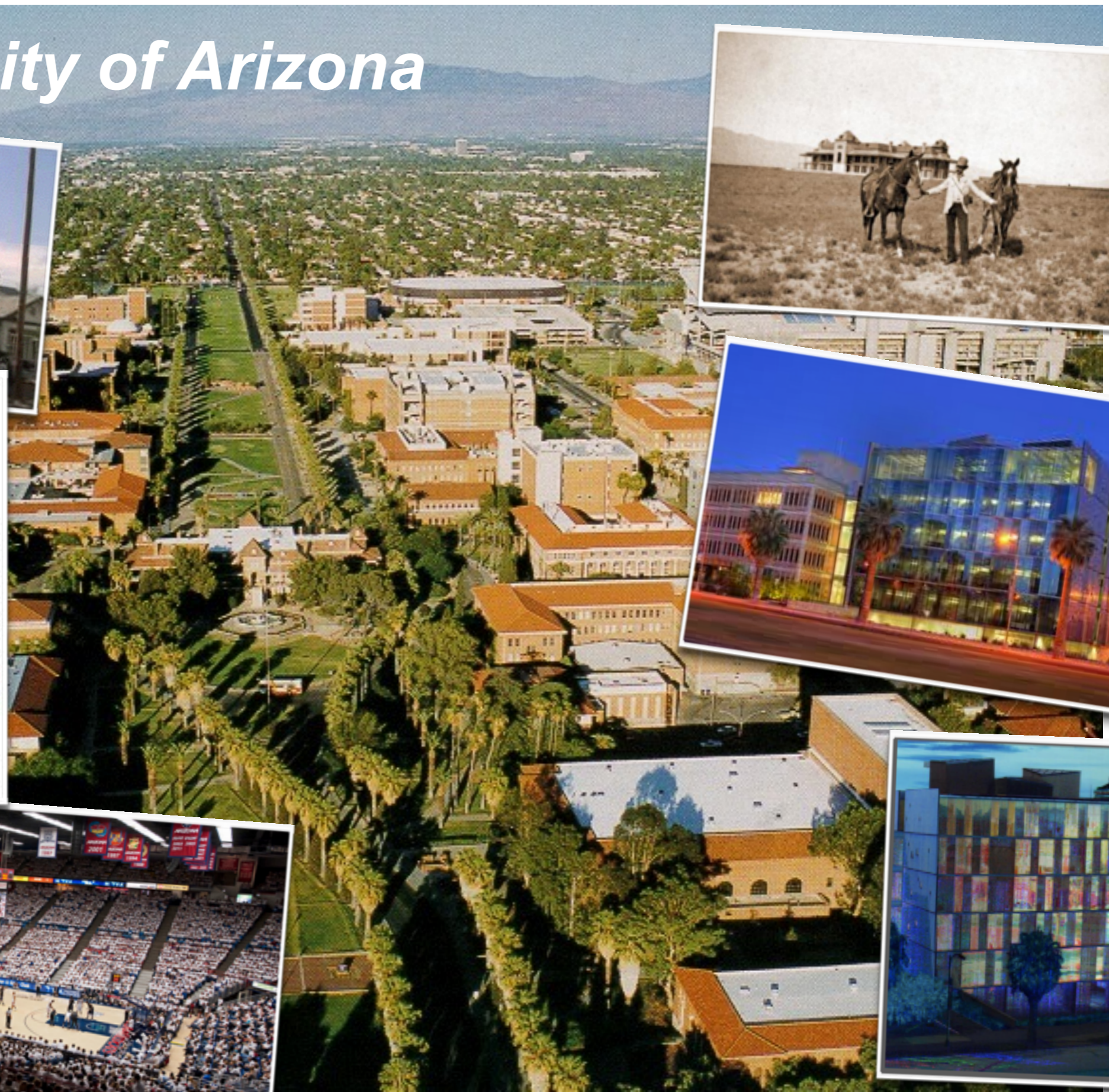
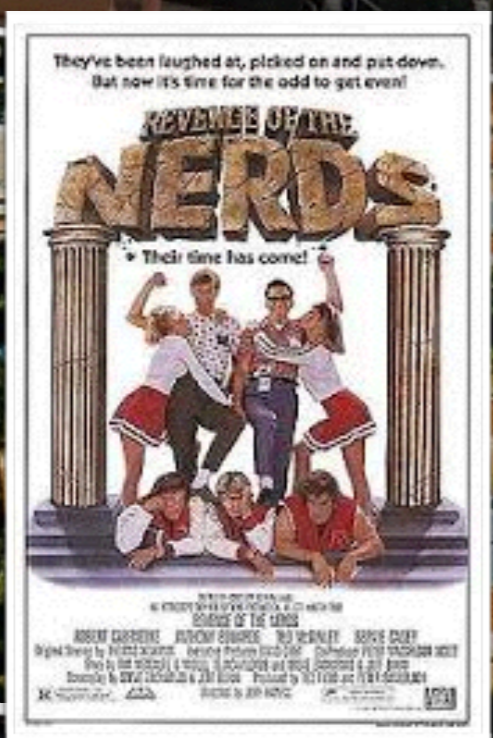
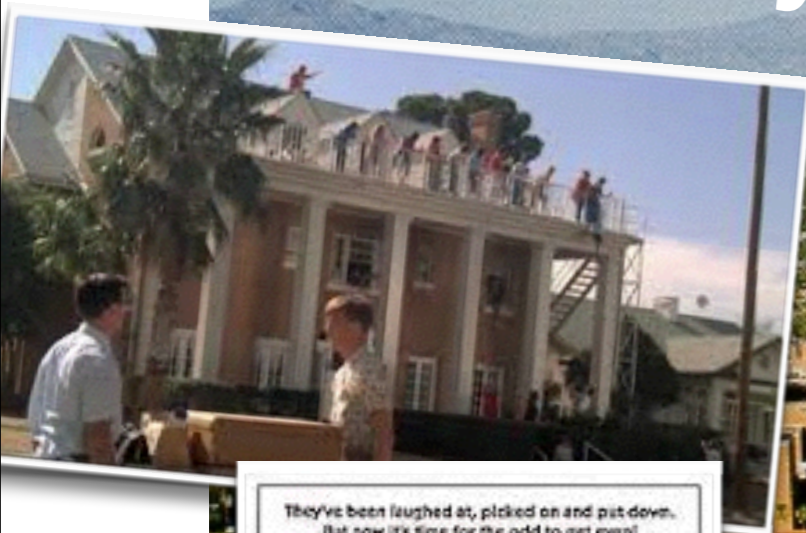


Relating the Statistics of the Angle of Linear Polarization (AoLP) to Measurement Uncertainty of the Stokes Vector

