Polarization algebra: decomposition of depolarizing Mueller matrices

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The challenges of experimental polarimetry

Fast, reliable and easy-to-use Mueller matrix polarimeters have made possible the characterization of complex structures:

- **anisotropic materials, e.g. liquid crystals,**
- **biological and biomedical samples,**
- **micro- and nano-structures, patterns, etc.**

classic physical approach

Experimental Mueller matrix M (generally depolarizing)	⇄	*Model* relating the physical properties of the structure to M

The model complexity is proportional to that of the structure !

So, what to do when a model is not readily available ?

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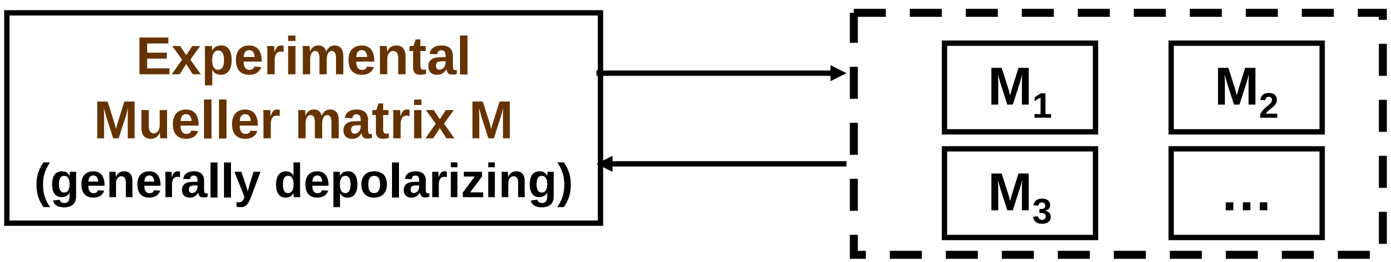
Why decompose?

- To reduce an arbitrary complex structure to a *set* of *simpler and familiar components* (*to simplify*)
- To get a better physical insight *without any preliminary information* (*to understand / to « feel »*)
- To obtain a *standard* « equivalent circuit » *parameterization* of any Mueller matrix (*to compare*)

phenomenological approach



Matrix decompositions are an universal phenomenological approach (whether a model exists or not)

The elementary building blocks

Basic polarization components:

name	**Diattenuator**	**Retarder**
action	change in *E amplitudes*	change in *E phases*
descriptor	a *symmetric* matrix $\mathbf{M_D}$	a *rotation* matrix $\mathbf{M_R}$
property	diattenuation vector D	retardance vector R
example	partial polarizer	waveplate

Example: ***plane surface*** **(ψ, Δ) :** $\mathbf{M_\psi\, M_\Delta = M_\Delta\, M_\psi}$

$$\begin{bmatrix} 1 & -\cos 2\psi & 0 & 0 \\ -\cos 2\psi & 1 & 0 & 0 \\ 0 & 0 & \sin 2\psi \cos\Delta & \sin 2\psi \sin\Delta \\ 0 & 0 & -\sin 2\psi \sin\Delta & \sin 2\psi \cos\Delta \end{bmatrix} = \begin{bmatrix} 1 & -\cos 2\psi & 0 & 0 \\ -\cos 2\psi & 1 & 0 & 0 \\ 0 & 0 & \sin 2\psi & 0 \\ 0 & 0 & 0 & \sin 2\psi \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\Delta & \sin\Delta \\ 0 & 0 & -\sin\Delta & \cos\Delta \end{bmatrix}$$

J. J. Gil and E. Bernabeu, *Optik* 76, 67 (1987)

Any *non-depolarizing* M is a sequence of these two blocks

The third block

name	**Depolarizer (diagonal or non-diagonal)**
action	non-deterministic change in *E amplitudes / phases*
descriptor	a *diagonal* $\mathbf{M}_{\Delta d}$ or a *special non-diagonal* $\mathbf{M}_{\Delta nd}$
property	depolarization index *DI* (depolarization)
example	suspension of scattering particles

two canonical forms

$$\mathbf{M}_{\Delta d} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{bmatrix} \qquad \mathbf{M}_{\Delta nd} = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{bmatrix}$$

