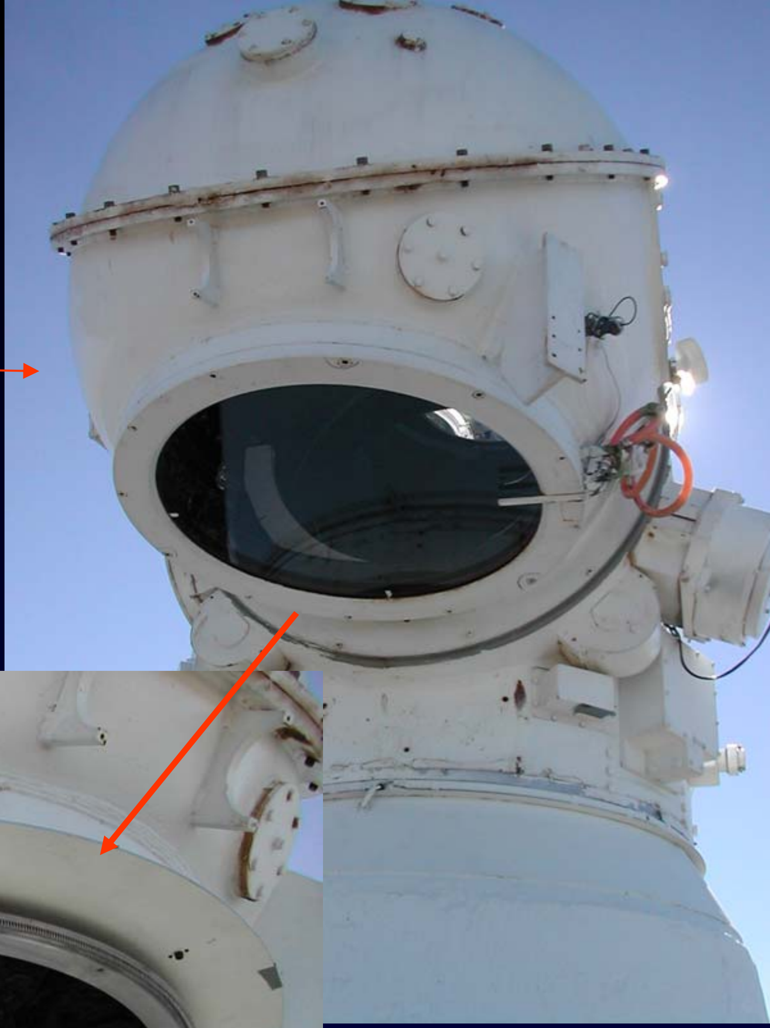
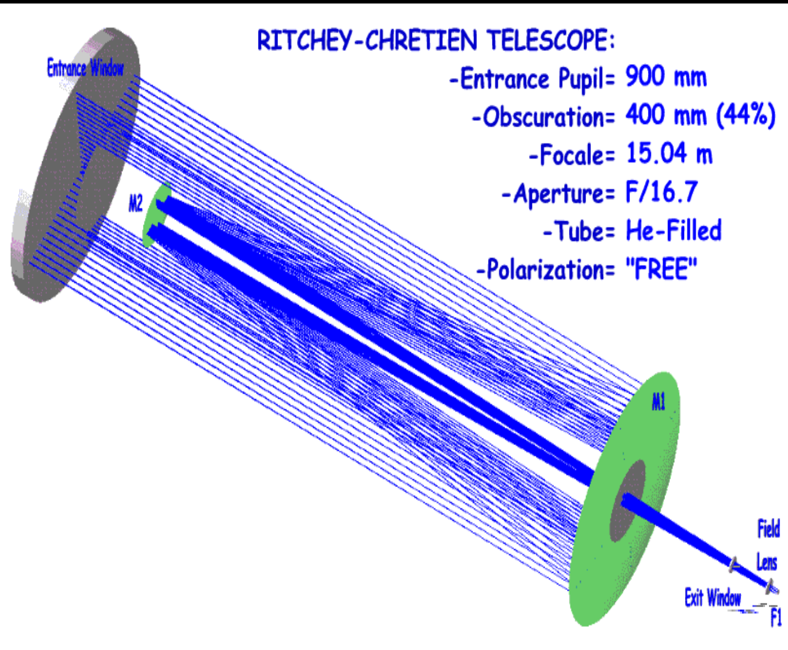
Polarimetry through a Nasmyth