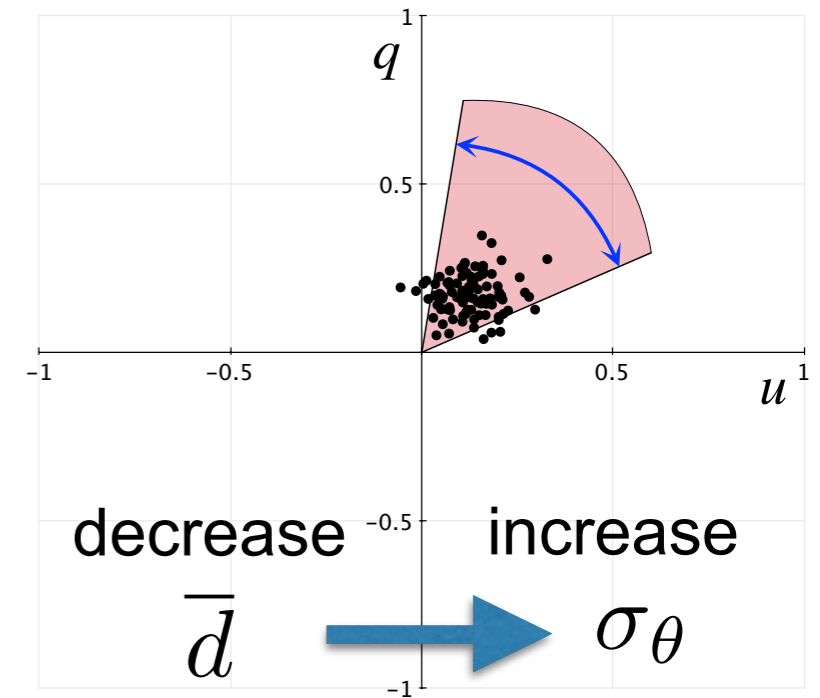
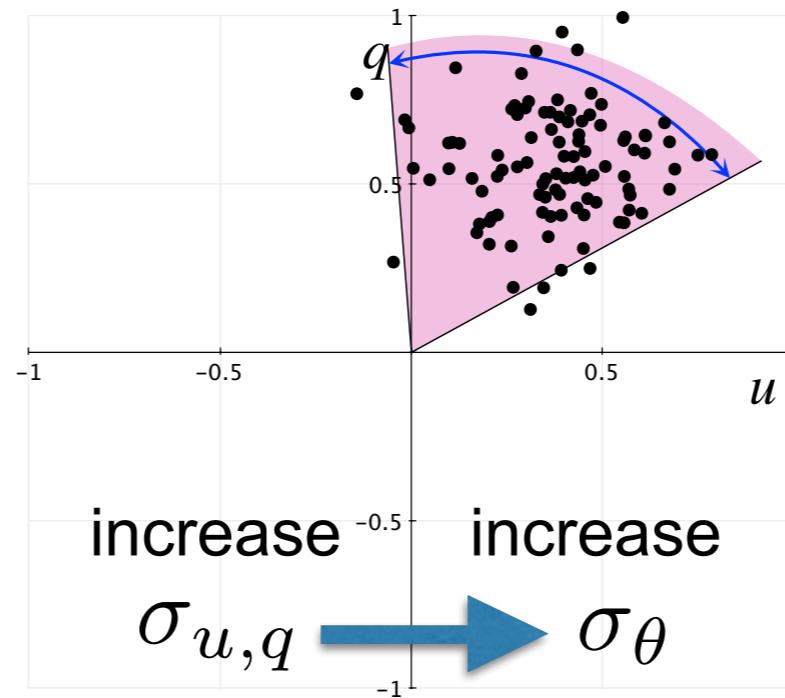
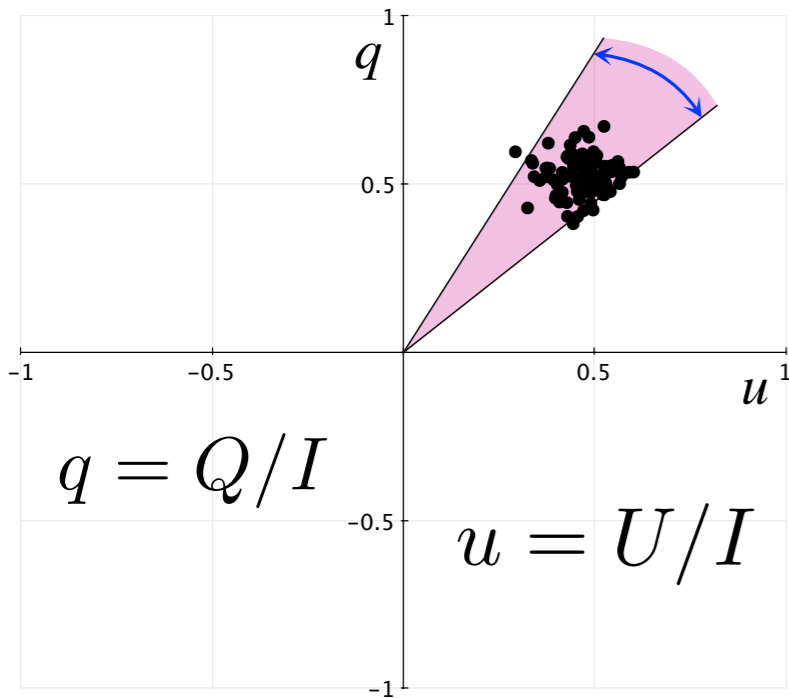
Meredith Kupinski
Assistant Research Professor
University of Arizona, College of Optical Sciences

Polarimetric Techniques & Technology Workshop
Lorentz Center, Leiden, the Netherlands
March 26, 2014

University of Arizona



Assumptions & Notation



$$\text{pr}(u) = \mathcal{N}(\bar{u}, \sigma_u^2)$$

$$\text{pr}(q) = \mathcal{N}(\bar{q}, \sigma_q^2)$$

$$\sigma_{u,q} = \sigma_q = \sigma_u$$

linear Stokes
parameters uncorrelated
Gaussian RVs

AoLP

$$\theta = \frac{1}{2} \text{atan}(u/q)$$

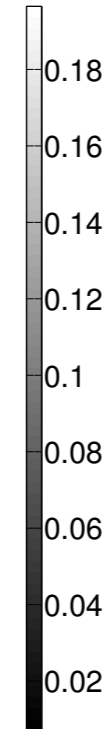
DoLP

$$\bar{d} = \sqrt{\bar{u}^2 + \bar{q}^2}$$

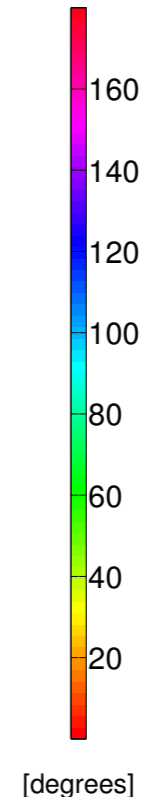
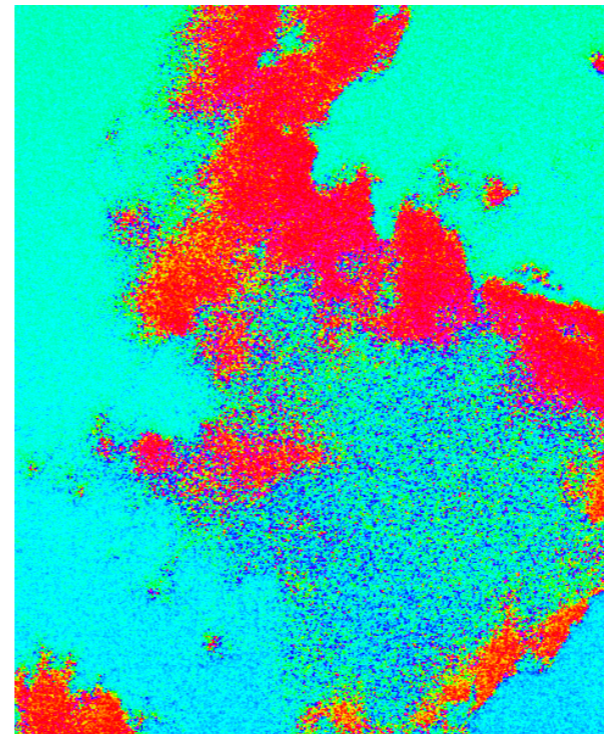
Utility of AoLP Statistical Analysis

- Hypothesis Testing: Are two samples different in AoLP?
- Quantitative: Given measurements what are confidence intervals on AoLP?