If $DOP_{in} = 1$
& $DOP_{out} < 1$
M: *depolarizing*

A.V. Gopala Rao et al. *J. Mod. Opt.* 45, 989 (1998); R. Ossikovski, *J. Opt. Soc. Am. A* 27, 123 (2010)

Any *depolarizing* M is a sequence containing a depolarizer (together with the other building blocks: diattenuator, retarder)

Poincaré sphere mapping by depolarizing Mueller matrices

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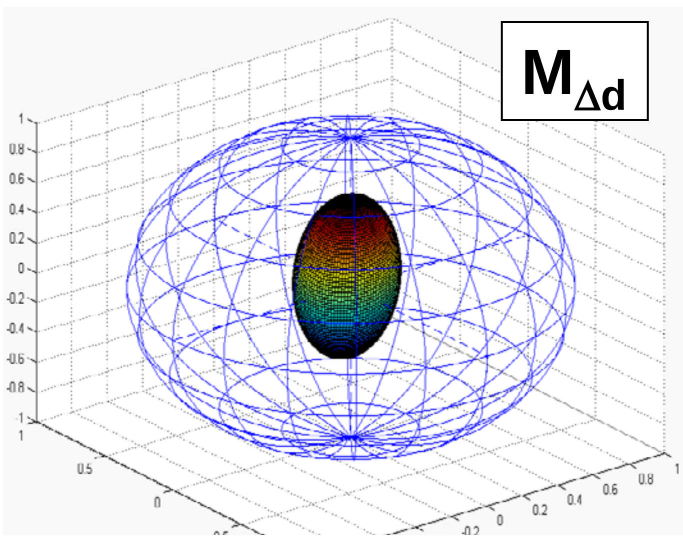
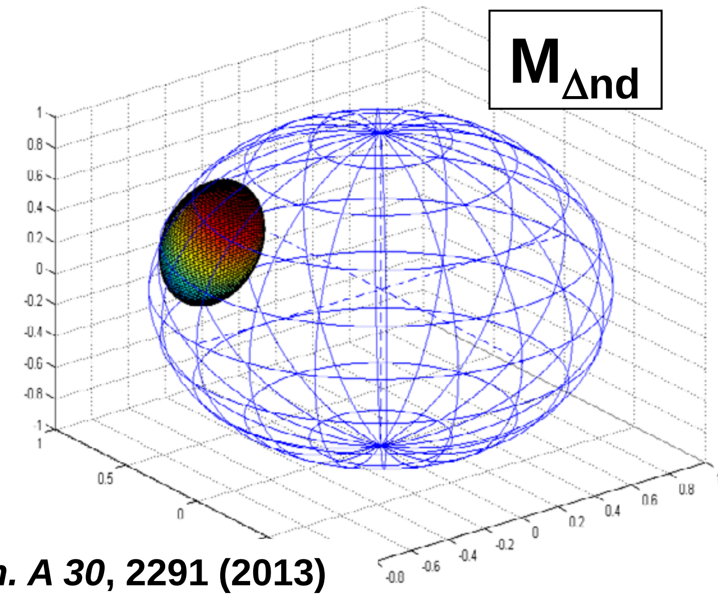
$S_{out}(\theta, \varphi) = M\, S_{in}(\theta, \varphi)$ -> DOP ellipsoid

$M \sim M_{\Delta d}$: *zero* or *two* contact points (*centered* ell.)

$M \sim M_{\Delta nd}$: *one* contact point (*eccentic* ell.)