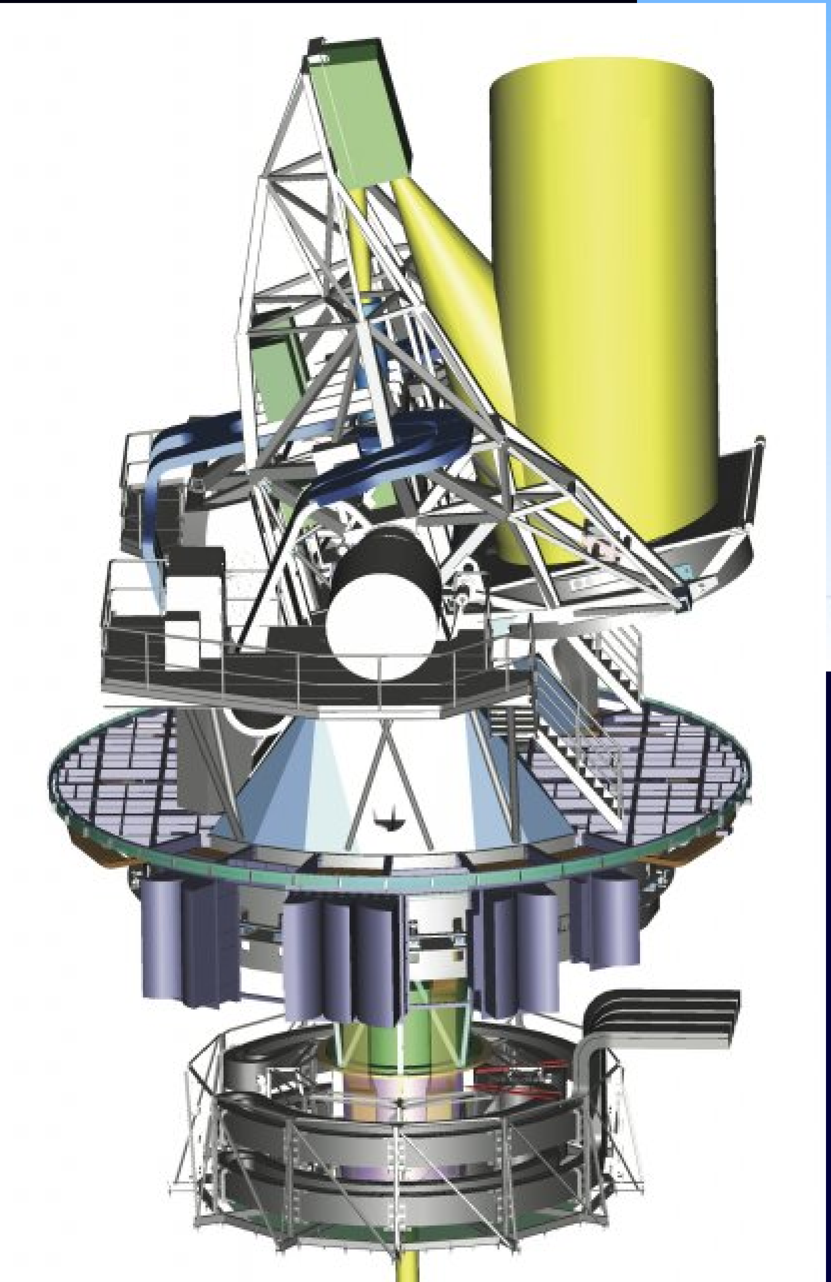
playing around with matrices

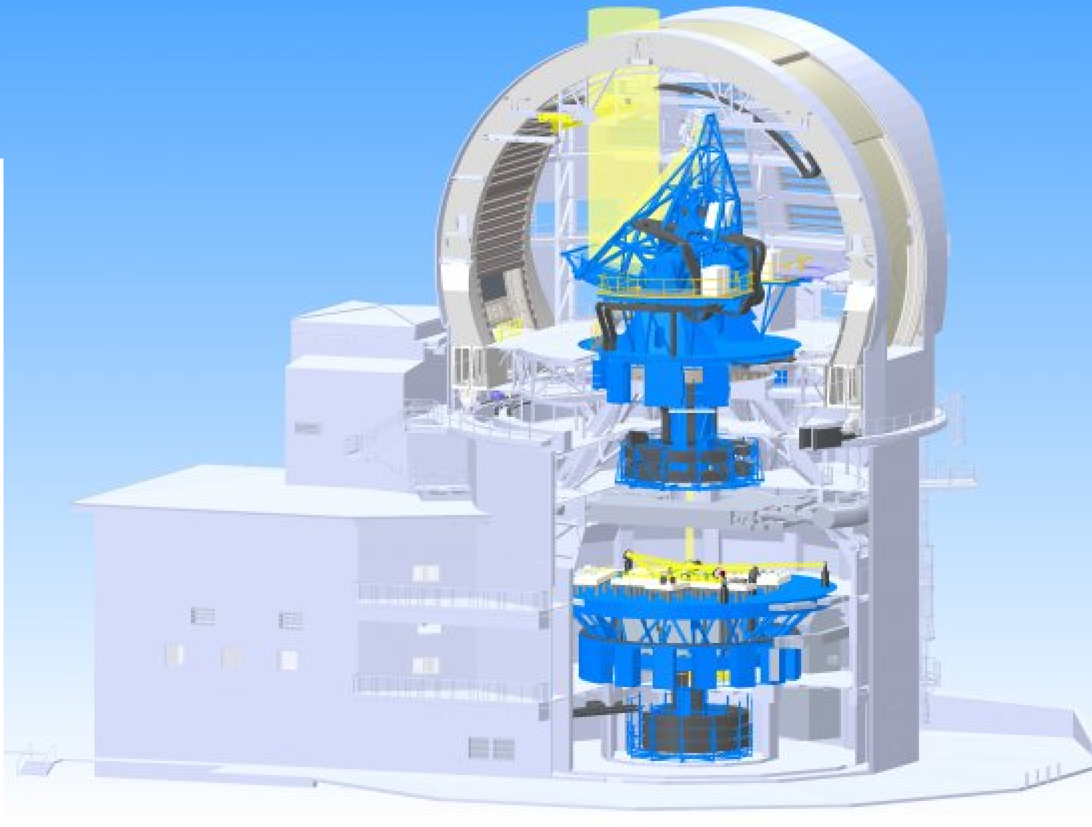
A. López Ariste

Dunn Solar Tower. New Mexico



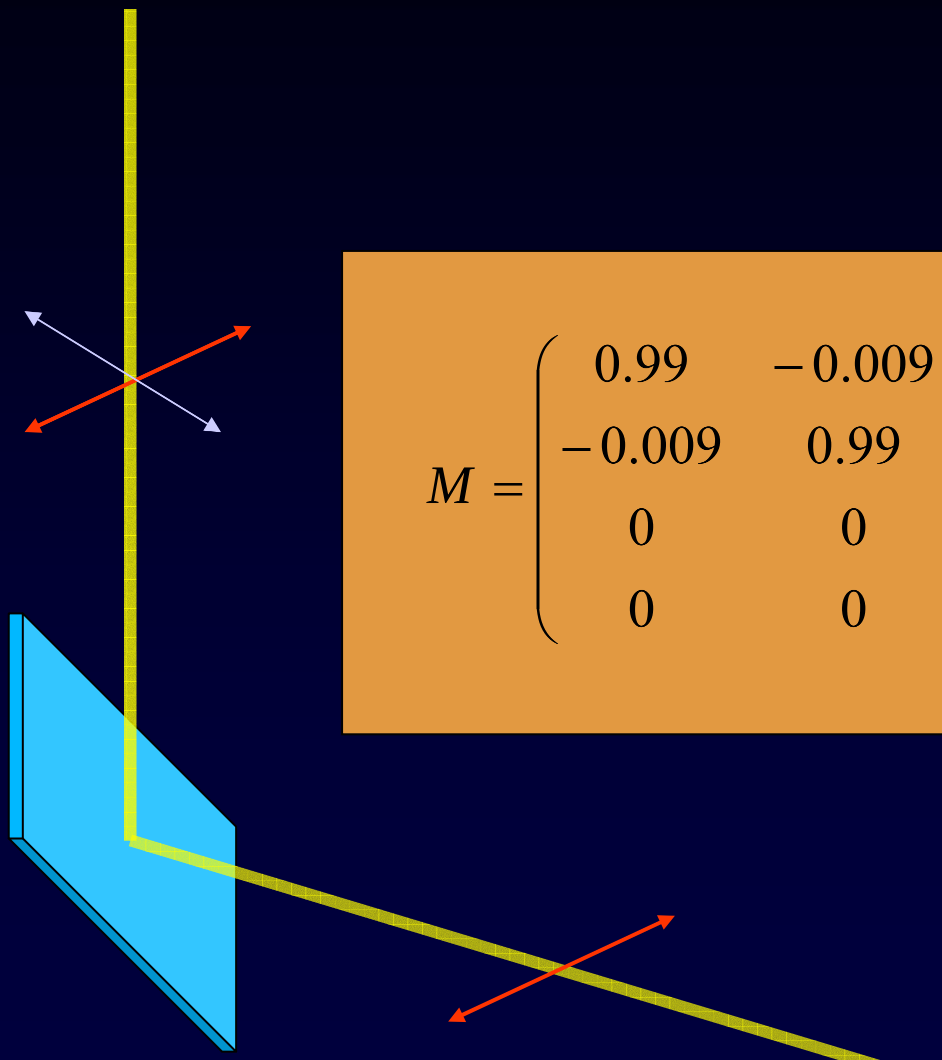

RITCHEY-CHRETIEN TELESCOPE:
-Entrance Pupil= 900 mm
-Obscuration= 400 mm (44%)
-Focale= 15.04 m
-Aperture= F/16.7
-Tube= He-Filled
-Polarization= "FREE"
Entrance Window
M2
M1
Field
Lens
Exit Window
F1



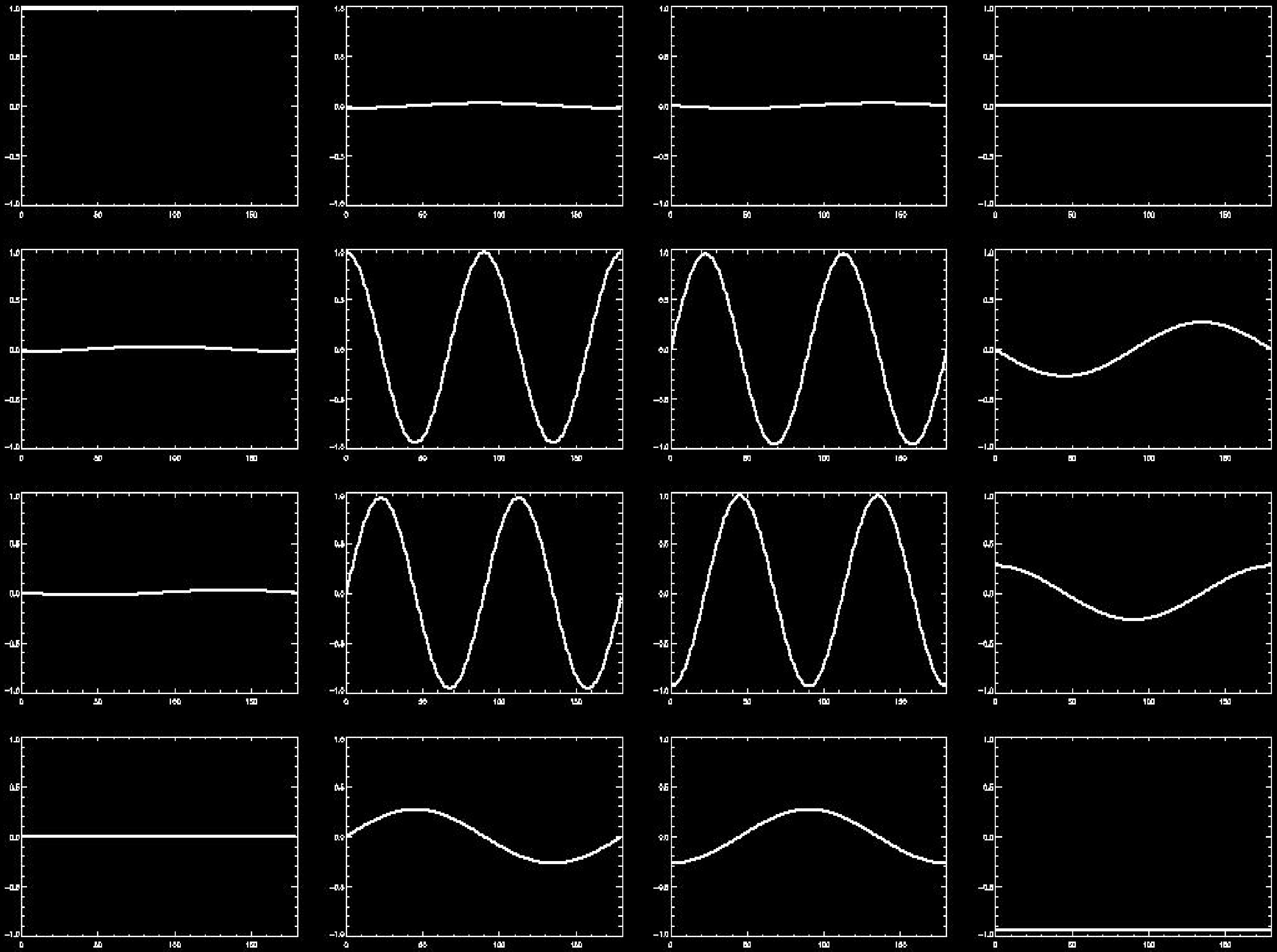


The ATST

A 4-m free-aperture solar telescope



$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

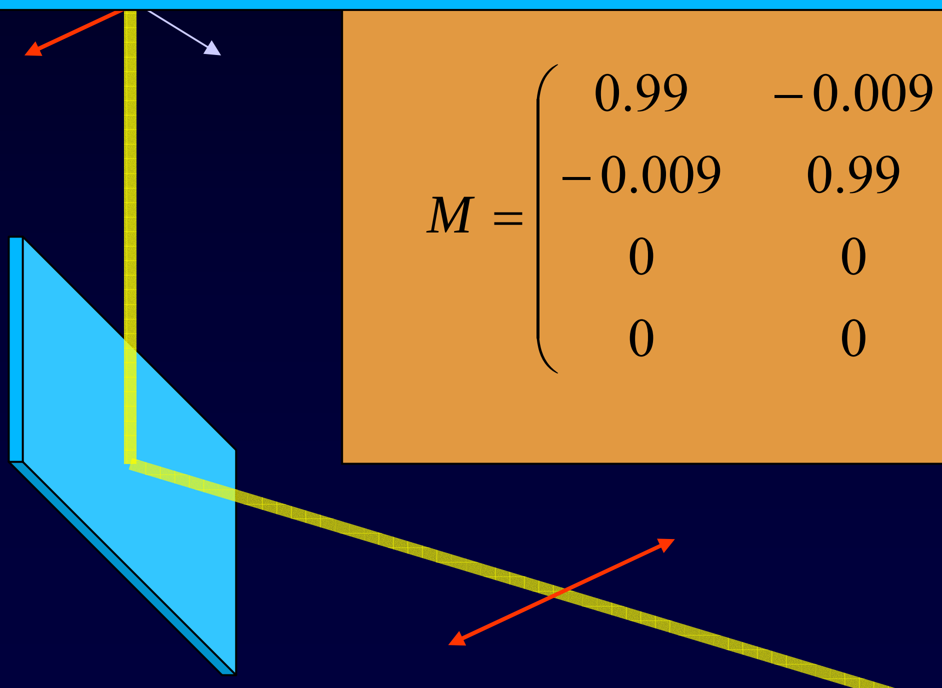


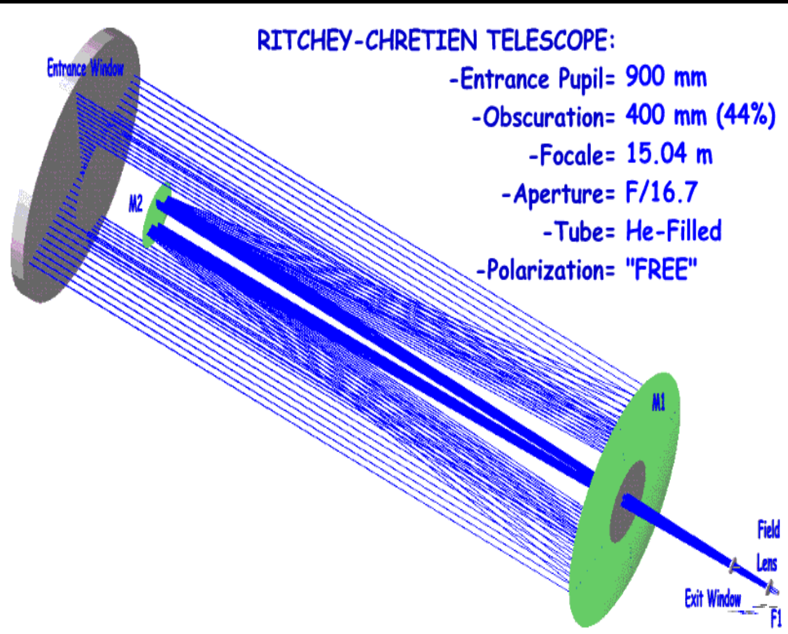
CLAIM:

We can do polarimetry through uncalibrated mirrors provided that

- we know their symmetries
- we have some rough info on their properties

$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

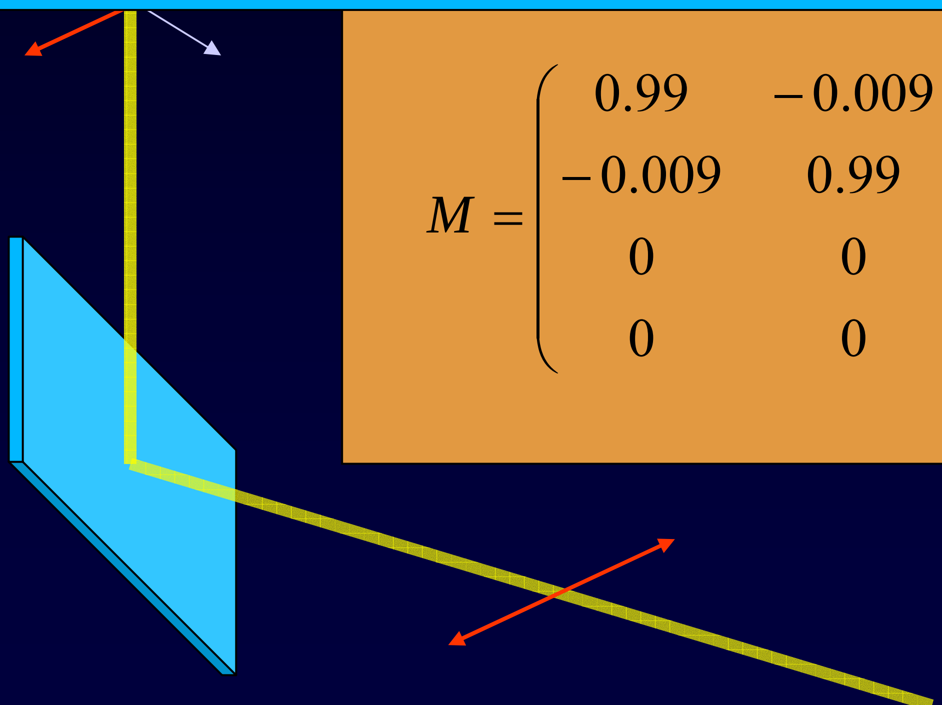



RITCHEY-CHRETIEN TELESCOPE:
-Entrance Pupil= 900 mm
-Obscuration= 400 mm (44%)
-Focale= 15.04 m
-Aperture= F/16.7
-Tube= He-Filled
-Polarization= "FREE"
Entrance Window
M2
M1
Field
Lens
Exit Window
F1