Are these measurements due to multiple scattering in the cloud or noise?



Intensity

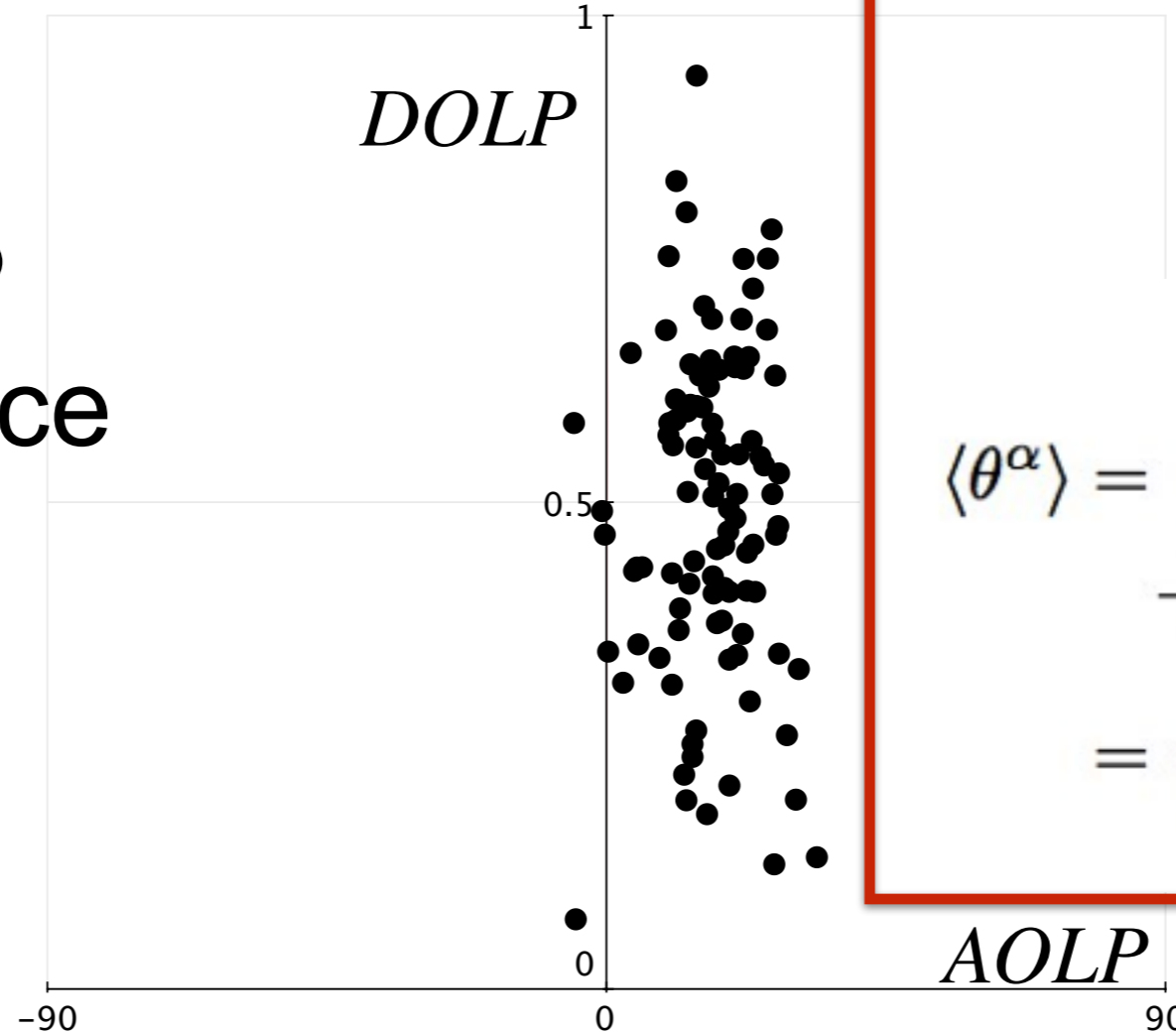


AoLP [°]

An 865 nm GroundMSPI cloud measurement acquired 08/16/2013 13:27(PST) at 32°N, 110°W. The resulting scattering angle over the FOV was 145°-159°.

Statistical Properties of AoLP

pdf of DoLP
Rayleigh-Rice



Analytic form for
AoLP moments

$$\begin{aligned} \langle \theta^\alpha \rangle &= \int_{-\pi/2}^{\pi/2} d\theta \text{pr}(\theta) \theta^\alpha \\ &= \frac{1}{2^{\alpha+1} \pi} \sum_{n=-\infty}^{\infty} B_n^\alpha e^{i n \bar{\phi}} f_n(\tilde{d}) \end{aligned}$$

pdf of AoLP
involves
error functions

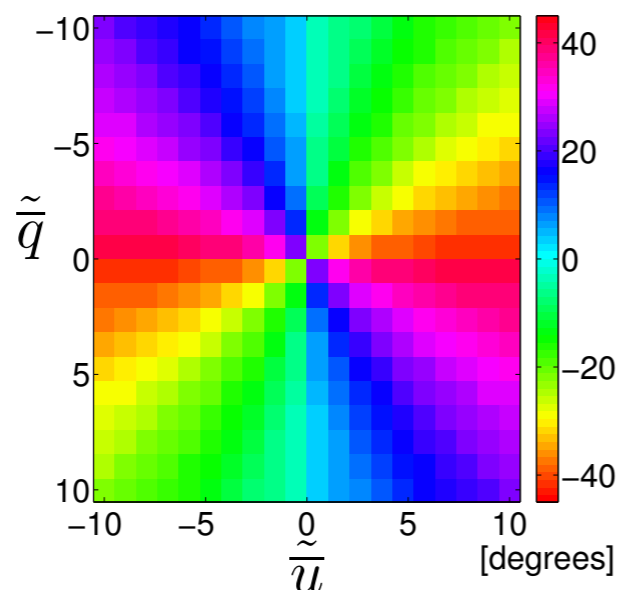
$$\begin{aligned} \text{pr}(\theta) &= \int_0^1 dp \text{pr}(\theta, p) = \\ &= \frac{1}{\pi} e^{-\frac{\tilde{d}^2}{2}} \int_0^\infty d\tilde{p} \tilde{p} e^{-\frac{\tilde{p}^2}{2}} e^{\tilde{p}(\tilde{q} \cos(2\theta) + \tilde{u} \sin(2\theta))} \end{aligned}$$

