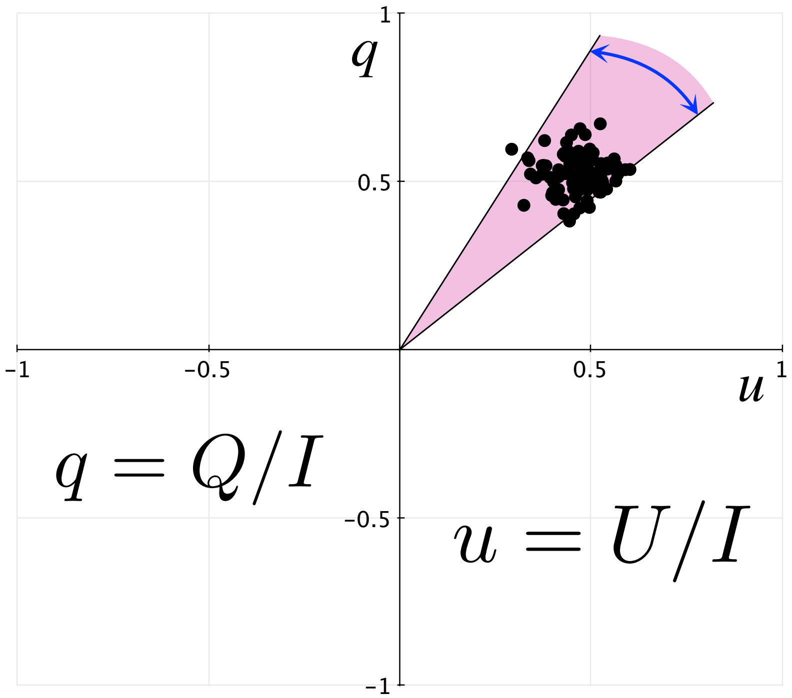
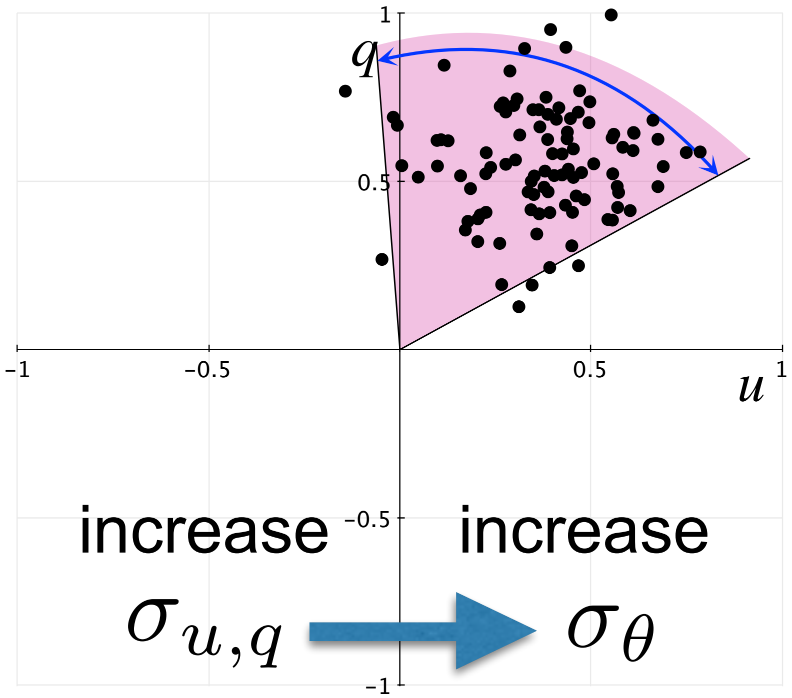
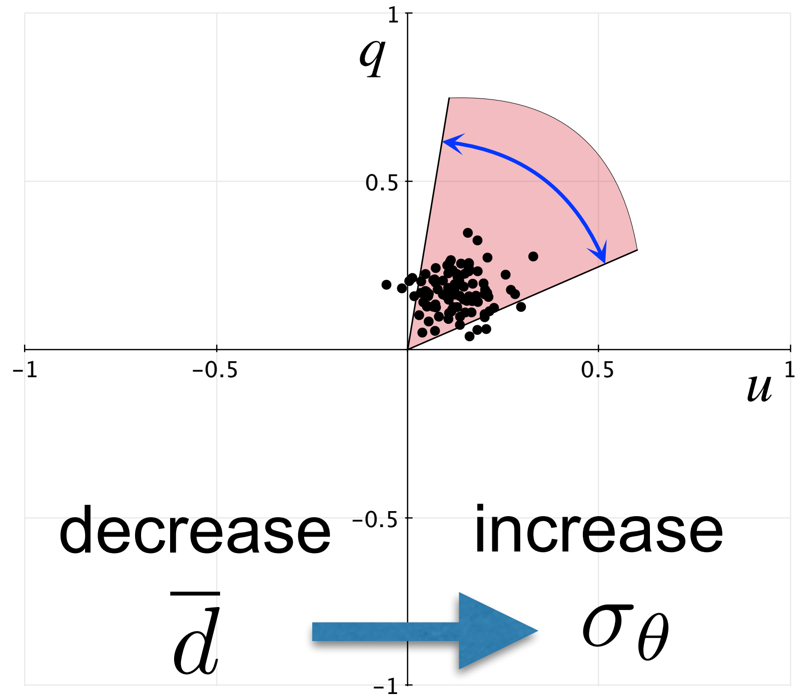
Assumptions & Notation