CLAIM:

We can do polarimetry through uncalibrated mirrors provided that

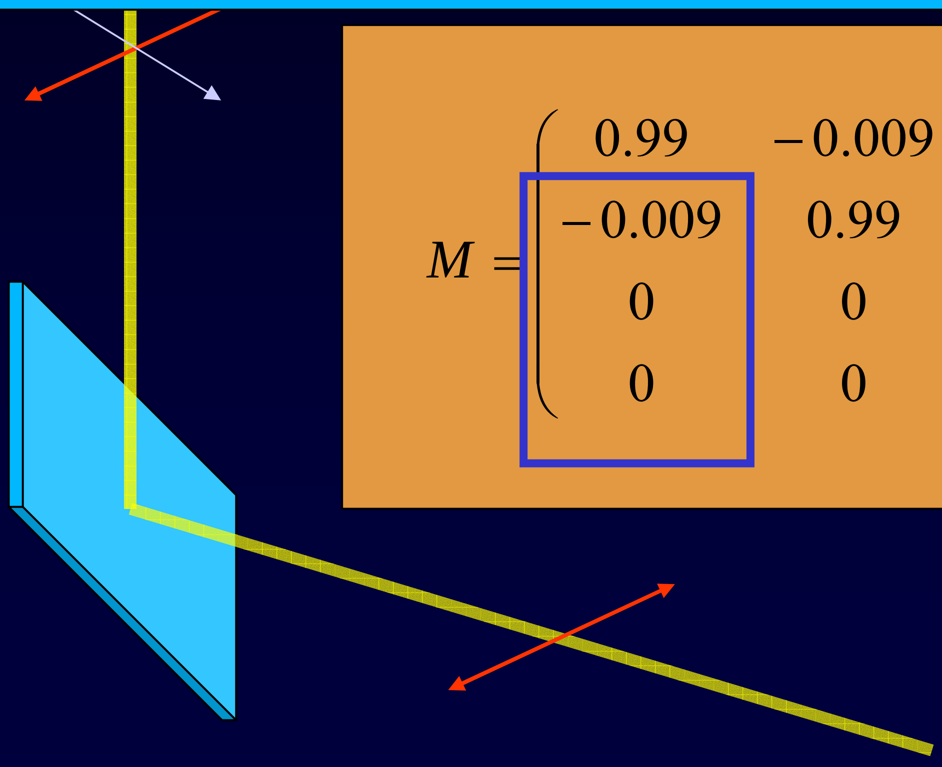
- we know their symmetries
- we have some rough info on their properties



$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

PROBLEMS TO BE ADDRESSED

1. Crosstalk I→Polarization. « The first column »
2. Existence of an optical solution: « Gadgetry »
3. Resiliance to errors: « Bettering classical calibration »



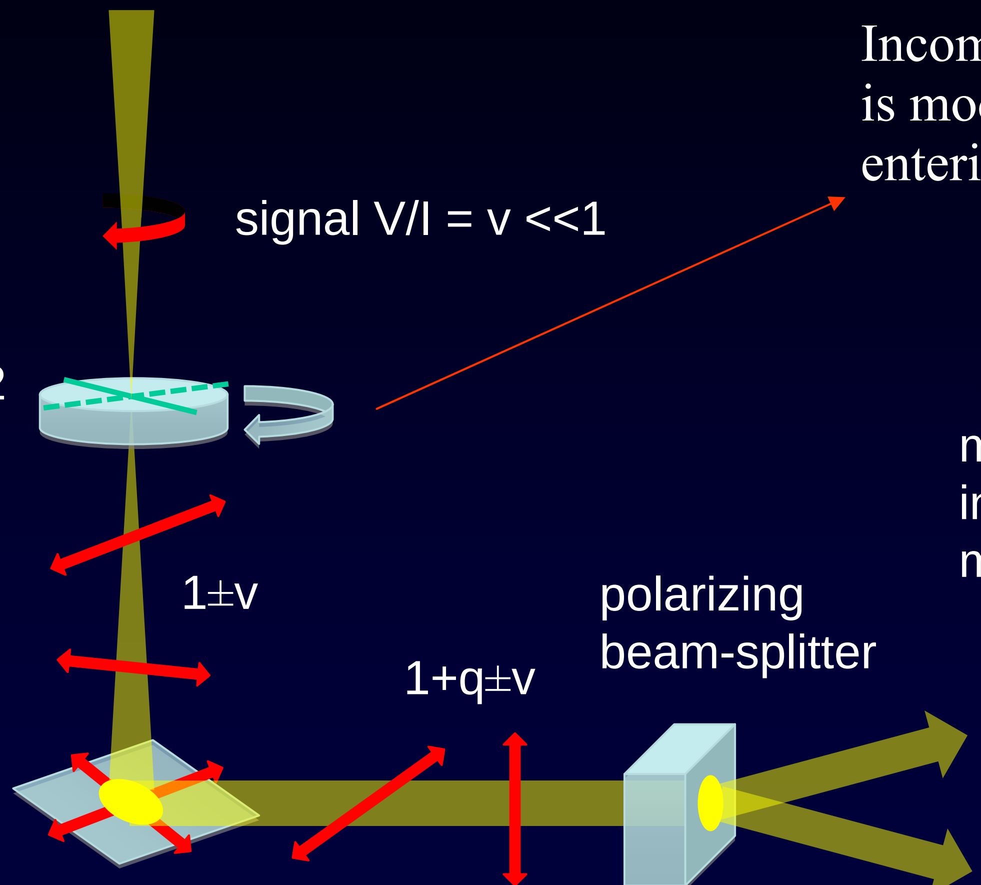
$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

Pre-conditioning
vs.
Post-conditioning

Can we modify the polarization state of light **BEFORE** it interacts with our telescope?

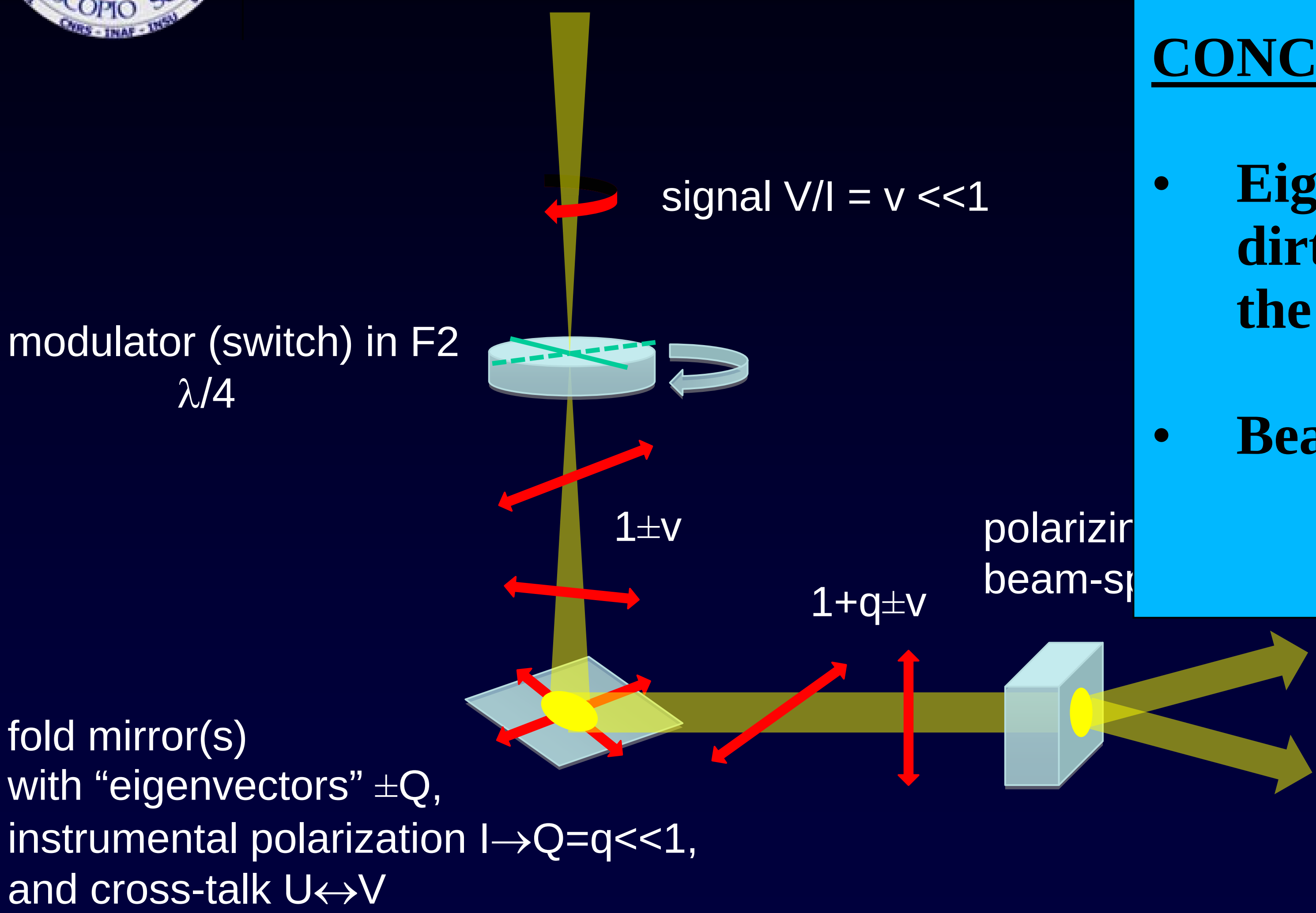
1. YES! Pre-conditioning is possible. Life is good
2. NO! Post-conditioning