Choices for Estimating Moments of AoLP

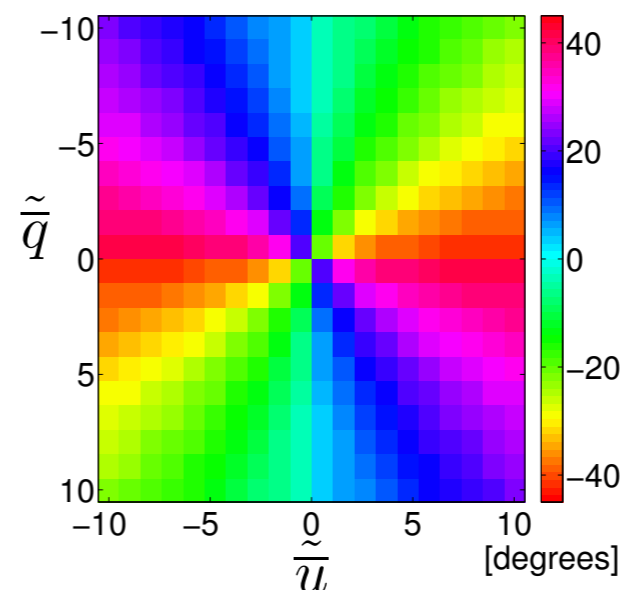
first moment

$$\bar{\theta}$$

Sample Statistics



Analytic Form

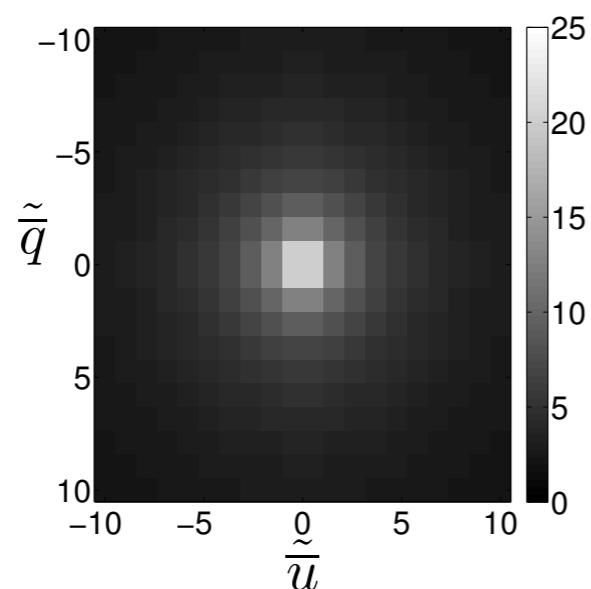


$$\bar{u} = \frac{\bar{u}}{\sigma_{u,q}}$$

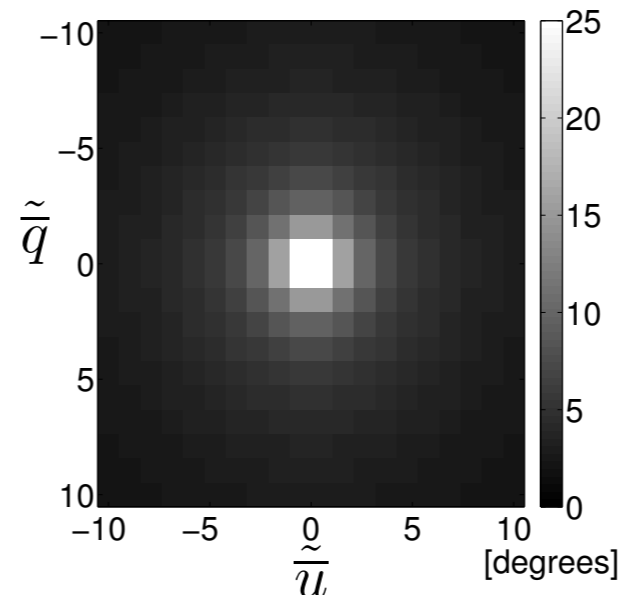
Propagation of Error

second moment

$$\sigma_{\theta} = \sigma_{AoLP}$$



Analytic Form



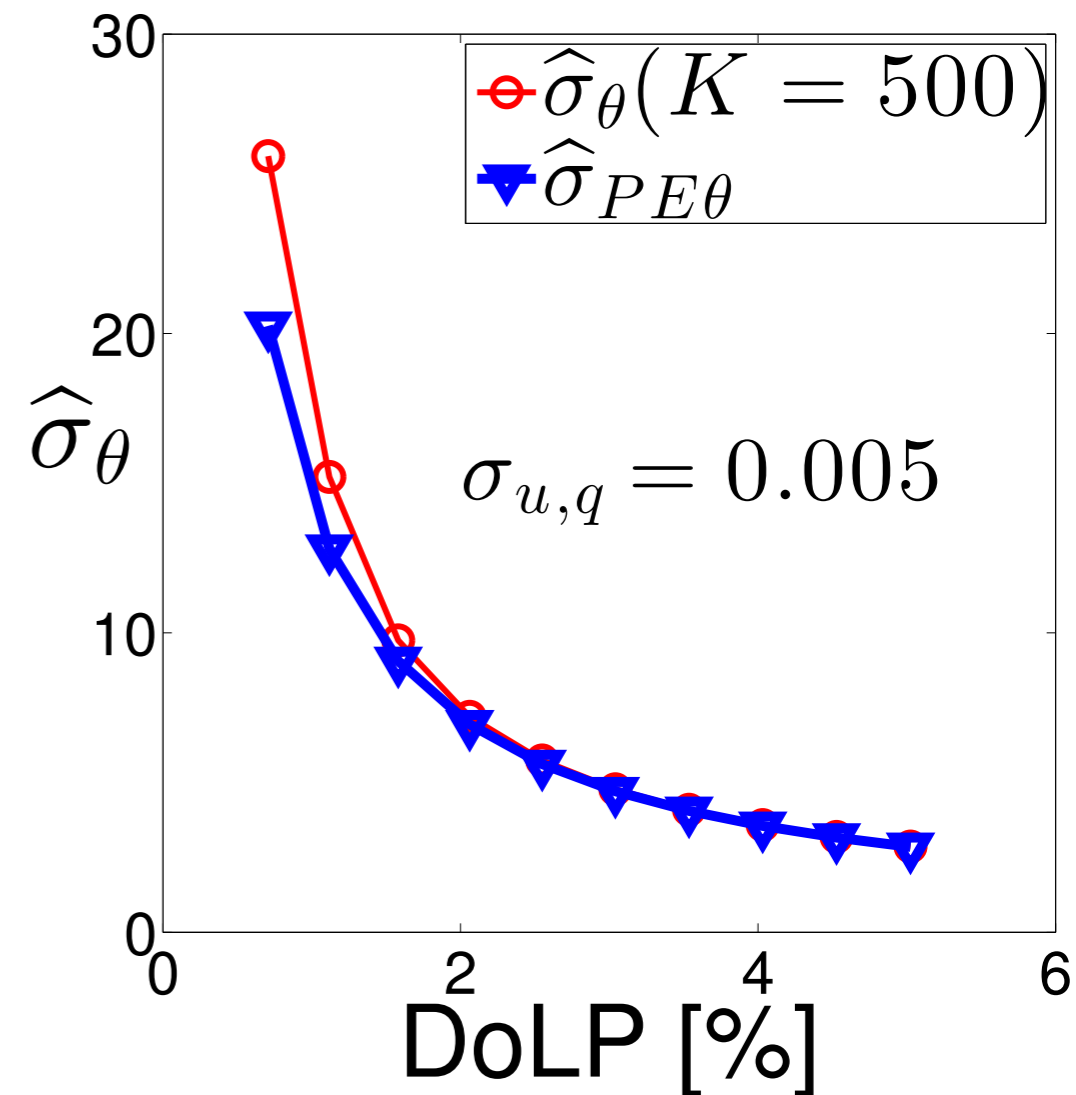
$$\bar{q} = \frac{\bar{q}}{\sigma_{u,q}}$$