$$\mathrm{pr}(u) = \mathcal{N}(\overline{u}, \sigma_u^2)$$

$$\mathrm{pr}(q) = \mathcal{N}(\overline{q}, \sigma_q^2)$$

$$\sigma_{u,q} = \sigma_q = \sigma_u$$

linear Stokes parameters uncorrelated Gaussian RVs

AoLP

$$\theta = \frac{1}{2}\mathrm{atan}(u/q)$$

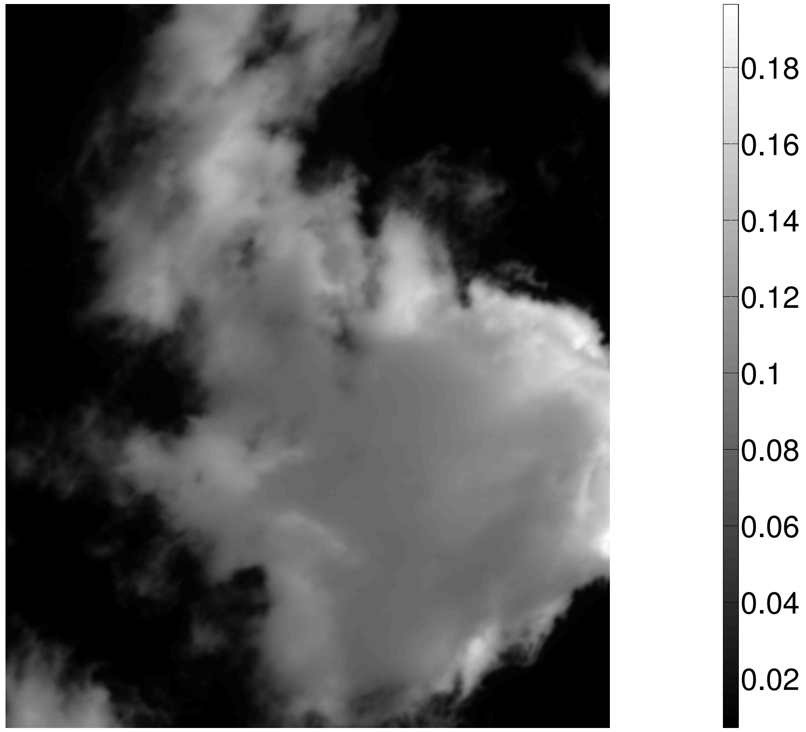
DoLP

$$\bar{d} = \sqrt{\overline{u}^2 + \overline{q}^2}$$

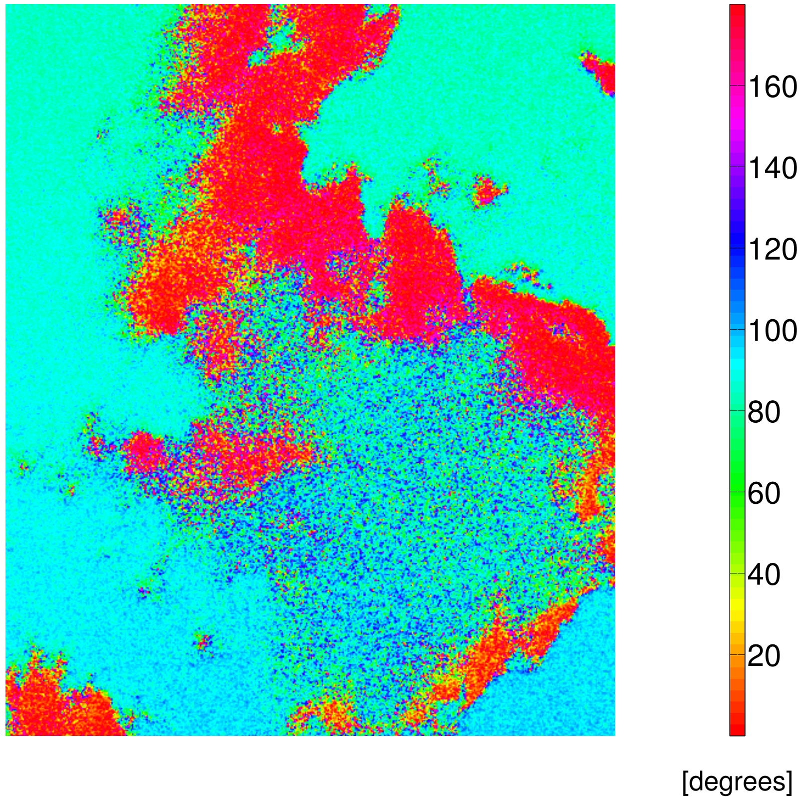
Utility of AoLP Statistical Analysis

- Hypothesis Testing: Are two samples different in AoLP?
- Quantitative: Given measurements what are confidence intervals on AoLP?

Are these measurements due to multiple scattering in the cloud or noise?