Illustrating pre-conditioning



From F. Snik

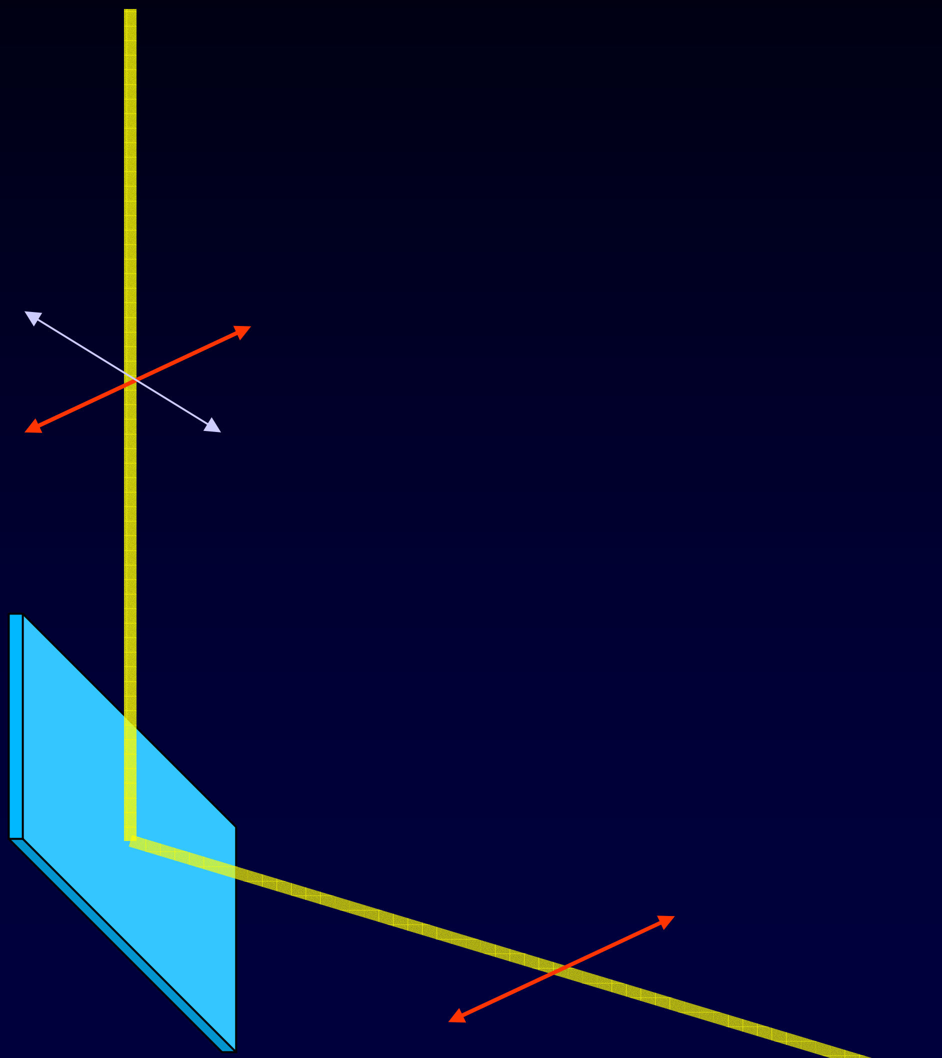
Illustrating pre-conditioning

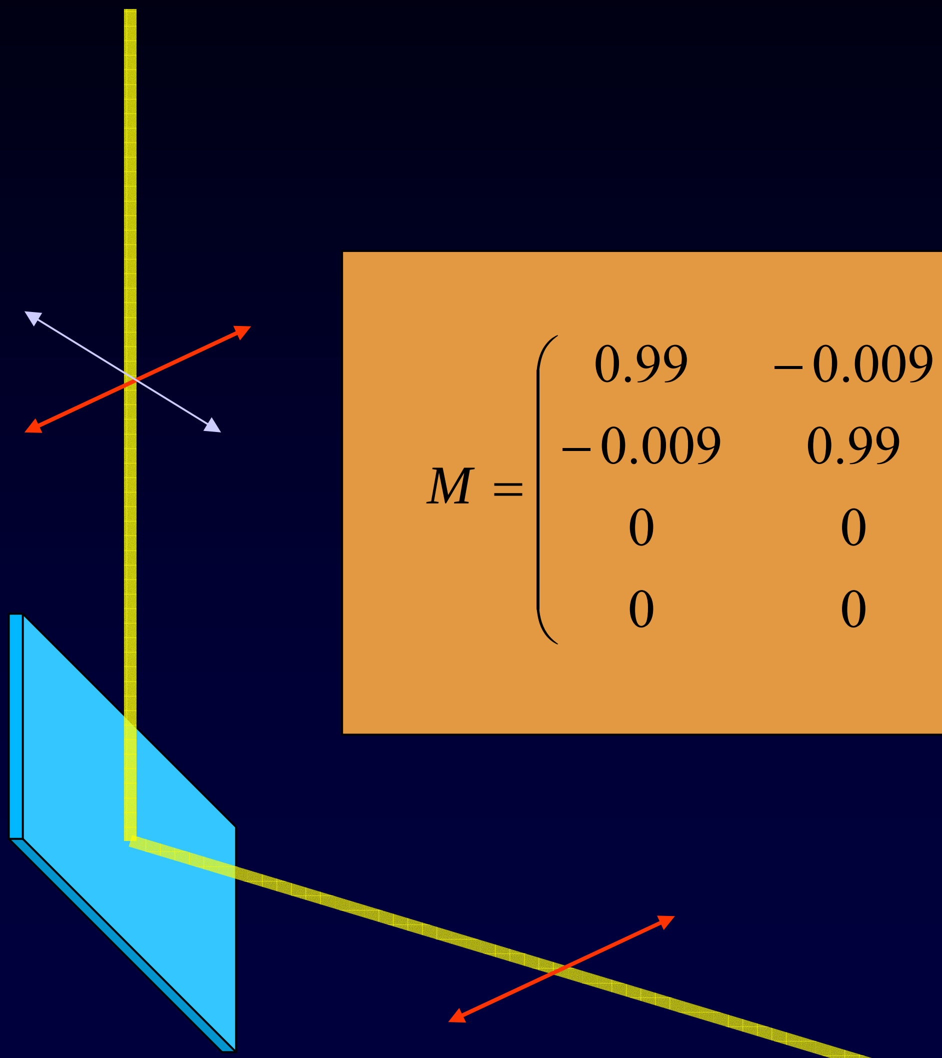


CONCEPTS:

- **Eigenvectors and dirty vectors of the system**
- **Beam exchange**

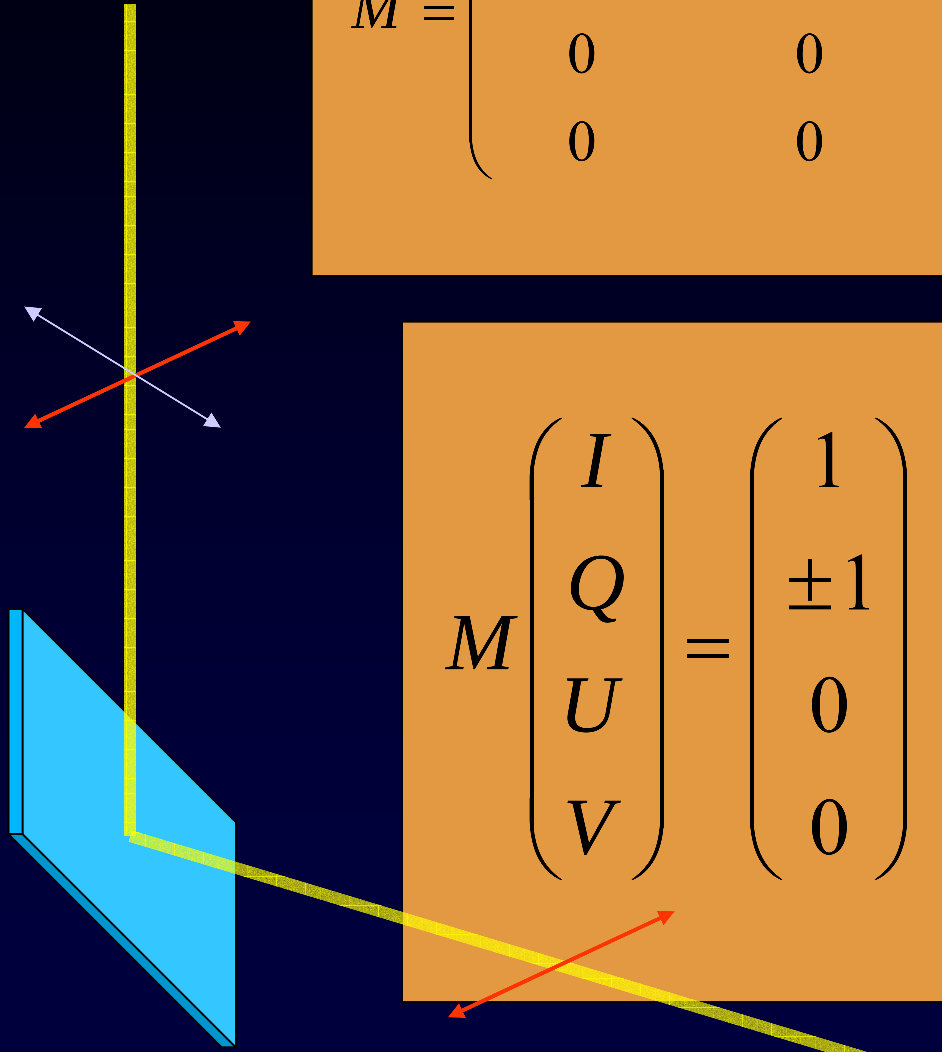
From F. Snik





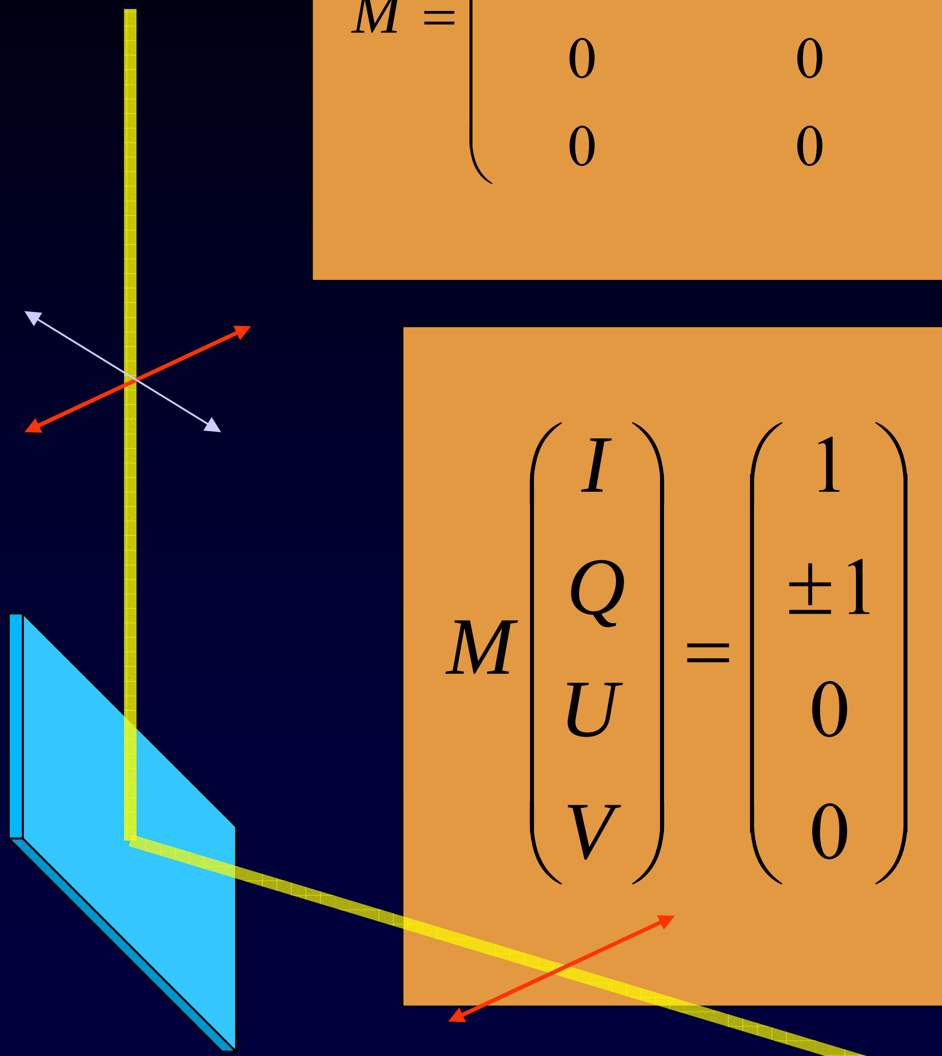
$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$