Propagation of Error for Estimating AoLP Variance

$$\sigma_{\theta}^2 \approx \sigma_u^2 \left(\frac{\partial \theta}{\partial u} \right)^2 + \sigma_q^2 \left(\frac{\partial \theta}{\partial q} \right)^2 .$$

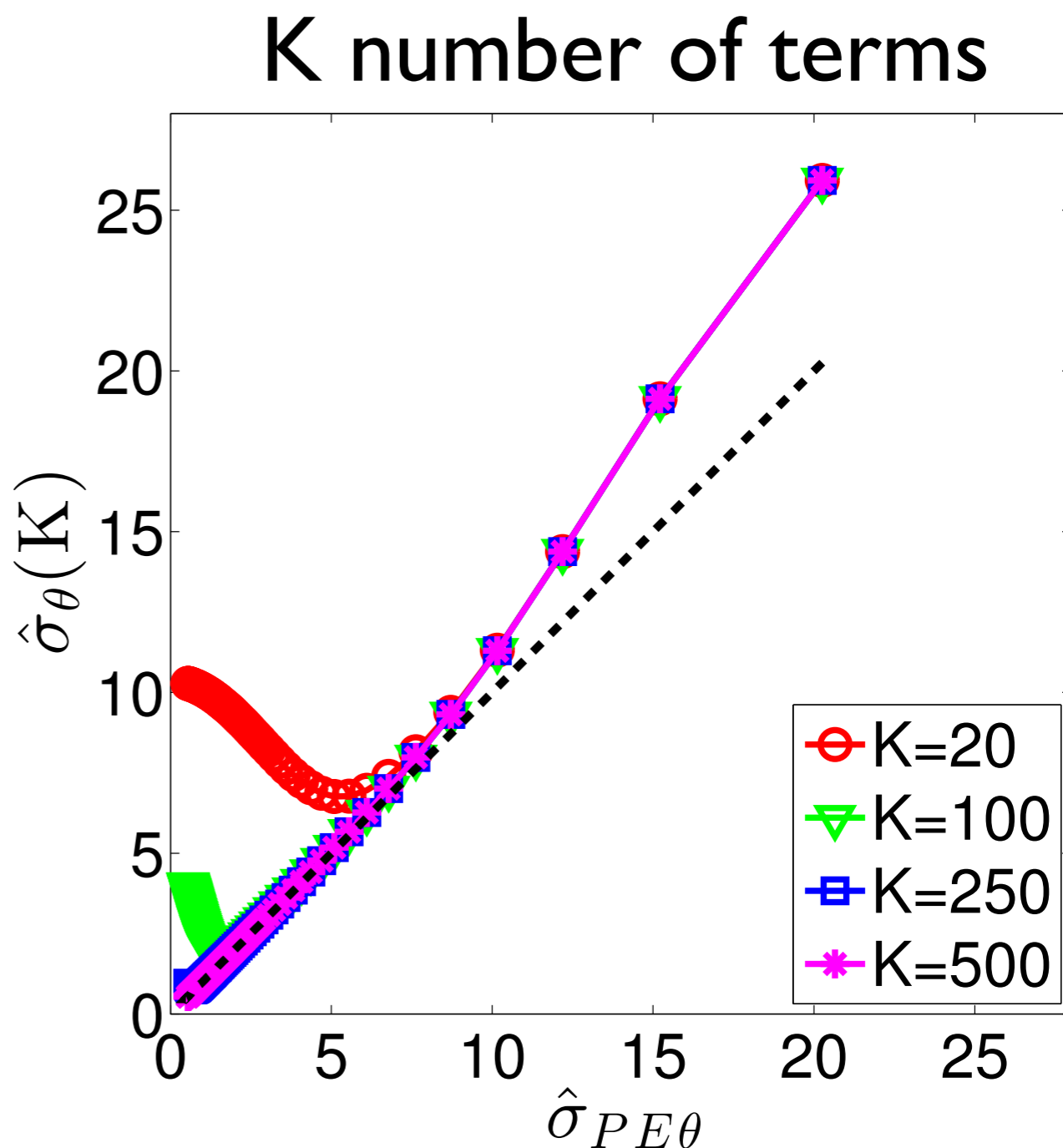
$$\hat{\sigma}_{PE\theta}^2 = \frac{1}{4\tilde{d}^2}$$

- Two methods differ for very low DoLP and high measurement precision
- Propagation of error underestimates variance at low DoLP
- Statistical testing is most useful when signals are weak



Convergence Properties of Analytic AoLP variance

- Analytic solution for AoLP moments is a Fourier Series of modulated Bessel functions. Converges around a few hundred terms



Multi-angle Spectro-Polarimetric Imager (MSPI)

Collaboratively designed/built with JPL
1st acquisition June 2010

Designed to prevent common polarimeter jitter artifacts

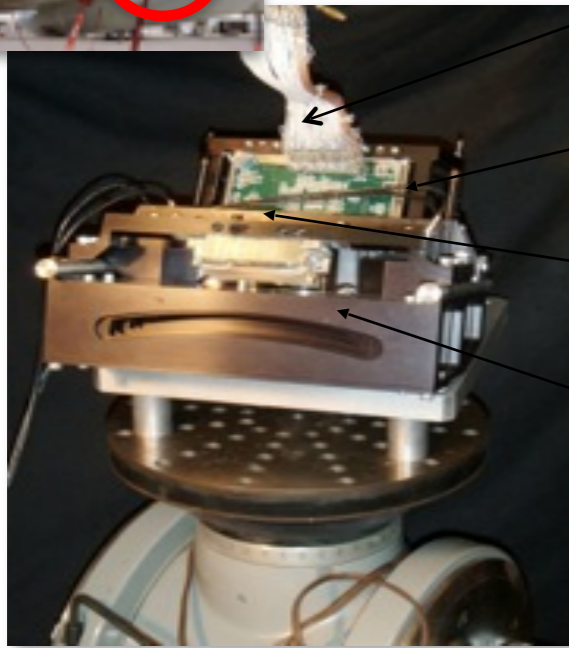
D. J. Diner, A. Davis, B. Hancock, G. Gutt, R. A. Chipman, and B. Cairns, "Dual-photoelastic-modulator-based polarimetric imaging concept for aerosol remote sensing," Appl. Opt. 46, 8428-8445 (2007).

Focal Plane Assembly board

Dual PhotoElastic Modulators (PEM)