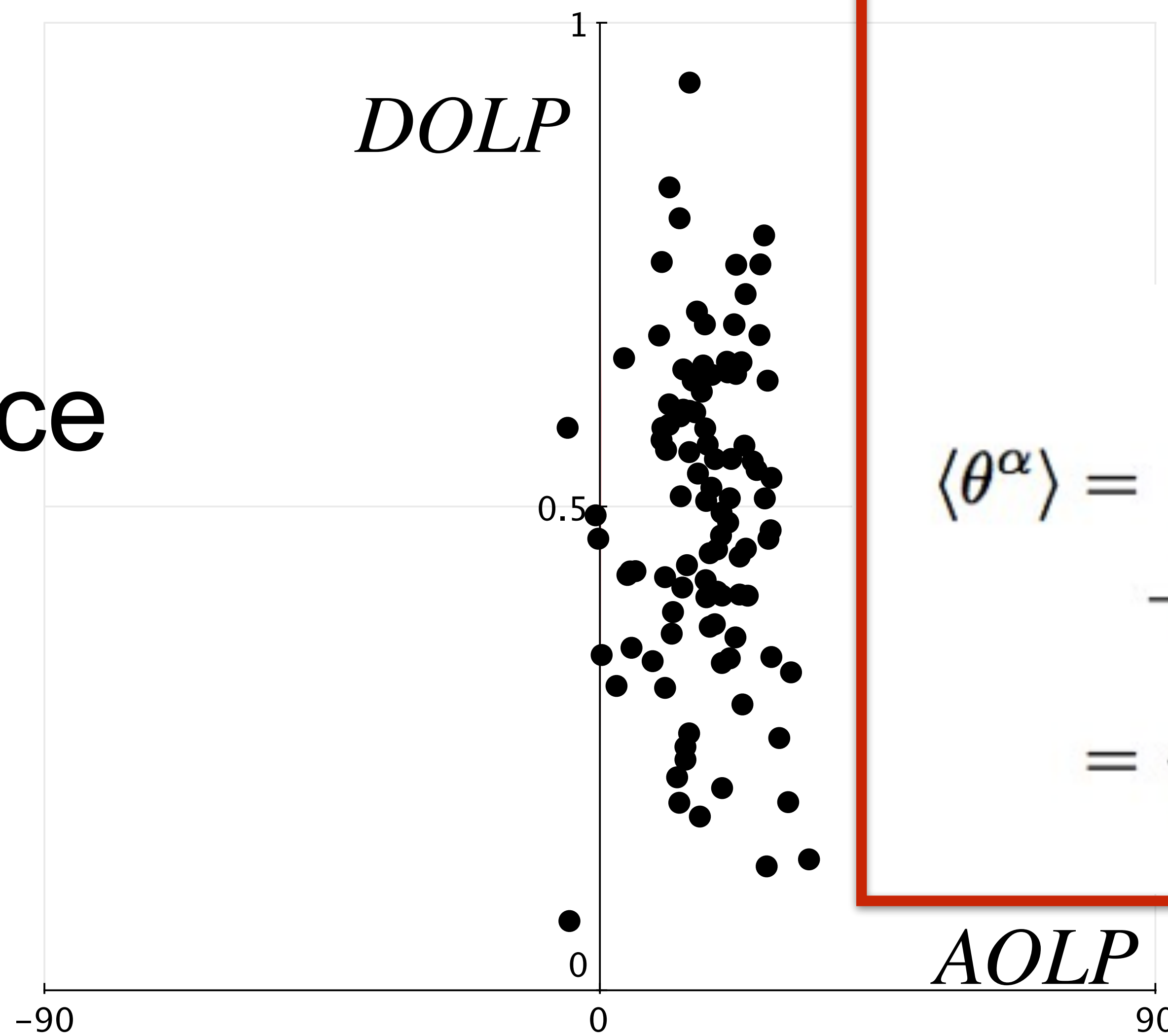
Intensity



AoLP [°]

An 865 nm GroundMSPI cloud measurement acquired 08/16/2013 13:27(PST) at 32°N, 110°W. The resulting scattering angle over the FOV was 145°-159°.

Statistical Properties of AoLP



pdf of DoLP
Rayleigh-Rice

Analytic form for AoLP moments

$$\langle \theta^\alpha \rangle = \int_{-\pi/2}^{\pi/2} d\theta \; \mathrm{pr}(\theta)\theta^\alpha$$

$$= \frac{1}{2^{\alpha+1}\pi} \sum_{n=-\infty}^{\infty} B_n^\alpha e^{i n \bar{\phi}} f_n(\tilde{\bar{d}})$$

pdf of AoLP
involves
error functions

$$\mathrm{pr}(\theta) = \int_0^1 dp \; \mathrm{pr}(\theta, p) =$$

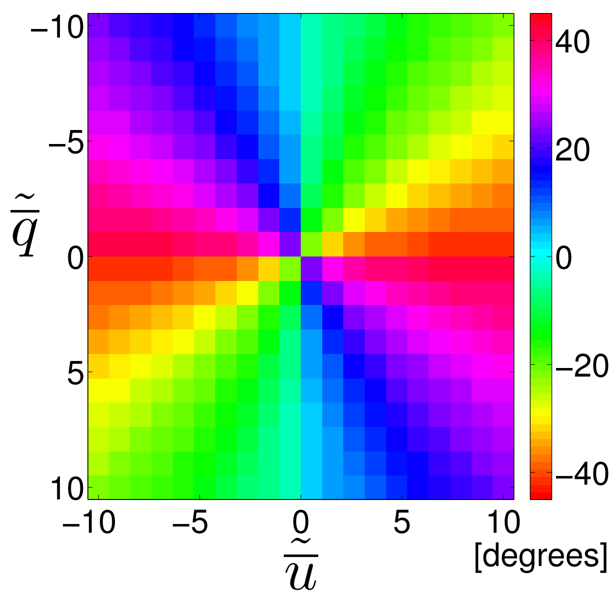
$$\frac{1}{\pi} e^{\frac{-\tilde{\bar{d}}^2}{2}} \int_0^\infty d\tilde{p}\, \tilde{p}\, e^{\frac{-\tilde{p}^2}{2}} e^{\tilde{p}(\tilde{\bar{q}}\cos(2\theta) + \tilde{\bar{u}}\sin(2\theta))}$$