R. Ossikovski, J. J. Gil , I. San José, *J. Opt. Soc. Am. A 30*, 2291 (2013)

Most generally, the DOP ellipsoid of a *depolarizing* M touches the Poincaré sphere in *zero*, *one* or *two* points: the existence of *a second (non-diagonal)* depolarizer type follows

« Nature-made » non-diagonal depolarizers

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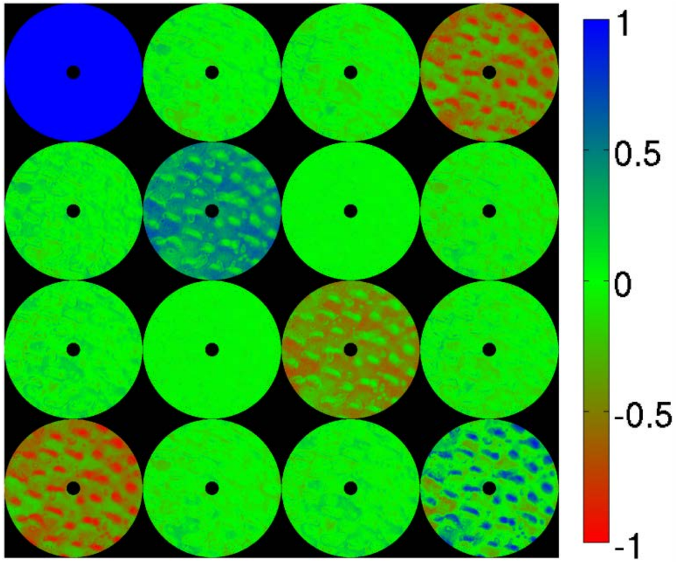
Cuticle area of *Cetonia aurata* beetle

two distinct zones : 1. perfect circular polarizer

2. non-diagonal depolarizer



$$\mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & -\alpha \\ 0 & \beta-\gamma & 0 & 0 \\ 0 & 0 & \gamma-\beta & 0 \\ -\alpha & 0 & 0 & 2\alpha-1 \end{bmatrix}$$

$$\left(\alpha \approx \beta \approx \tfrac{1}{2} \quad \gamma \approx 0\right)$$

R. Ossikovski et al., *Opt. Lett.* 34, 974 (2009); 34, 2426 (2009)

The incoherent mixture of a perfect (linear or circular) polarizer and a plane mirror produces a non-diagonal depolarizer

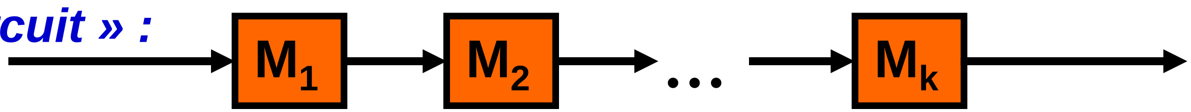
Product (serial, multiplicative) decompositions

Lu-Chipman (forward):	$M_{\Delta f}\ M_R\ M_D$
Reverse:	$M_D\ M_R\ M_{\Delta r}$
Symmetric:	$M_{D2}\ M_{R2}\ M_{\Delta}\ M_{R1}\ M_{D1}$

S.-Y. Lu and R.A. Chipman, *J. Opt. Soc. Am. A* 13, 1106 (1996)
R. Ossikovski et al., *Opt. Lett.* 32, 689 (2007)
R. Ossikovski, *J. Opt. Soc. Am. A* 26, 1109 (2009)

Note: unlike M_{Δ}, $M_{\Delta f}$ and $M_{\Delta r}$ are *not elementary depolarizers* (i.e., they can be further decomposed down to $M_{\Delta} \equiv M_{\Delta d}$ or $M_{\Delta nd}$)

« equivalent circuit » :



The product decompositions represent any depolarizing M as a series combination of the three basic components