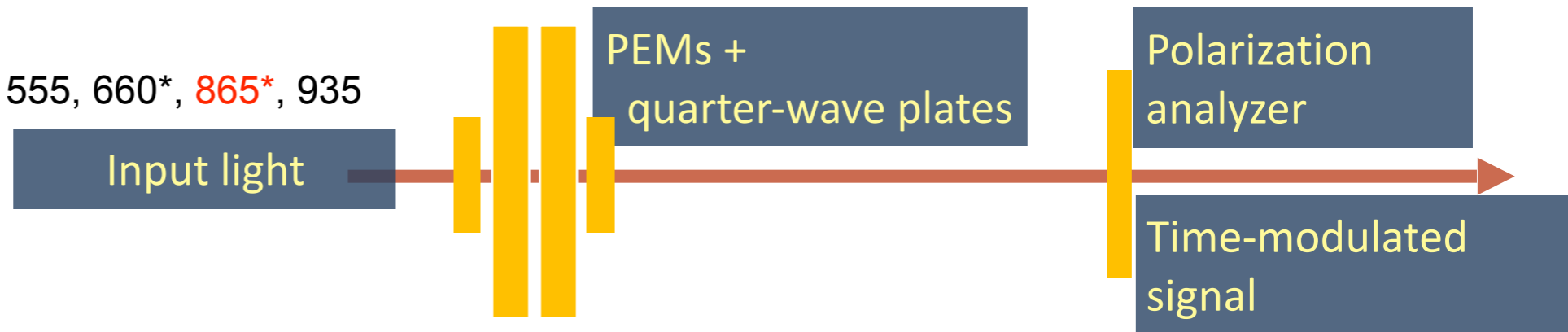
Back side of mirror 2

Entrance aperture with baffles



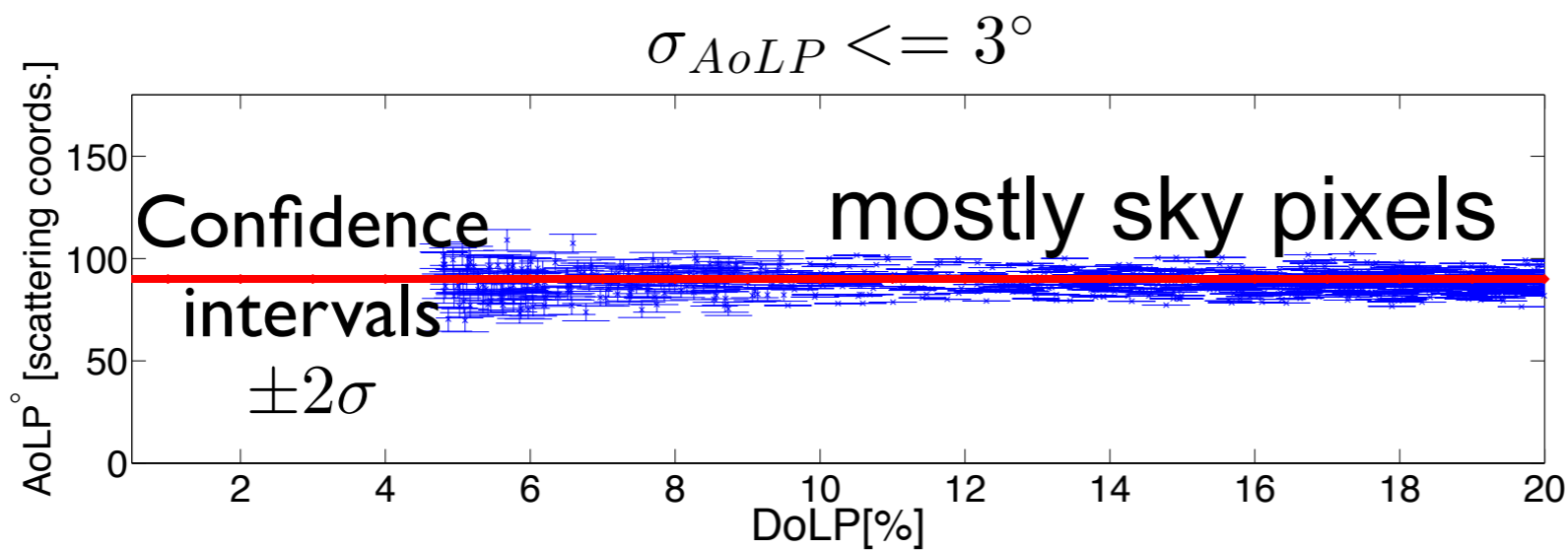
MSPI Specs ***** MSPI Hardware Schematic

- pushbroom acquisition
- wavebands: 355, 380, 445, 470*, 555, 660*, 865*, 935 (* polarization band)
- polarimetric uncertainty: < 0.5%
- field of view: $\pm 15^\circ$ IFOV 0.02°
- ground resolution: 125m – 2.2km