Choices for Estimating Moments of AoLP

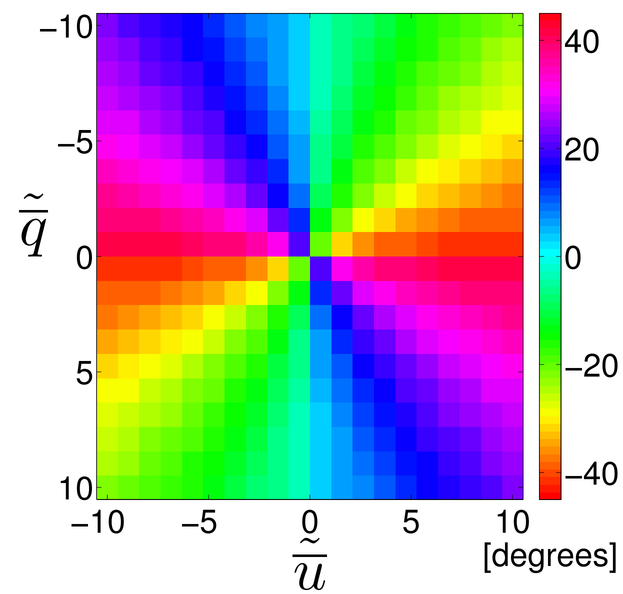
first moment

$$\bar{\theta}$$

Sample Statistics



Analytic Form

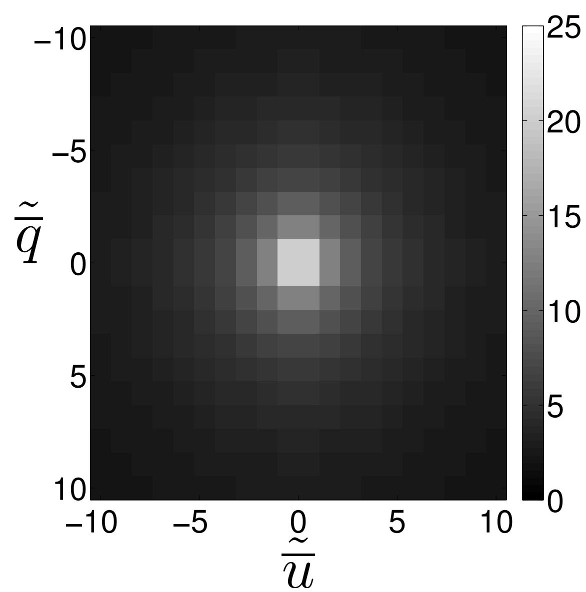


$$\tilde{u} = \frac{\bar{u}}{\sigma_{u,q}}$$

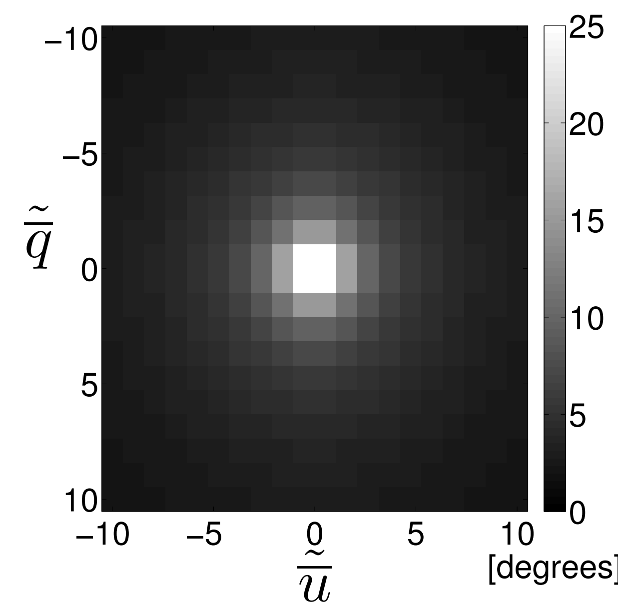
second moment

$$\sigma_\theta = \sigma_{AoLP}$$

Propagation of Error



Analytic Form



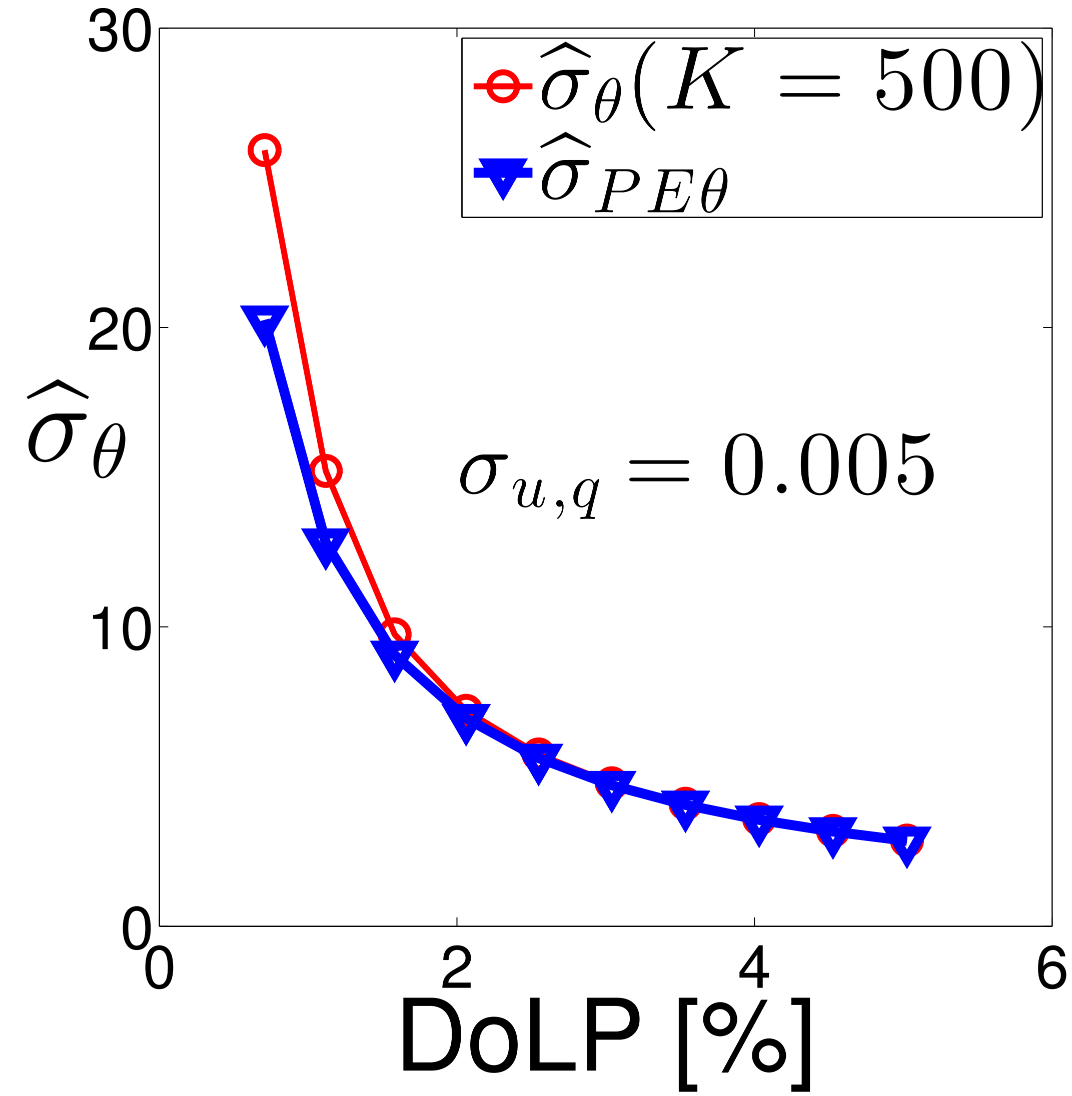
$$\tilde{q} = \frac{\bar{q}}{\sigma_{u,q}}$$

Propagation of Error for Estimating AoLP Variance

$$\sigma_\theta^2 \approx \sigma_u^2 \left(\frac{\partial \theta}{\partial u}\right)^2 + \sigma_q^2 \left(\frac{\partial \theta}{\partial q}\right)^2 .$$