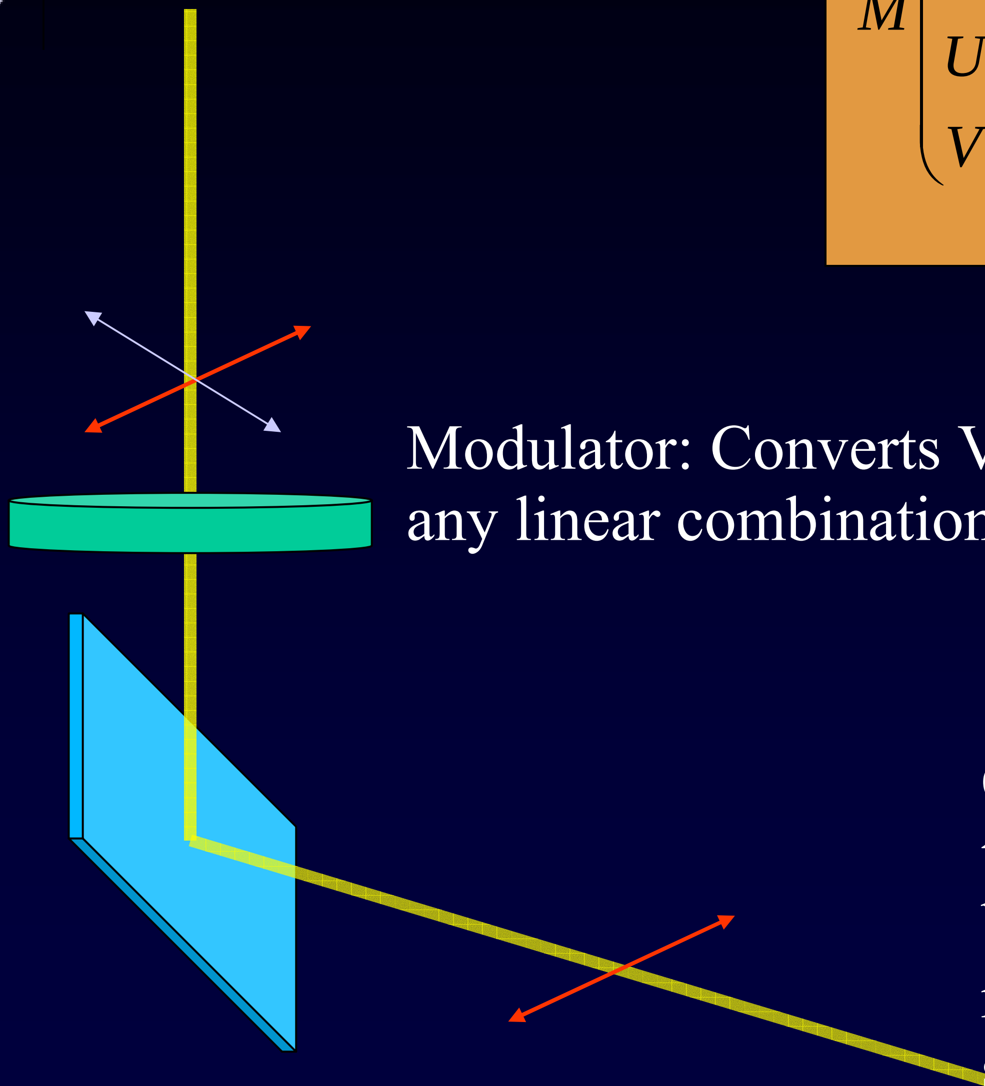
$$M \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$



$$M\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} \quad \Bigg| \quad \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix}$$

$$M\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix}$$



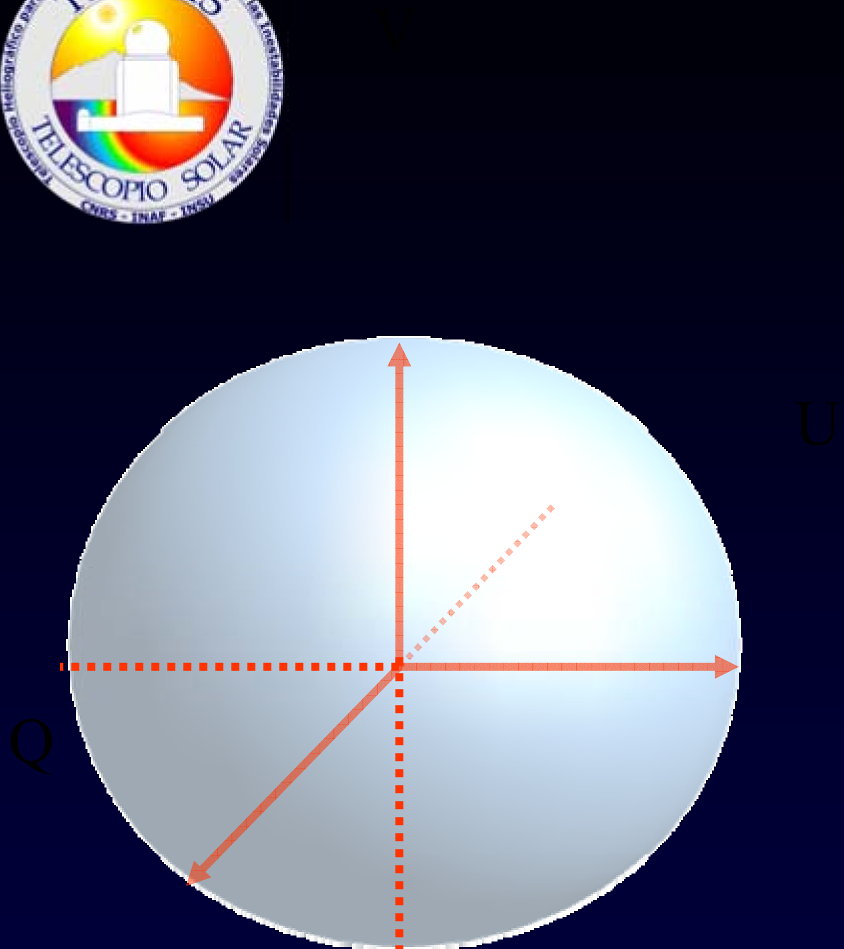
Modulator: Converts V (or Q or U or any linear combination) into Q.

Q carries the information to be measured, undeemed by the pass of the mirror, ready to be analyzed by a separator

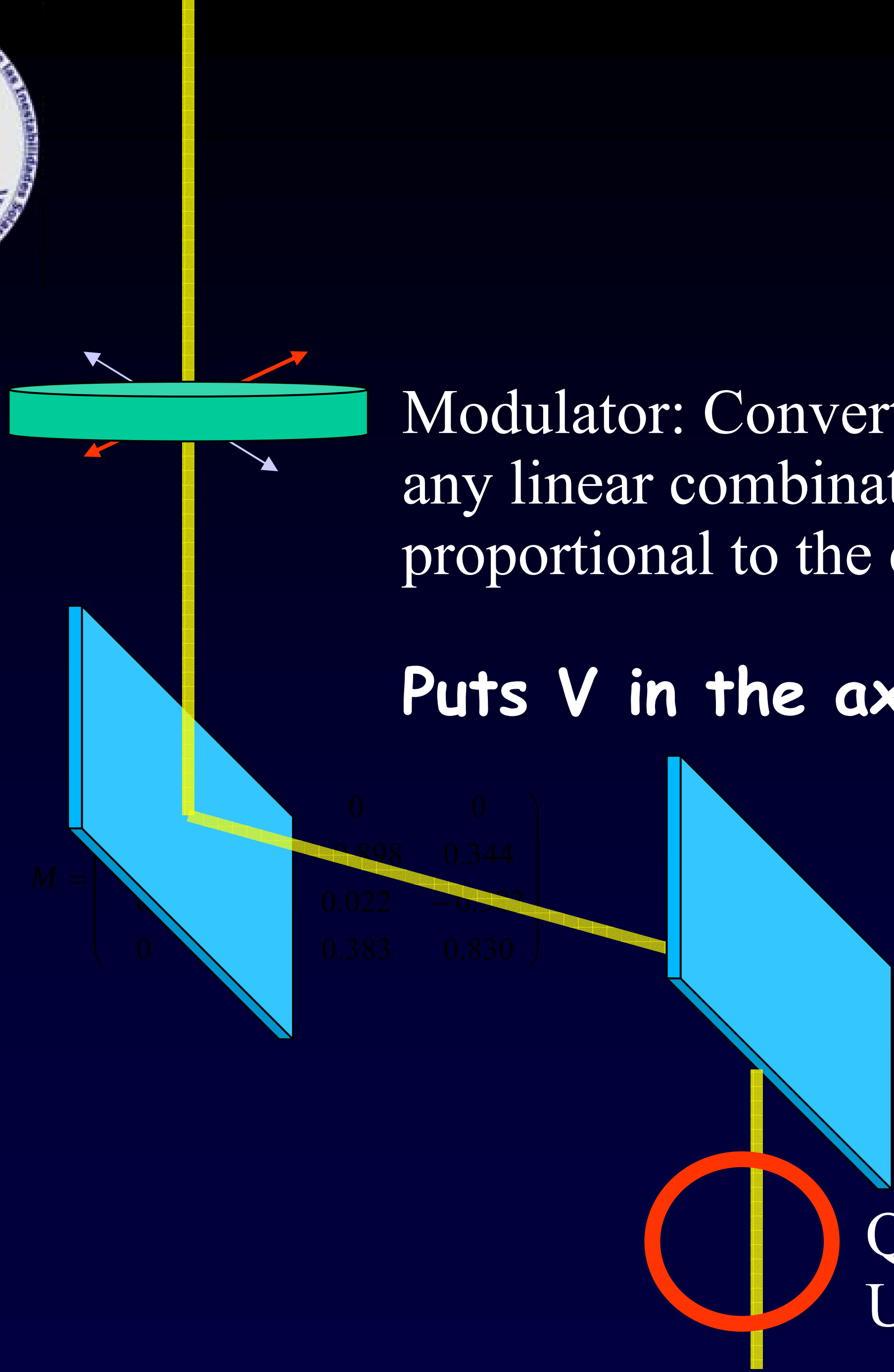
Pre-conditioning

Rotate the incoming polarization so that the Stokes parameter to be measured is projected onto the eigenvector of the system

RECIPE FOR EIGENPOLARIMETRY IN THE POINCARE SPHERE:



- Compute/Measure the 3x3 Mueller sub-matrix
- It corresponds to a rotation in the Poincaré sphere
- Per Euler's theorem, a rotation has a unique axis: Find it
- Rotate (modulator) your desired Stokes parameter into that axis.
- Recover it on the other side as Q