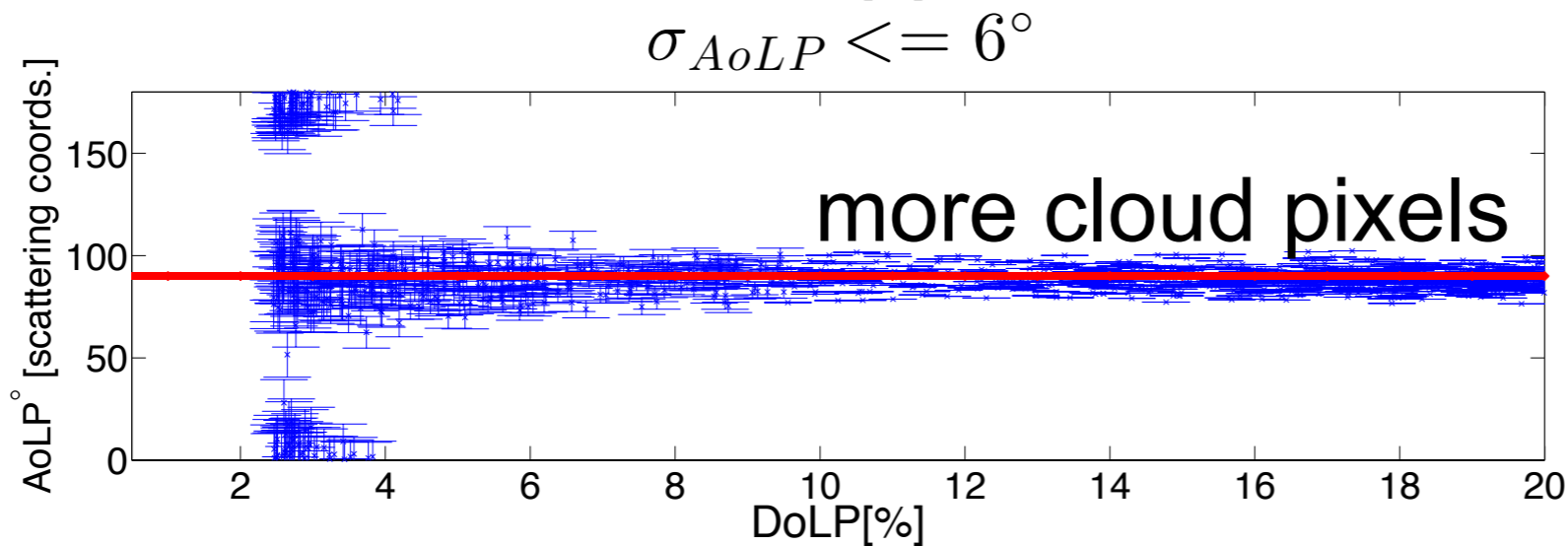


AoLP Uncertainty Thresholding

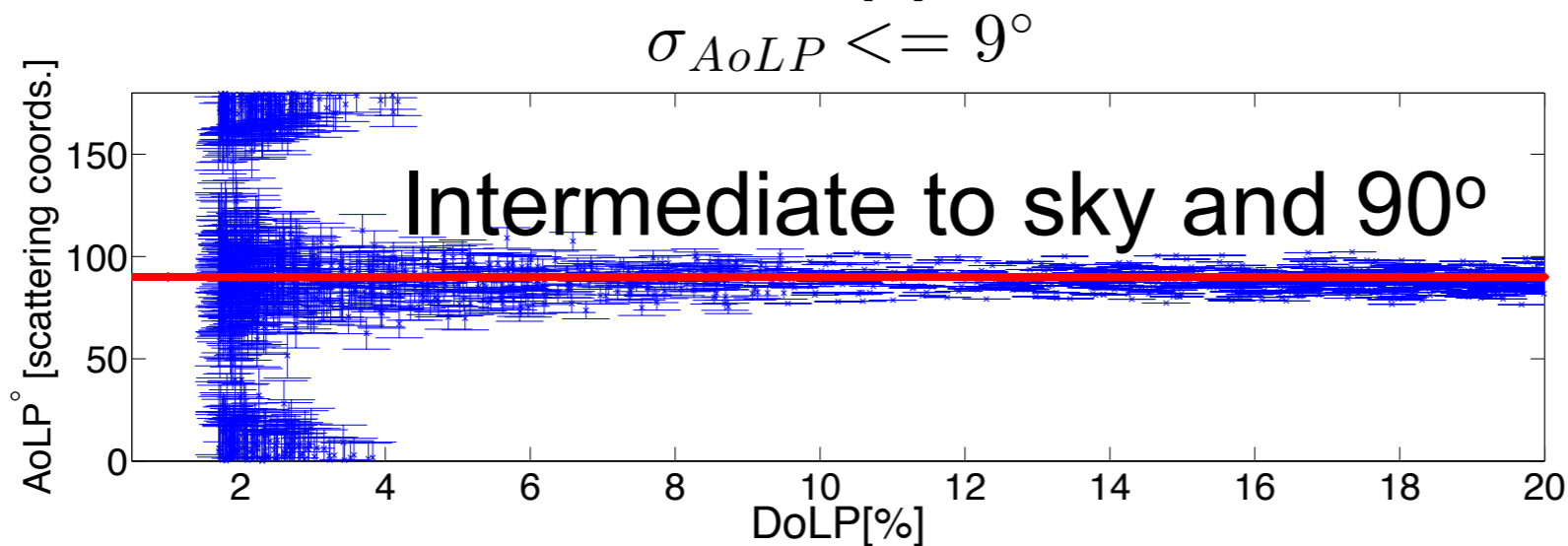
40 %
of image



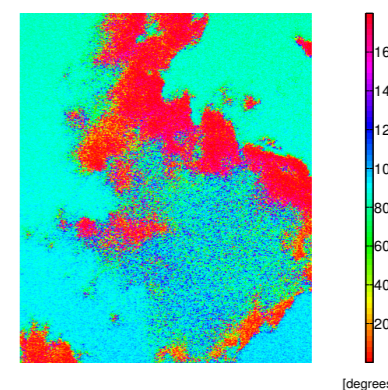
48 %
of image



60 %
of image

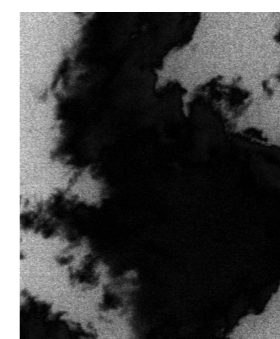


AoLP [$^\circ$]



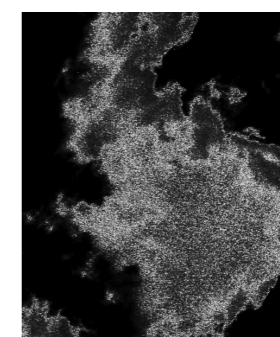
M
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DoLP [%]



$\sigma_{u,q} = 0.005$ A PRIORI

σ_{AoLP} [$^\circ$]



CALCULATE

Detection of Multiple Scattering

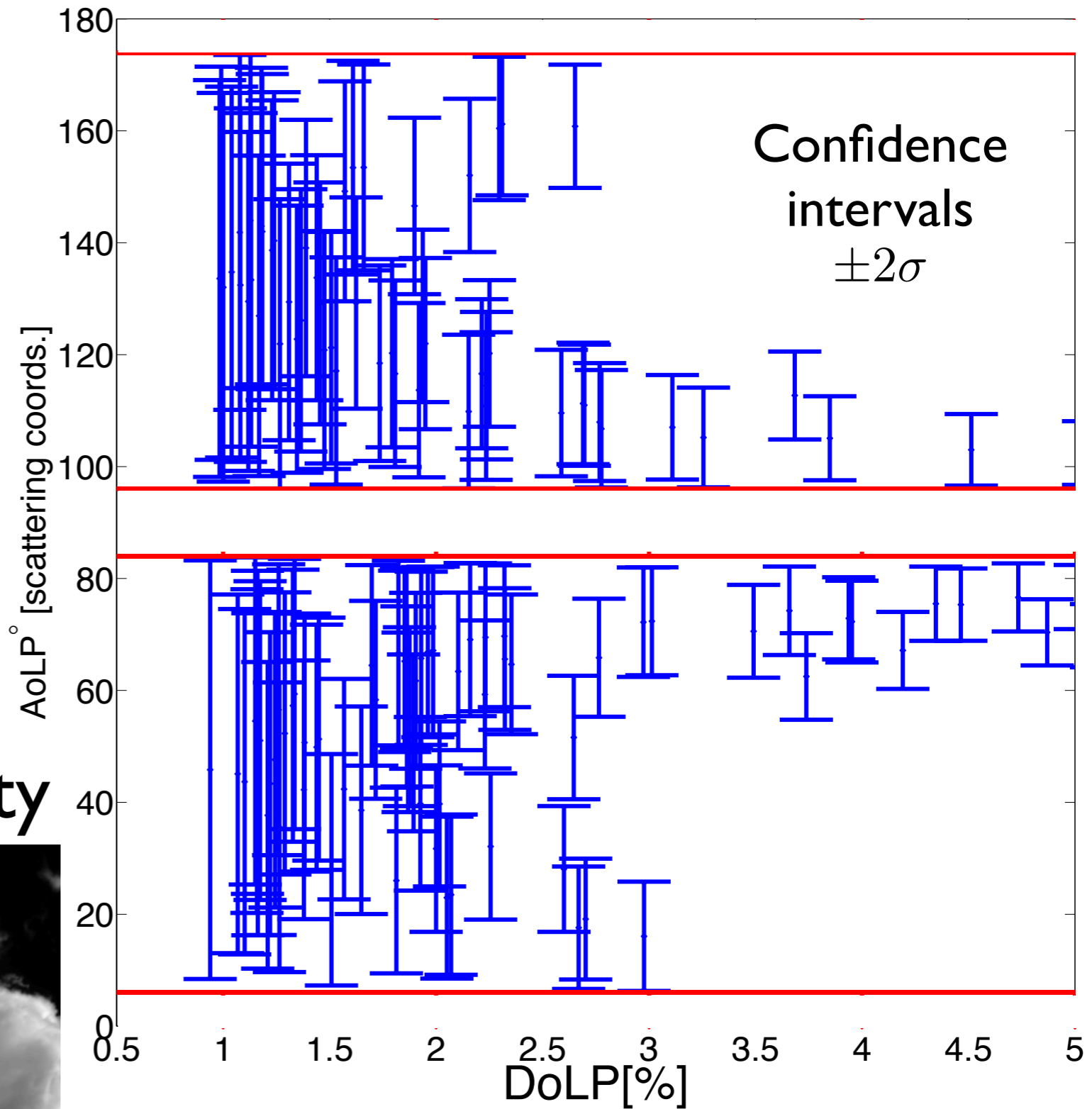
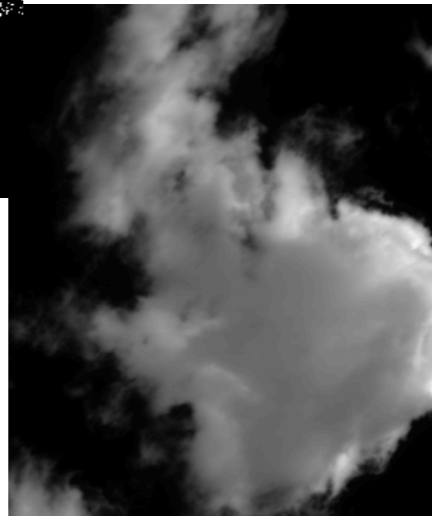
$$\text{Sky } \sigma_{AoLP} = 90^\circ \pm 6^\circ$$