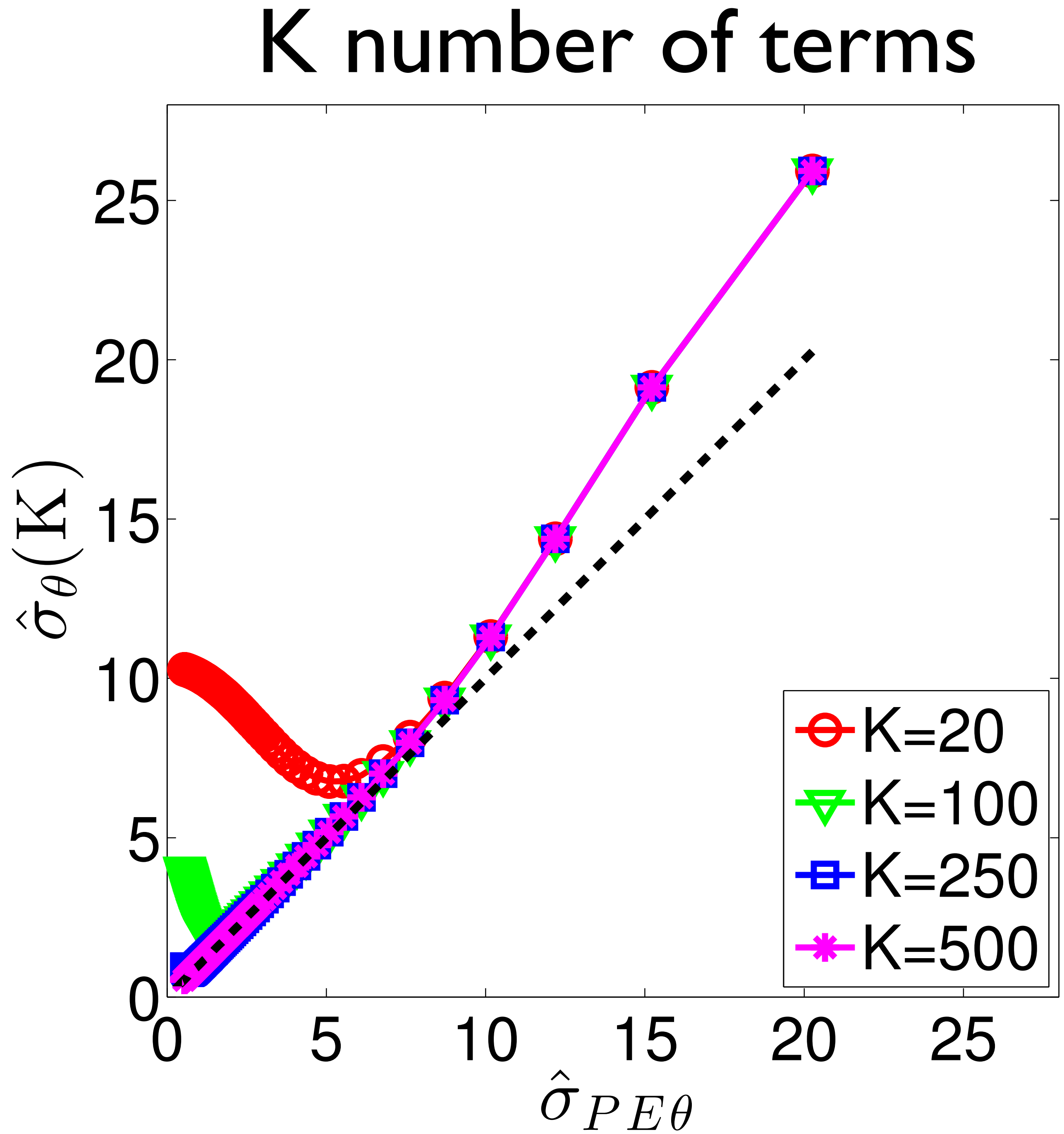
$$\hat{\sigma}^2_{PE\theta} = \frac{1}{4\tilde{d}^2}$$

- Two methods differ for very low DoLP and high measurement precision
- Propagation of error underestimates variance at low DoLP
- Statistical testing is most useful when signals are weak



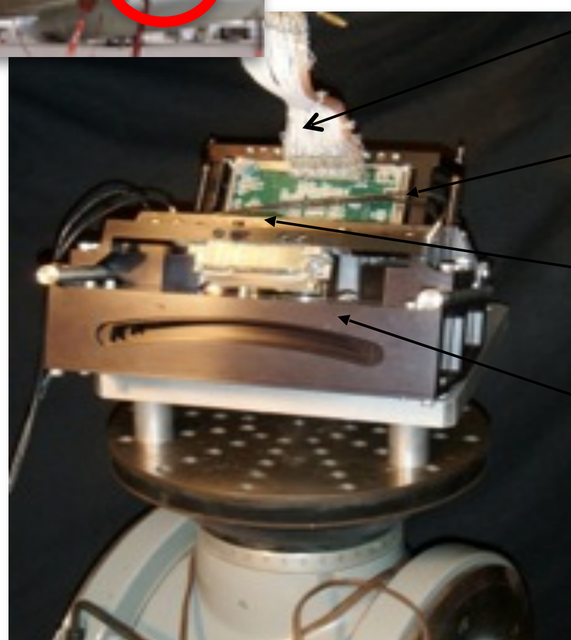
Convergence Properties of Analytic AoLP variance

- Analytic solution for AoLP moments is a Fourier Series of modulated Bessel functions. Converges around a few hundred terms



Multi-angle Spectro-Polarimetric Imager (MSPI)

Collaboratively designed/built with JPL
1st acquisition June 2010



Focal Plane Assembly board

Dual PhotoElastic Modulators (PEM)

Back side of mirror 2

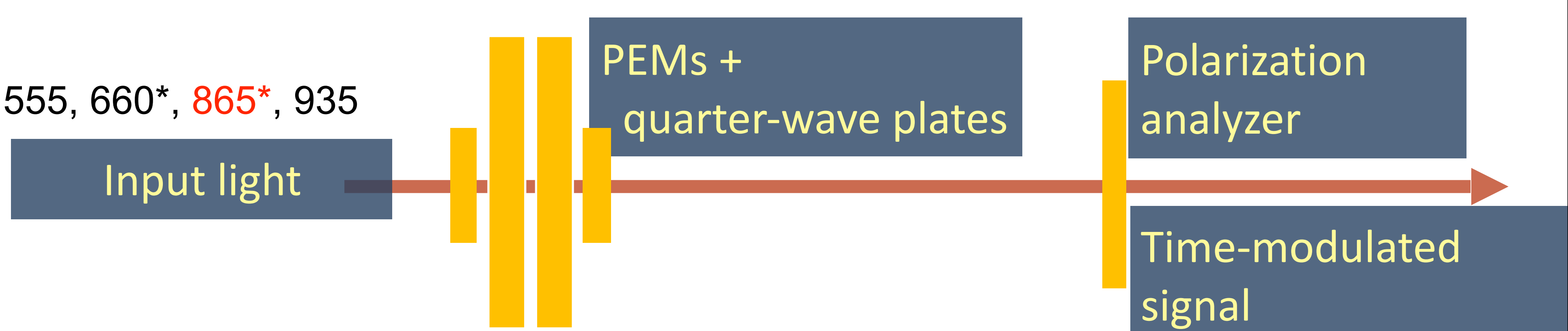
Entrance aperture with baffles

Designed to prevent common polarimeter jitter artifacts

D. J. Diner, A. Davis, B. Hancock, G. Gutt, R. A. Chipman, and B. Cairns, "Dual-photoelastic-modulator-based polarimetric imaging concept for aerosol remote sensing," Appl. Opt. 46, 8428-8445 (2007).