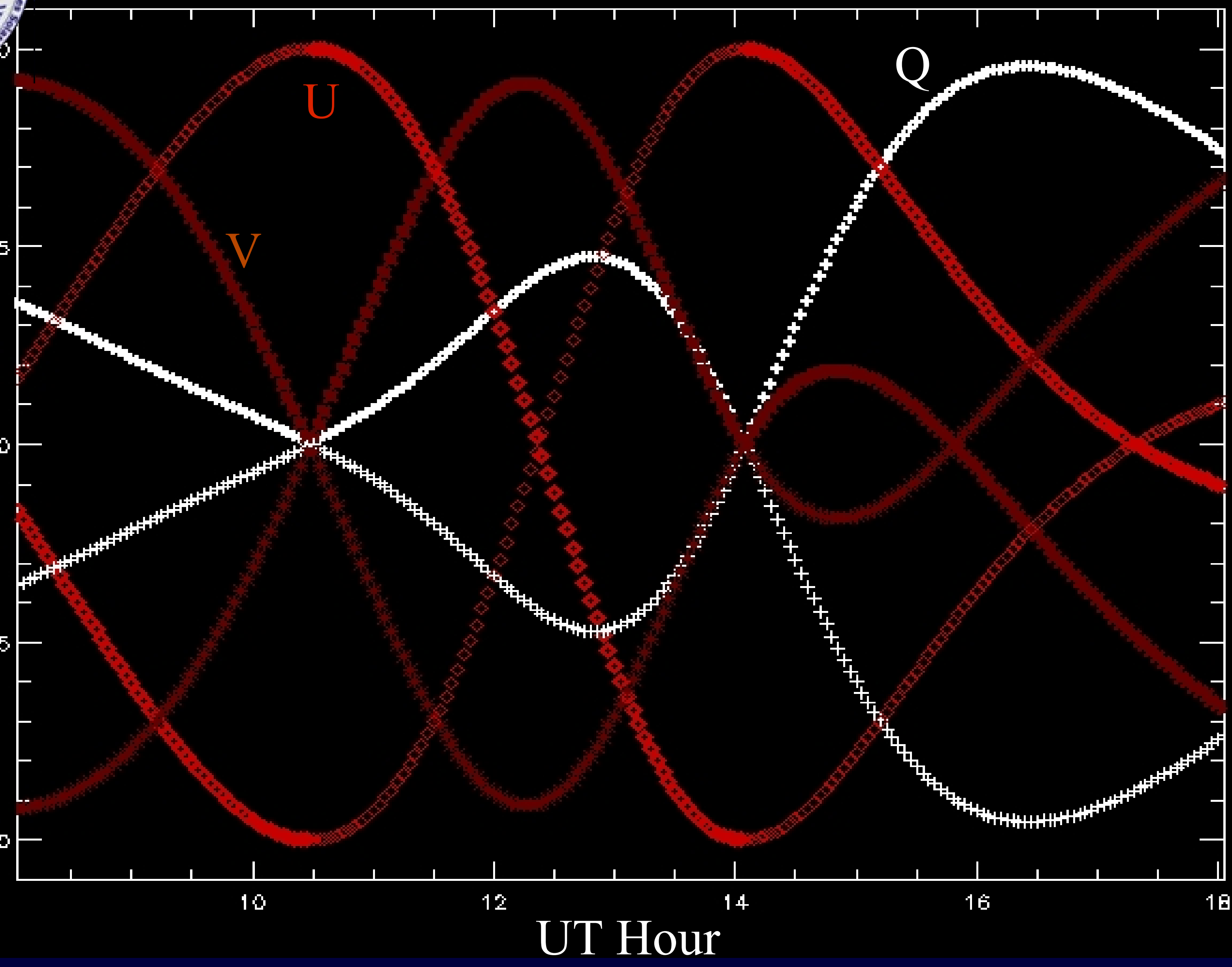
Modulator: Converts V (or Q or U or any linear combination) into a vector proportional to the eigenvector:

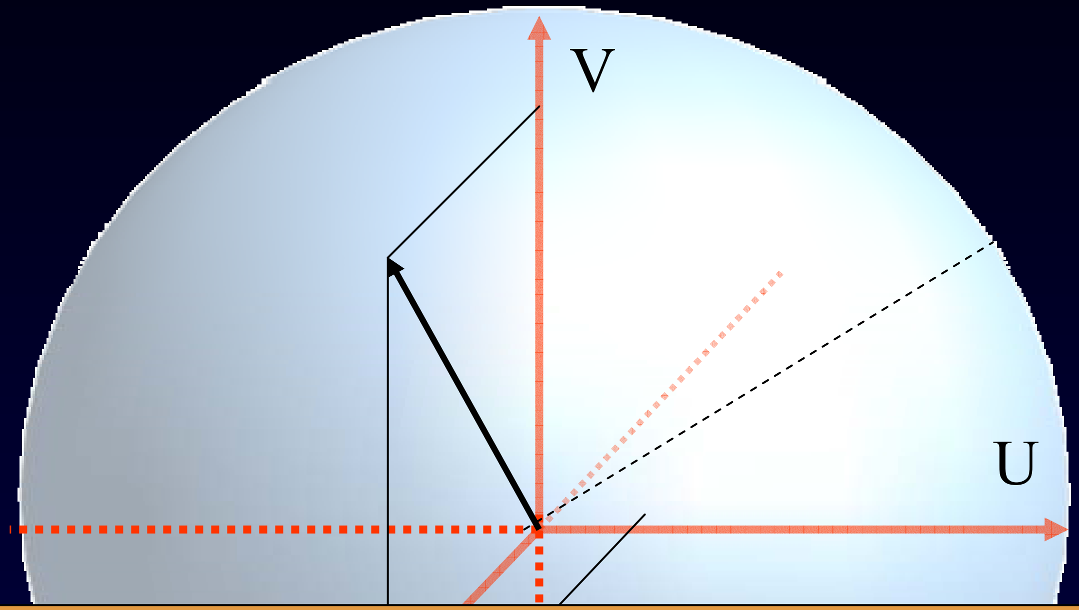
Puts V in the axis

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ 0.173 \\ -0.916 \\ 0.352 \end{pmatrix}$$

Q carries your measure
U and V carry the dirty vector

THEMIS, Jan 25th 2008

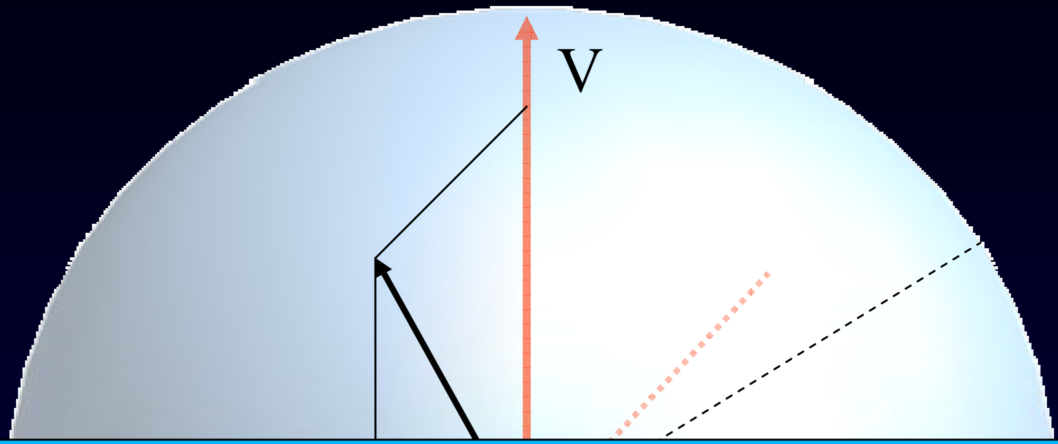




Retarders rotate the vector in the Poincaré sphere.

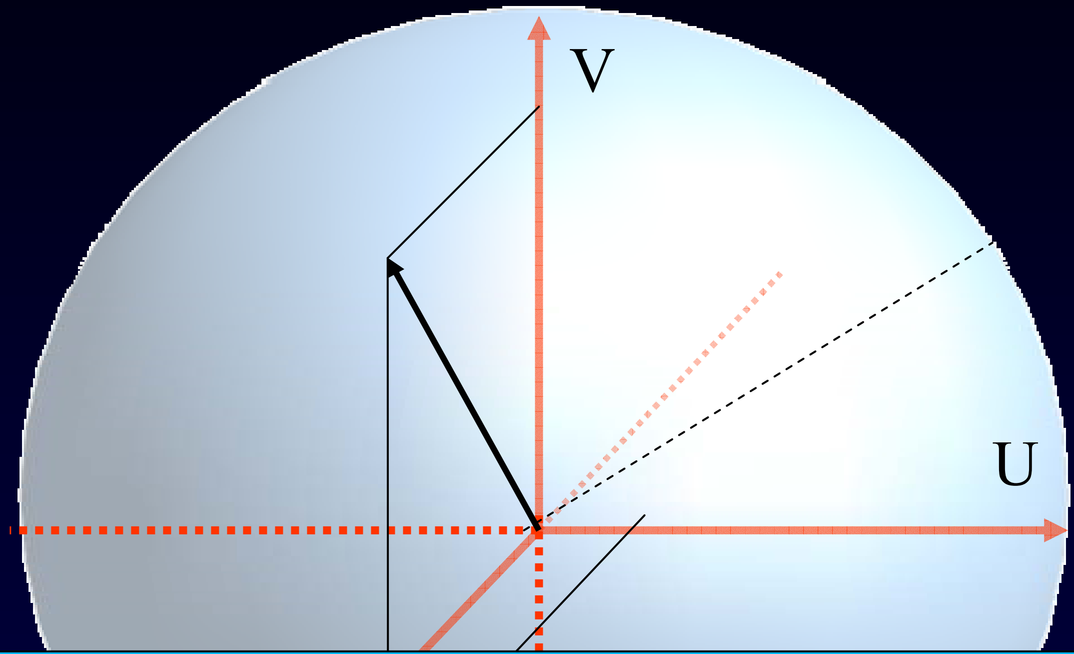
We are dealing with 3D rotations.

We are dealing with O(3).



Any 3D rotation has an axis of rotation (Euler's theorem)

The eigenvector is actually an AXIS, any vector parallel to the AXIS, **whatever its modulus**, goes unchanged through the transformation



Rotation in 3-dimensions is an orthogonal transformation:
If V goes into the eigenvector/axis, Q and U will go into orthogonal directions in the Poincaré sphere

Clean vectors and dirty vectors

$$\tilde{M}\begin{pmatrix} I_{\pm} \\ \pm Q_{loxo} \\ \pm U_{loxo} \\ \pm V_{loxo} \end{pmatrix} = \begin{pmatrix} I_{\pm} \\ \pm Q \\ \pm U \\ \pm V \end{pmatrix}$$

NON-ZERO!!!

Q ≠ I and I++Q amd I-Q cannot be substracted, they are Contaminated by the dirty vector

Clean vectors and dirty vectors

$$\tilde{M}\begin{pmatrix} I_\pm \\ \pm Q_{loxo} \\ \pm U_{loxo} \\ \pm V_{loxo}\end{pmatrix} = \begin{pmatrix} I_\pm \\ \pm Q \\ \pm U \\ \pm V\end{pmatrix}$$

Polarization has already been
measured, it is encoded in intensity
The dirty vector is a result of bad
rotation into the system eigenvector
Simple BEAM-EXCHANGE solves
The problem. A few photons are lost
To the measure: diminution in
efficiency is to be expected

CALIBRATION ERRORS EQUAL TO LOSS OF PHOTONS AND NO ERROR IN POLARIMETRY

3x3 sub-matrix

$$M = \begin{pmatrix} 0.977 & 0 & 0 & 0 \\ 0 & 0.169 & -0.898 & 0.344 \\ 0 & 0.898 & 0.022 & -0.383 \\ 0 & 0.344 & 0.383 & 0.830 \end{pmatrix}$$

4x4 true Mueller matrix

$$M=\begin{pmatrix} 0.977 & -0.013 & 0.010 & -0.003 \\ -0.013 & 0.169 & -0.898 & 0.344 \\ -0.010 & 0.898 & 0.022 & -0.383 \\ -0.003 & 0.344 & 0.383 & 0.830 \end{pmatrix}$$