Polarimetric imaging by symmetric decomposition

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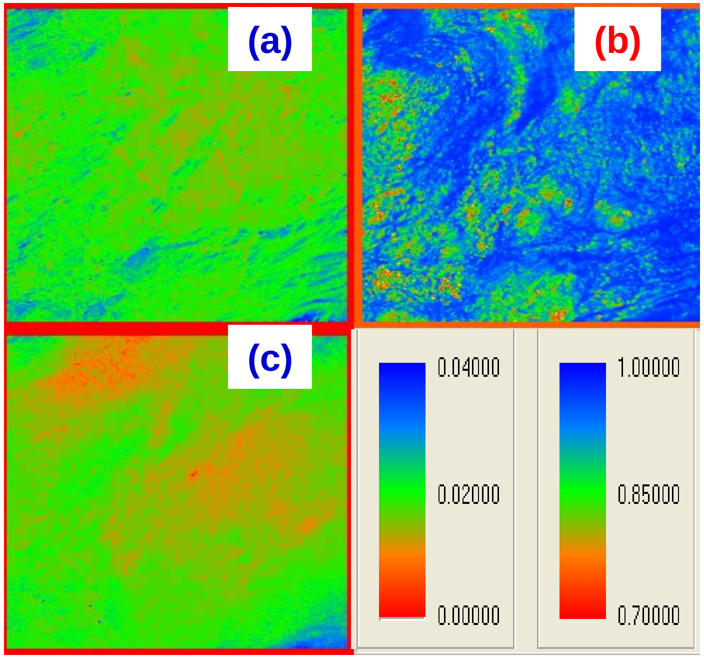
Meat slice sample: diattenuation – depolarization images



(a) Symmetric: 2nd factor ($\mathbf{M_{D2}}\ M_{\Delta d}\ M_{D1}$)
(b) Symmetric : depol ($M_{D2}\ \mathbf{M_{\Delta d}}\ M_{D1}$)
(c) Symmetric : 1st factor ($M_{D2}\ M_{\Delta d}\ \mathbf{M_{D1}}$)

***Observation*:**
(a) has *sharper* contrast than (c)

***Explanation*:**
Meat slice behaves *roughly* as $M_D\ M_{\Delta d}$

R. Ossikovski et al., *phys. stat. sol. (a)* 205, 720 (2008)

The *non-depolarizing* and the *depolarizing* components contain information about *different* sample properties

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Sum (parallel, additive) decompositions

eigenvalue / arbitrary

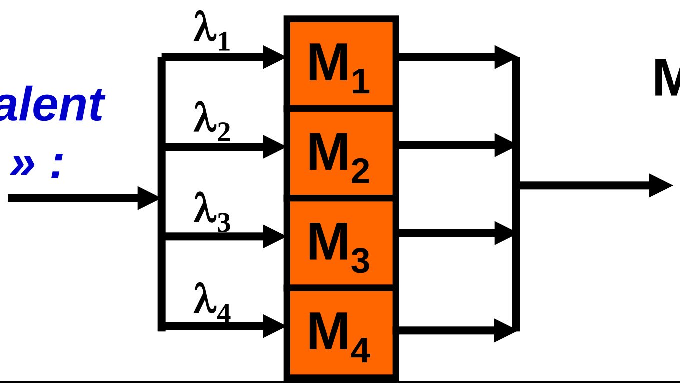
Cloude / Gil	$\lambda_1 M_1 + \lambda_2 M_2 + \lambda_3 M_3 + \lambda_4 M_4$, $\lambda_k > 0$

S.R. Cloude, *Optik* 75, 26 (1986)

J.J. Gil and I. San José, *J. Opt. Soc. Am. A* 30, 1078 (2013) and refs. therein

***Note*: the sum decompositions require *no* depolarizers**
(depolarization through incoherent addition)

« equivalent circuit » :



M_k : *non-depolarizing* components

(λ_i : eigenvalues of the coherency matrix of M)

The sum decompositions represent any depolarizing M as a parallel combination of non-depolarizing components M_k

Matrix filtering by virtual experiment method

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Polarimeters do not measure M, but rather *light intensities I*,

$$I = s_o^T M s_i$$

produced by in & out Stokes vectors s_i & s_o interacting with M

« virtual intensities » I *« filtered » non-depolarizing M_{nd}*

Minimize the norm $$\left| \mathbf{s}_o^T \mathbf{M} \mathbf{s}_i - \mathbf{s}_o^T \mathbf{M}_{nd} \mathbf{s}_i \right|^2$$

where

s_i and s_o are any two regular tetrahedron sets of states on the Poincaré sphere to get M_{nd} defined by a (7-parameter) Jones matrix J (to be determined)

R. Ossikovski, *Opt. Lett.* 37, 578 (2012)

H. Hu, R. Ossikovski, F. Goudail, *Opt. Express* 21, 5117 (2013)

Best method in presence of additive Gaussian and Poissonian noises appearing as « depolarization »