MSPI Specs ************************************* MSPI Hardware Schematic

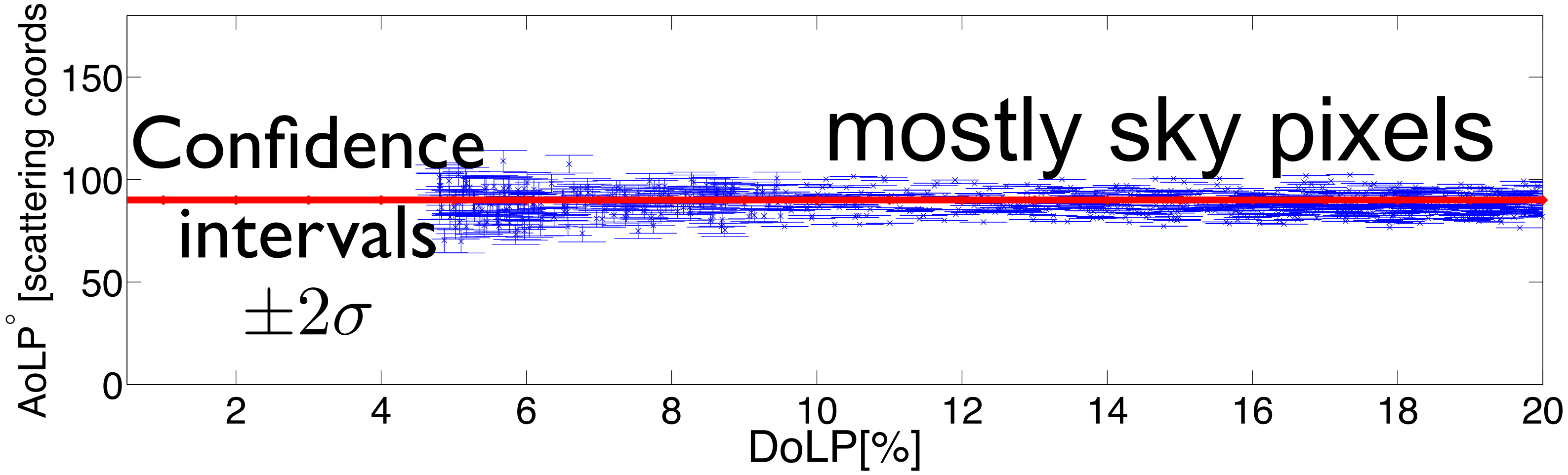
- pushbroom acquisition
- wavebands: 355, 380, 445, 470*, 555, 660*, 865*, 935
(* polarization band)
- polarimetric uncertainty: < 0.5%
- field of view: ±15° IFOV 0.02°
- ground resolution: 125m – 2.2km