Existence of axis in the 4x4 case

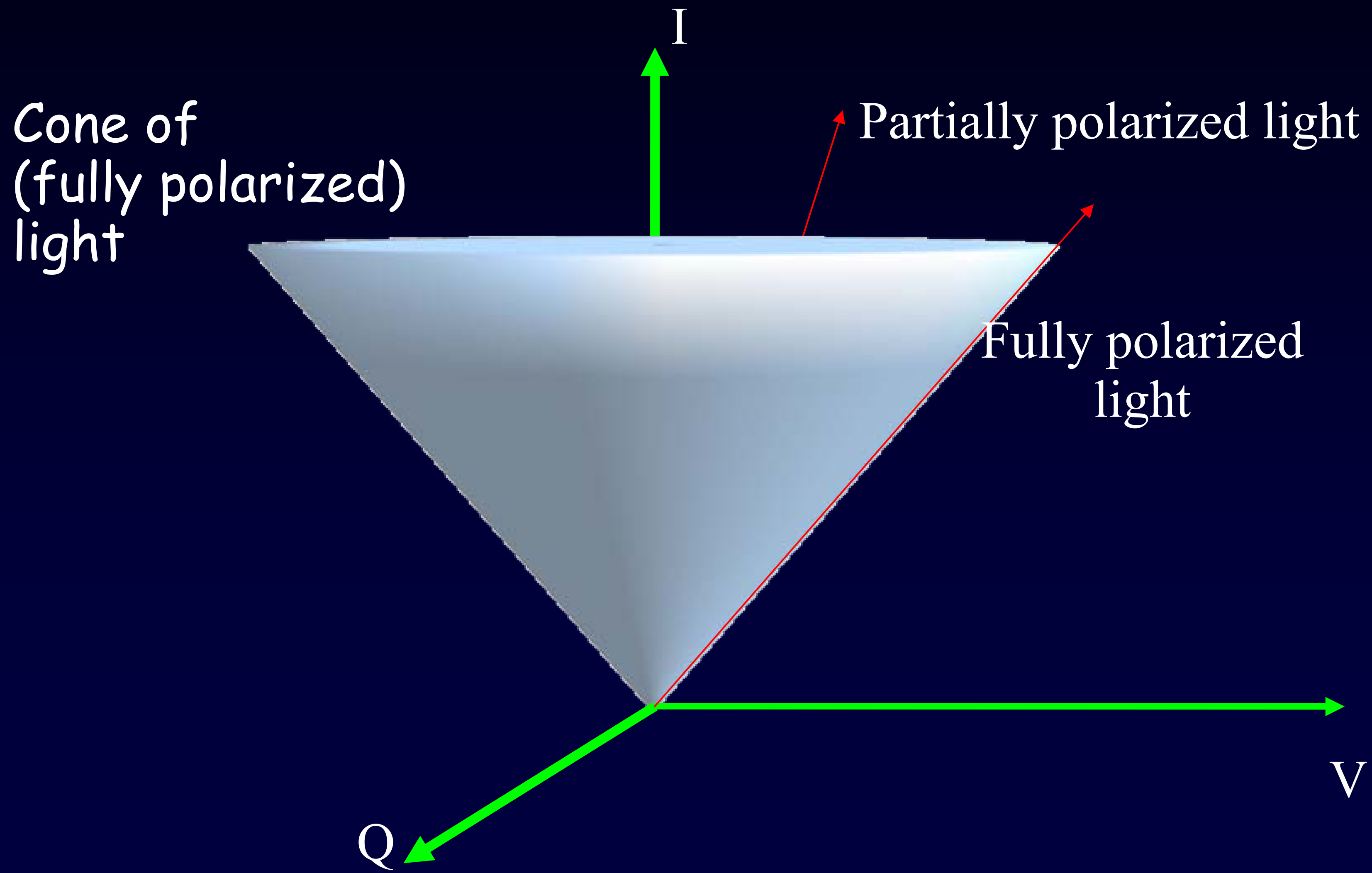
$$\exists\ \vec{I} \quad so \quad that \quad M\vec{I} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} ?$$

Facts and requirements:

- M is a valid Mueller matrix: it belongs to the $SO^{+}(3,1)$ group, the group of the proper orthochronous Lorentz transformation (perhaps closed with the limit algebra, but let us hope it is not required!)

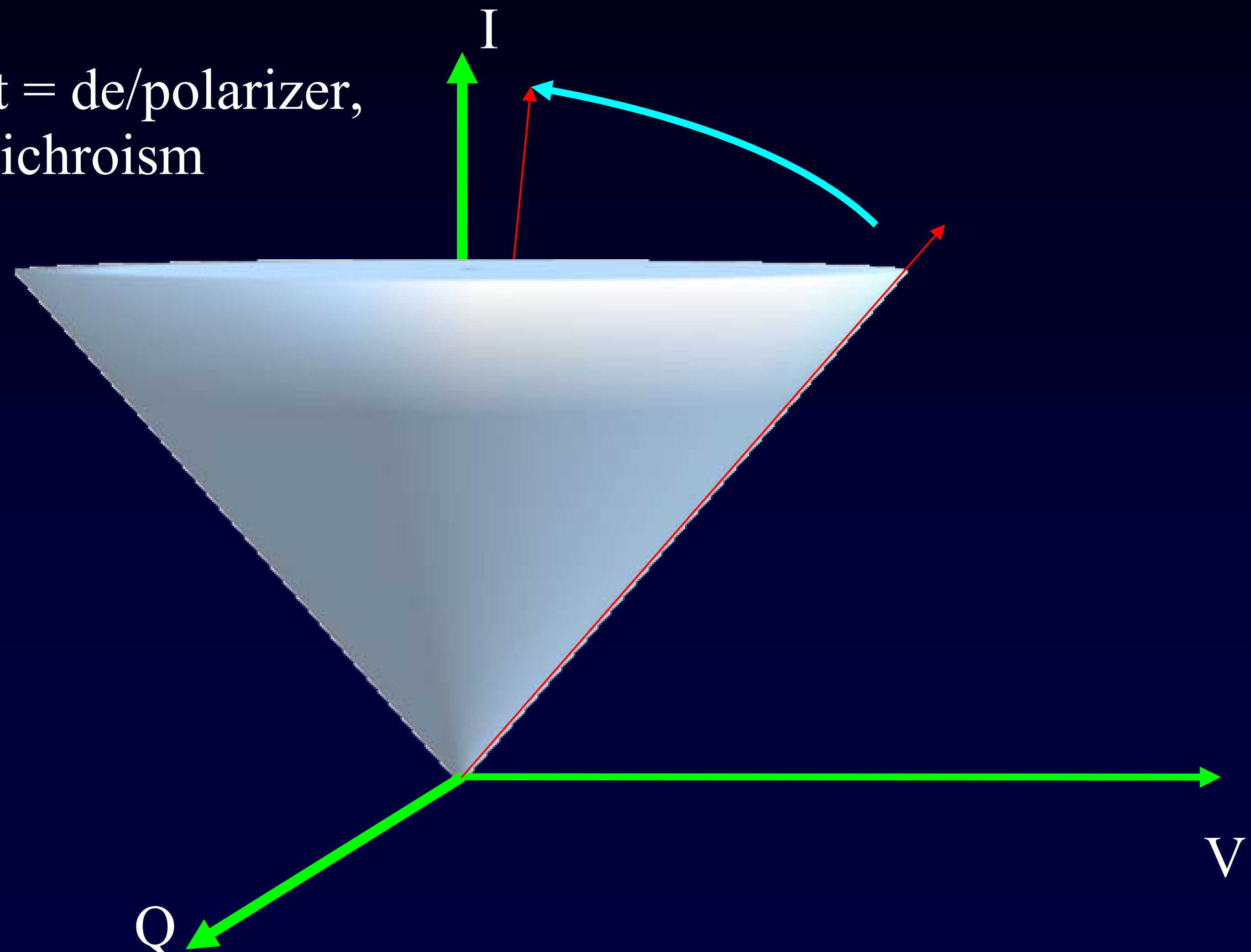
$$\left\| \vec{I} \right\| = I^2 - Q^2 - U^2 - V^2$$

The Minkowski space



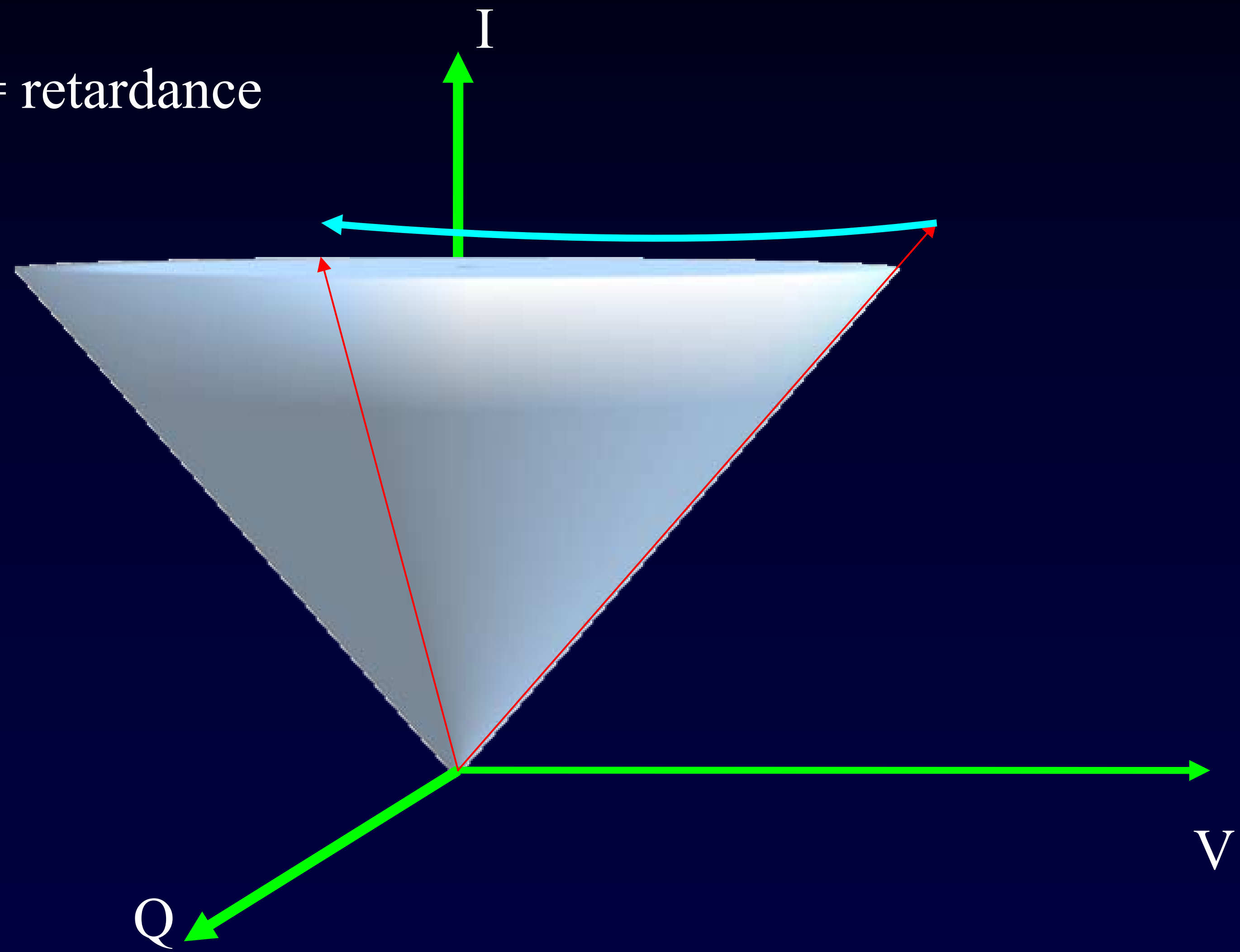
The cone of (fully polarized) light

Lorentz boost = de/polarizer, attenuators, dichroism



The cone of (fully polarized) light

3-d rotation = retardance



There are 3 conjugacy classes in the $SO^{+}(3,1)$ group:

- Parabolic transformations : Unphysical

There are 3 conjugacy classes in the $SO^{+}(3,1)$ group:

- ~~Parabolic transformations : Unphysical~~

There are 3 conjugacy classes in the $SO^+(3,1)$ group:

- ~~Parabolic transformations : Unphysical~~

- Elliptic transformations = 3-d rotations.
  - Imaginary eigenvalues exp (ia)
  - Unphysical eigenvectors (not valid Stokes vectors) BUT valid axis of rotation

There are 3 conjugacy classes in the $SO^{+}(3,1)$ group:

~~• Parabolic transformations : Unphysical~~

•Elliptic transformations = 3 … ns.