Continuous media: elementary polarization properties

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Defined from the differential matrix **m** *of a non-depolarizing Mueller matrix* ***M:***

$$\mathbf{m} = \begin{bmatrix} 0 & LD & LD' & CD \\ LD & 0 & CB & -LB' \\ LD' & -CB & 0 & LB \\ CD & LB' & -LB & 0 \end{bmatrix}$$

Dichroic / diattenuation (D) properties

Birefringent / retardance (B) properties

L: linear, 0° - 90°

L': linear, 45° - 135°

C: circular, R - L

e. g.

$$CB = \frac{2\pi\, l}{\lambda}(n_R - n_L)$$

$$CD = \frac{2\pi\, l}{\lambda}(k_R - k_L)$$

m *is G-antisymmetric, i.e.* **m = - G m^{T}G**
(**G** = *diag*(1, -1, -1, -1): Minkowski metric)

Propagation eq. for ***M*** *along z*

$$\frac{d\mathbf{M}}{dz} = \mathbf{m}\,\mathbf{M}$$

R. M. A. Azzam, *J. Opt. Soc. Am.* 68, 1756 (1978)

For a homogenenous non-depolarizing ***M:***

i.e. no z-dependence of m

$$\mathbf{M} = \exp(\mathbf{m}\, l) \Rightarrow \mathbf{m}\, l = \ln \mathbf{M}$$

(*l: optical path length*)

Fully describe the polarimetric response of a continuous medium

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Differential decomposition: the Mueller matrix logarithm

$$\mathbf{L} = \ln \mathbf{M} \qquad \mathbf{L} = \mathbf{L}_m + \mathbf{L}_u$$

L_m: *non-depolarizing* part (6 p.) L_u: *depolarizing* part (9 p.)

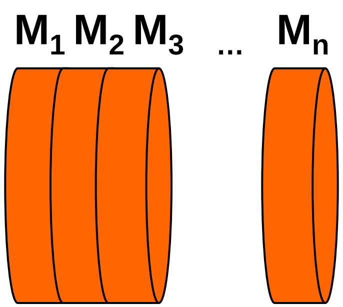
R. Ossikovski, *Opt. Lett.* 36, 2330 (2011)
generalizing R.M.A. Azzam, *J. Opt. Soc. Am.* 68, 1756 (1978)
R. Ossikovski, *Opt. Lett.* 37, 220 (2012)
N. Ortega-Quijano *et al*., *Opt. Express* 20, 1151 (2012)

L_m contains the *mean values* of the polarization properties

L_u contains the *uncertainties* of the properties (depolarizations)

« equivalent circuit » :

M_k : *depolarizing* infinitesimal slabs



$$\begin{bmatrix} 0 & \Delta p_1 & \Delta p_2 & \Delta p_3 \\ -\Delta p_1 & \alpha_1 & \Delta p_4 & \Delta p_5 \\ -\Delta p_2 & \Delta p_4 & \alpha_2 & \Delta p_6 \\ -\Delta p_3 & \Delta p_5 & \Delta p_6 & \alpha_3 \end{bmatrix}$$

α_i: three *anisotropic* depolarizations: LΔ, LΔ’, CΔ

The Mueller matrix logarithm represents any depolarizing M as *continuously depolarizing* along the optical path

Useful properties of the decompositions

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Product (serial) decompositions …

make possible the localization of the depolarization

separate the depolarizing and the non-depolarizing parts of M

Sum (parallel) decompositions …

allow to recover a physically realizable / nondepolarizing estimate of M from a non-realizable / depolarizing experimental M

Both classes of decompositions …

permit to retrieve the polarization properties of the medium from a depolarizing (experimental or simulated) M

The decompositions complement each other and generate different parameterizations describing different physical aspects of M

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Conclusions and outlook

The decompositions appear as a powerful tool for the analysis of experimental Mueller matrices without any model or preliminary information

They are not just plain data-reduction methods but rather allow for a deeper physical insight

We are only in the beginning of experimental polarimetry, so use them and abuse them to get the most of them!

THANK YOU !