AoLP Uncertainty Thresholding

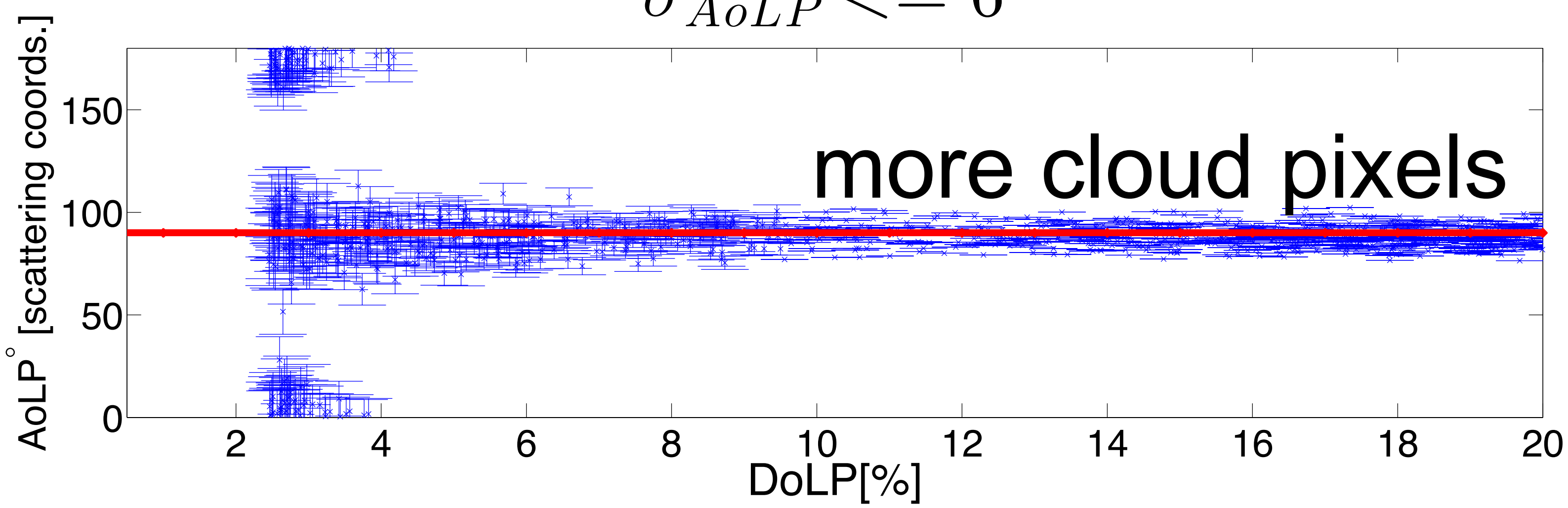
40 %
of image

$\sigma_{AoLP} <= 3^\circ$



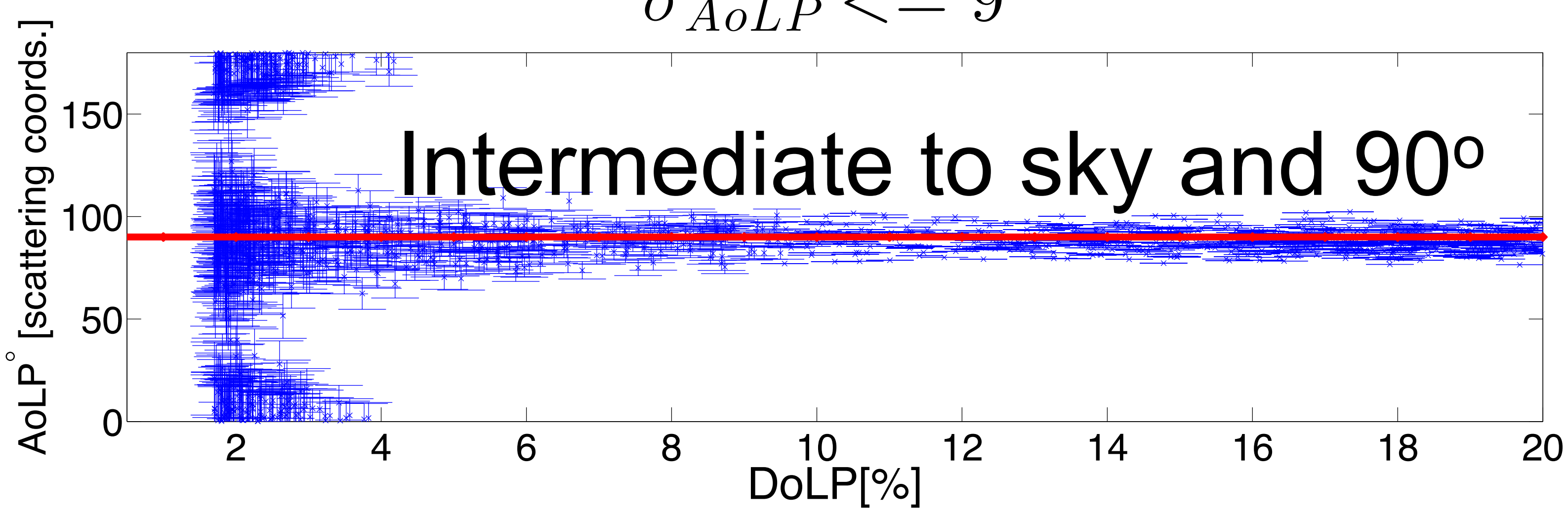
48 %
of image

$\sigma_{AoLP} <= 6^\circ$

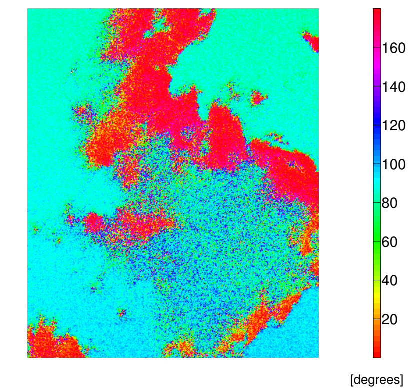


60 %
of image

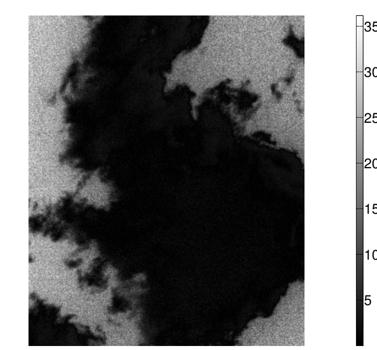
$\sigma_{AoLP} <= 9^\circ$



AoLP [°]



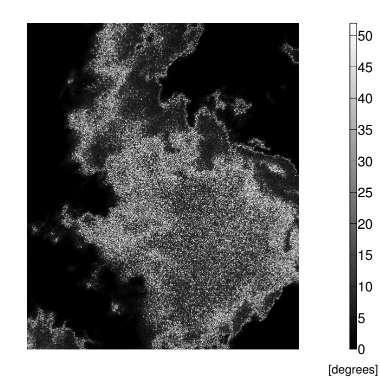
DoLP [%]



MEASUREMENTS

$\sigma_{u,q}$= 0.005 *A PRIORI*

σ_{AoLP} [°]