Binary Mask of Pixel Locations



4 % of image

Intensity



CONCLUSIONS

- New method for calculating moments of AoLP from uncorrelated Gaussian measurements of the linear Stokes parameters
- Compared new analytic method to propagation of error and sample methods of estimating moments
- Example of AoLP statistical analysis for identifying multiple scattering events in MSPI cloud images



Improving an Optical Nephelometer for Student Inquiry of Air Quality

Matt Haverty (mhaverty@amphi.com), Amphitheater High School; Angel Lee, Cheyenne Eagle-Butte High School; Prof. Kupinski, University of Arizona

Research in Optics for K-14 Educators and Teachers (ROKET)

an NSF-sponsored Research Experience for Teachers (RET) Program

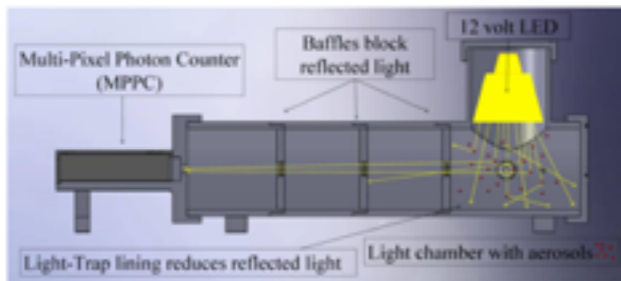


Abstract

This project started in 2009 with the purpose of engineering and constructing a portable, low-cost nephelometer for use in inquiry-based environmental science projects. Although the prototype provided my students with valuable optics, engineering, air quality, experimental design, and data analysis lessons, design flaws hampered its ability to quantitatively analyze air quality. Version 2.0 addresses those flaws and provides students with an opportunity to not only analyze air quality, but quantify molecule size in air samples.

Prototype 1.0

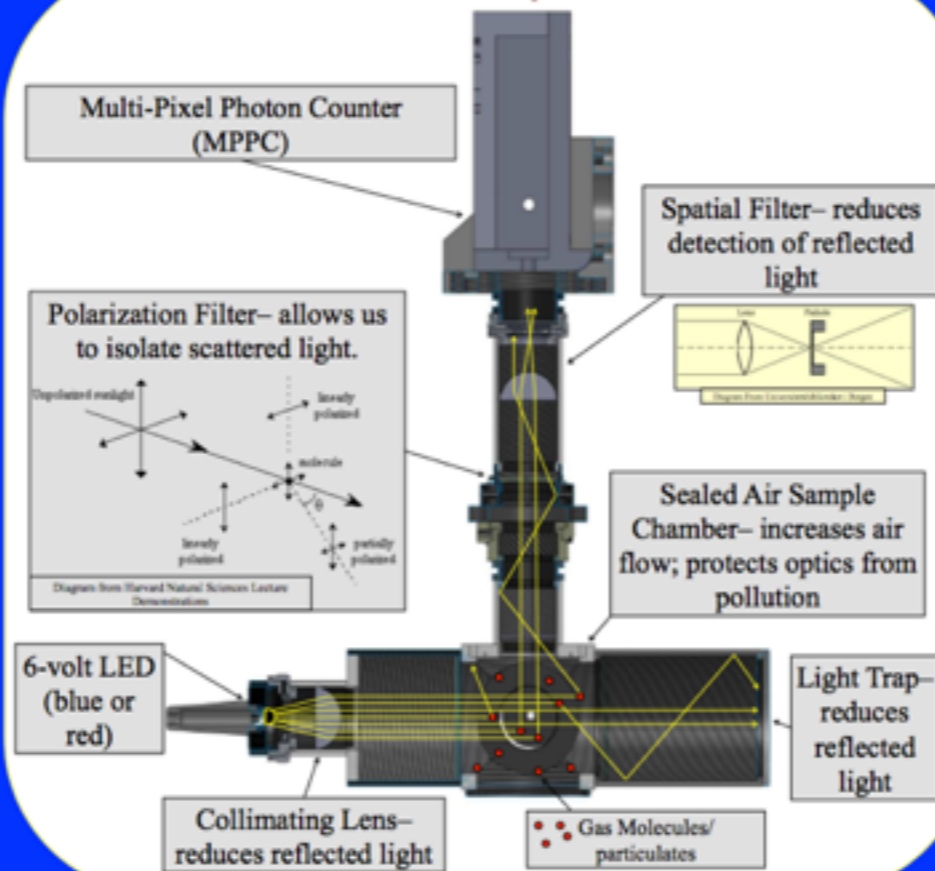
The Nephelometer measures the photons scattered from an air chamber. The amount of scatter depends on the composition of the gas in the chamber and the amount of other particulate matter.



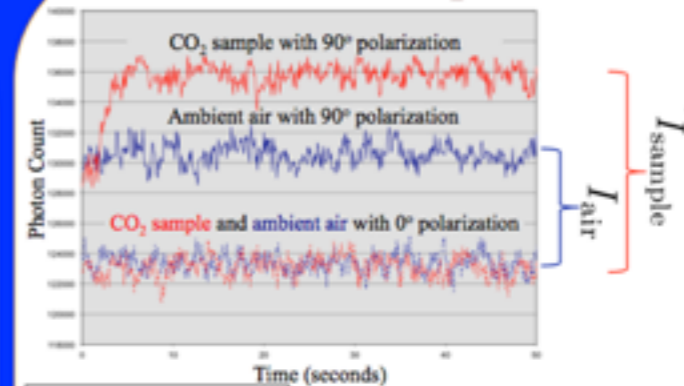
Prototype 1.0 Shortcomings:

- Construction material not light-tight
- Inability to discern stray light measurements from desired scattered light reading
- Broad spectrum LED does not allow for quantitative assessment of particles
- Air delivery system inadequate

Version 2.0 Improvements



Results: CO₂ Test



Effective Molecular Diameter from Rayleigh Scattering Equation: $d_{\text{sample}} = d_{\text{air}} \left[\frac{I_{\text{sample}}}{I_{\text{air}}} \right]^{1/6} \approx 1.05 \text{ nm}$

- At 0° polarization scattered light is blocked. Measurement consists of stray light.
- At 90° polarization scatter is accepted along with stray light.
- Difference between 90° and 0° measurements is an estimate of the amount of scattered light from gas sample.
- Comparison between ambient air and sample gas allows assessment of molecular diameter.

Objectives: Students will...

- Explain the operation of the nephelometer and answer the question: "Why is the sky blue?"
- Explain the relationships between the size of particulate matter and both its health effects and its scattering characteristics
- Conduct and report an inquiry experiment using the optical nephelometer

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Visual Timeline for Project



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- UA senior design project to improve performance and packaging
- Teachers create original curriculum for student-led experiments
- Dissemination at American Indian Sci. Eng. Soc. (AISES) conference

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