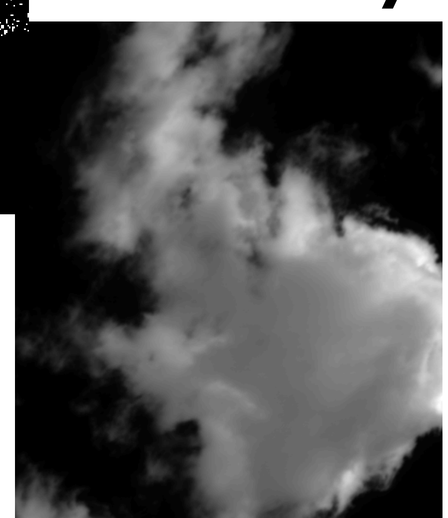
CALCULATE

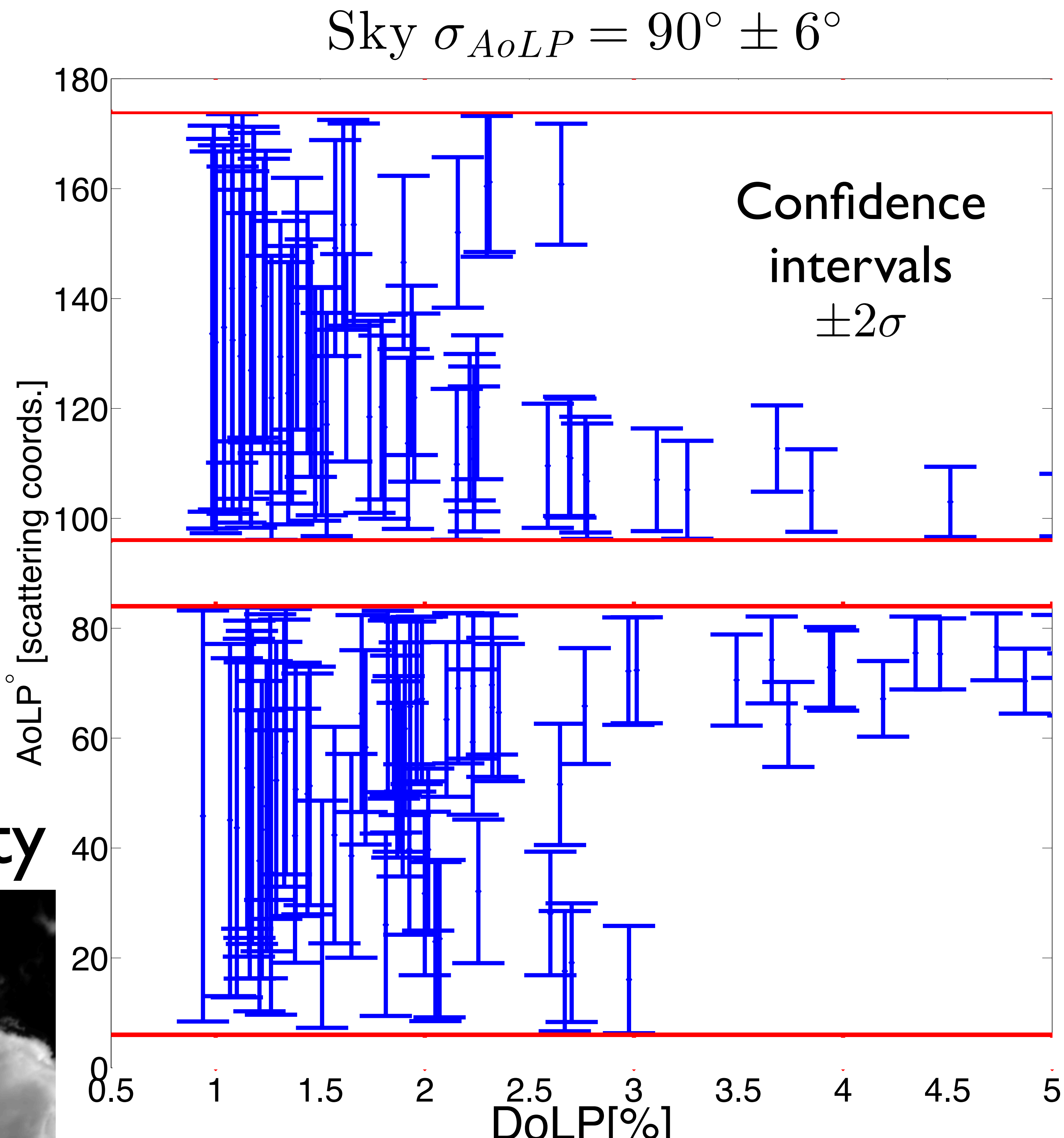
Detection of Multiple Scattering

Binary Mask of Pixel Locations

4 % of image

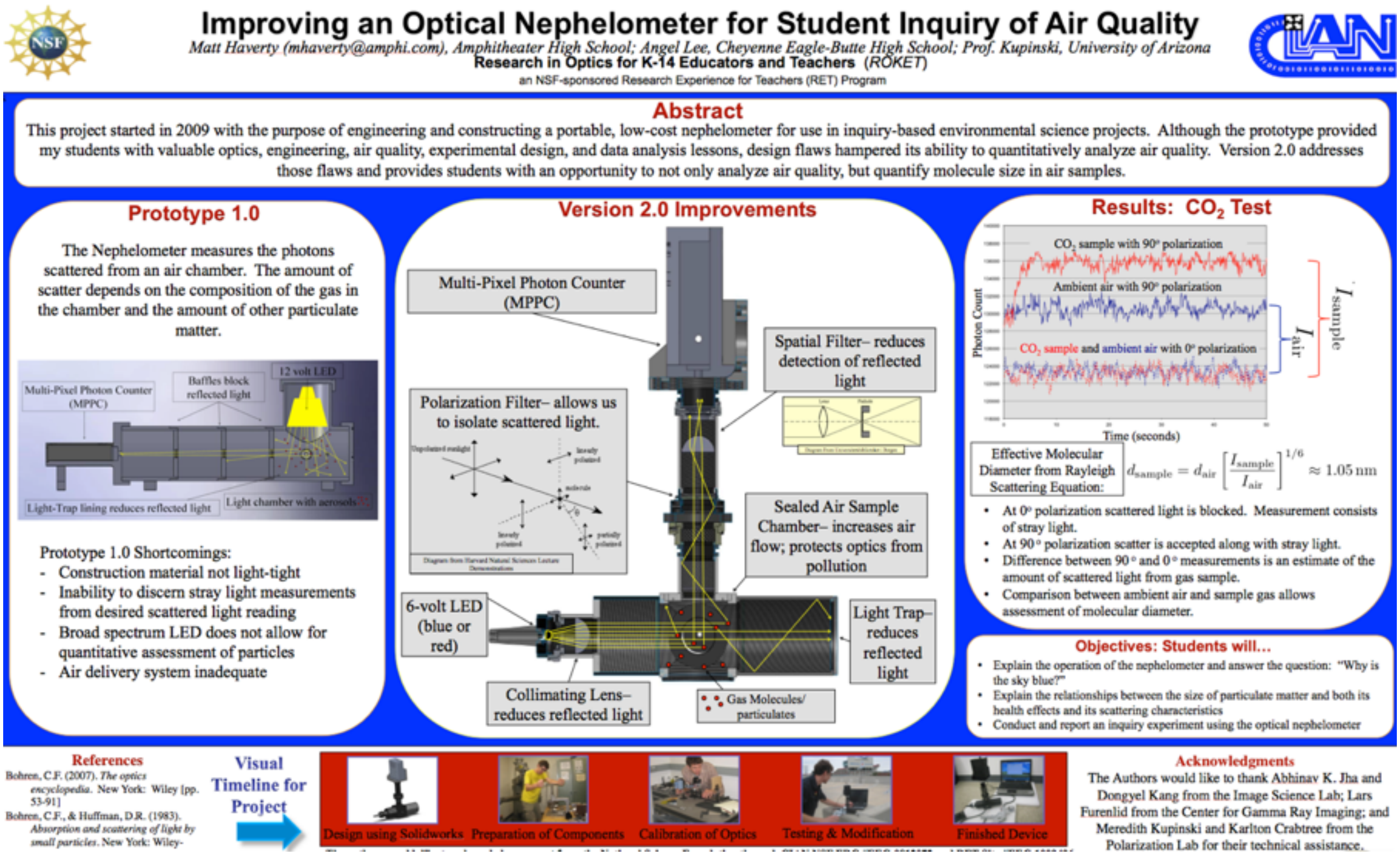
Intensity





CONCLUSIONS

- New method for calculating moments of AoLP from uncorrelated Gaussian measurements of the linear Stokes parameters
- Compared new analytic method to propagation of error and sample methods of estimating moments
- Example of AoLP statistical analysis for identifying multiple scattering events in MSPI cloud images



- UA senior design project to improve performance and packaging
- Teachers create original curriculum for student-led experiments
- Dissemination at American Indian Sci. Eng. Soc. (AISES) conference

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