- Imaginary eigenv…
- Unphysical … (not valid Stokes vectors) … is of rotation

3x3 case: We just explored it

There are 3 conjugacy classes in the $SO^{+}(3,1)$ group:

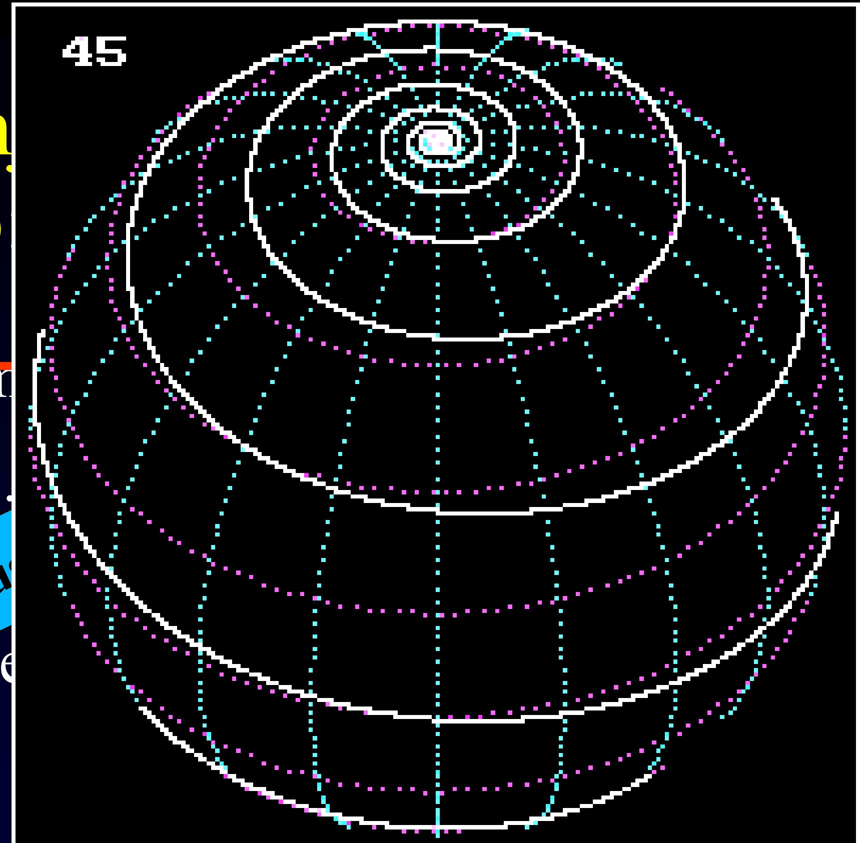
- ~~Parabolic transformations : Unphysical~~

•Elliptic transformati… …d rotations.
  - Imaginar… …alues exp (ia)
  - Un… …eigenvectors (not valid Stokes ve…s)

3x3 case: We just explored it

•Loxodromic transformations
  - Eigenvalues ±1, exp(a)
  - Eigenvectors = Stokes vectors. In particular the eigenvalues ±1 are associated to **fully polarized vectors**

There are 3 con
$SO^{+}(3,1)$ group



- ~~Parabolic transform~~

•Elliptic transformat

- Imaginar
- Un eige
  ve)

3x3 case: We ju

•Loxodromic transformations

- Eigenvalues ±1, exp(a)
- Eigenvectors = Stokes vectors. In particular the eigenvalues ±1 are associated to **fully polarized vectors**

Recipe for 4x4 Mueller matrix

If your entrance/solar Stokes vector is a **fully polarized vector**, it can be rotated (modulator) so that the Stokes parameter Desired is parallel to the loxodromic axis of The Mueller matrix of the system.

The Stokes parameter will so travel un-deemed through The system and will exit as Q polarization in front of the analyzer

Recipe for 4x4 Mueller matrix

If your entrance/solar Stokes vector is a **fully polarized vector**, it can be rotated (modulator) so that the Stokes parameter

Solving the Xtalk problem (the first column) requires to fully polarize your entrance light.

It can be done, while keeping the information…
..but not today

Post-conditioning

- Light polarizations cannot be modified before it enters our telescope
- The Stokes vector cannot be projected onto the eigenvector of the system

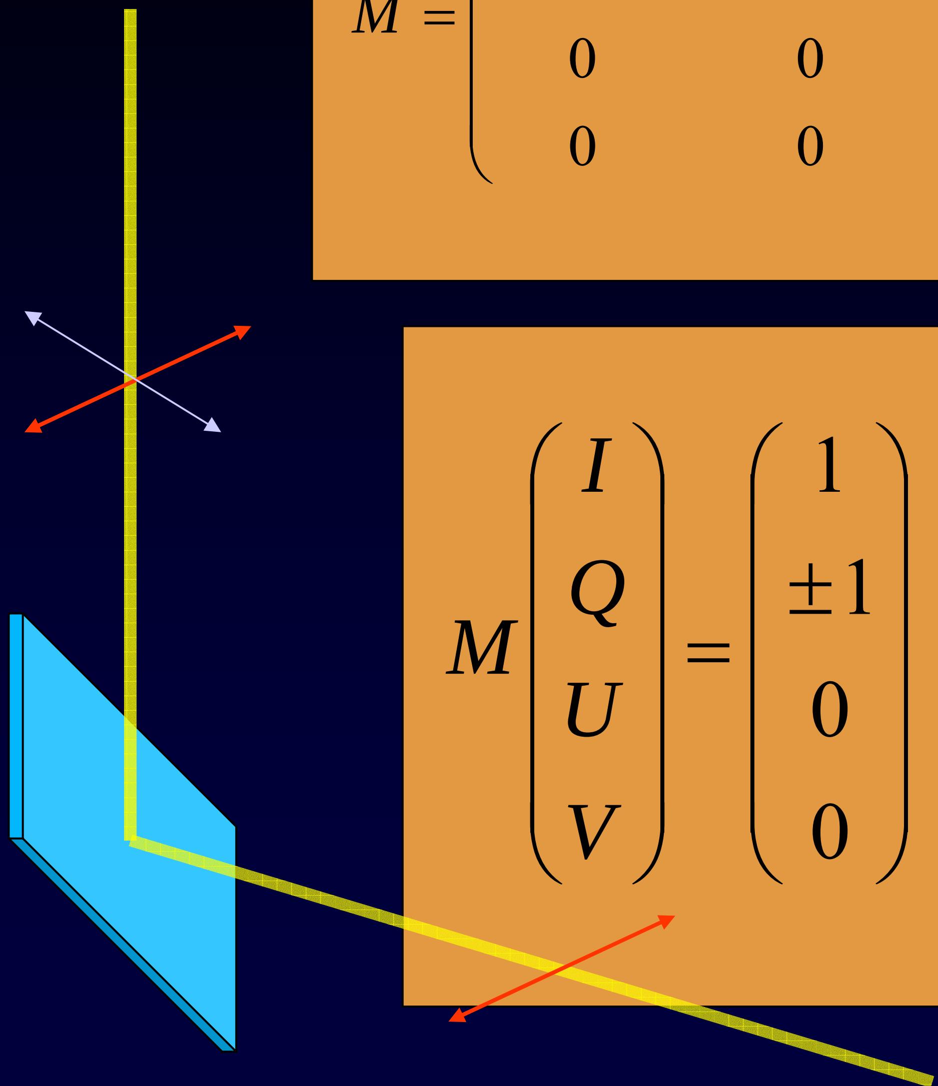
Post-conditioning

- Light polarizations cannot be modified before it enters our telescope

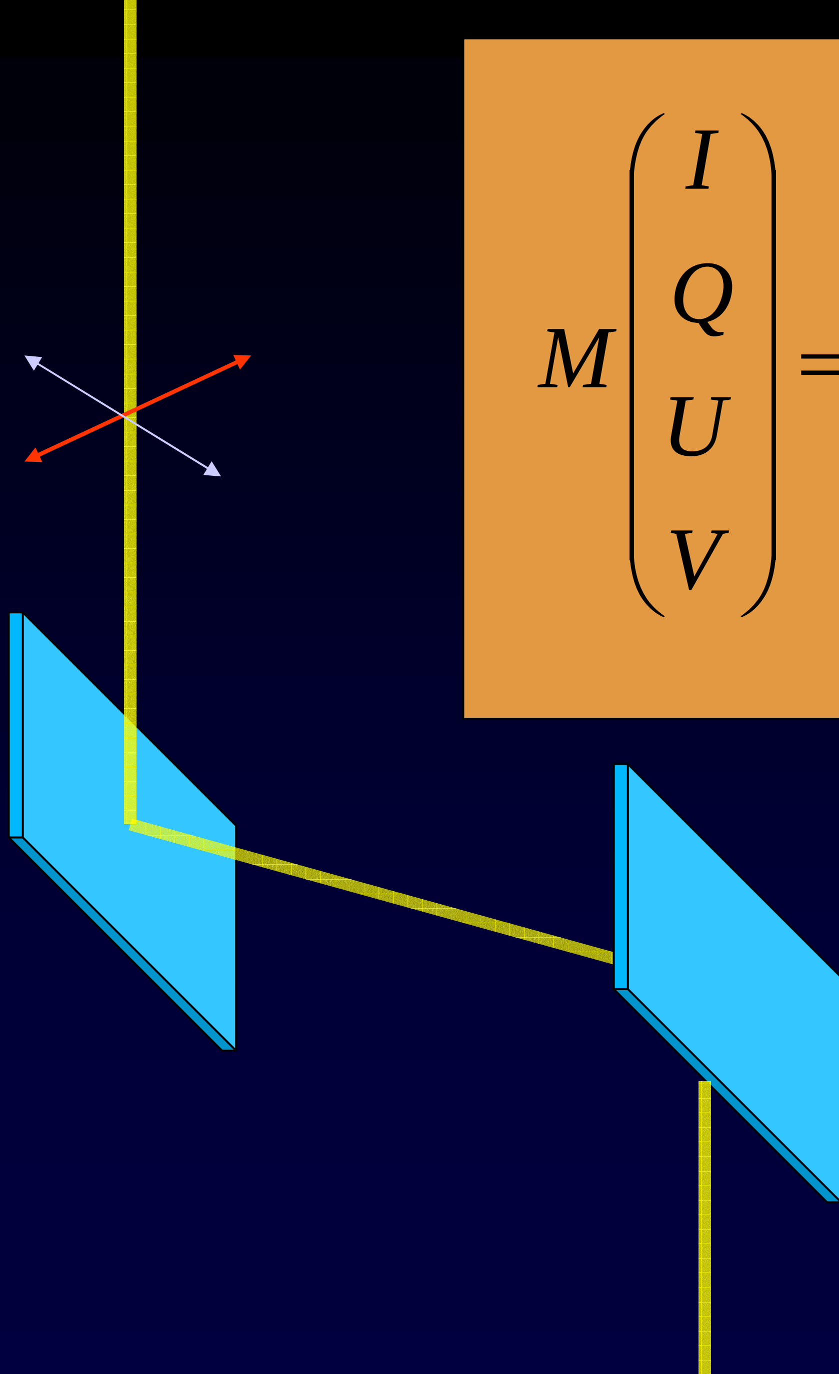
if the mountain won't come to Muhammad…

Since you cannot project onto the eigenvector, **change** the eigenvector

$$M = \begin{pmatrix} 0.99 & -0.009 & 0 & 0 \\ -0.009 & 0.99 & 0 & 0 \\ 0 & 0 & -0.935 & 0.323 \\ 0 & 0 & -0.323 & -0.935 \end{pmatrix}$$

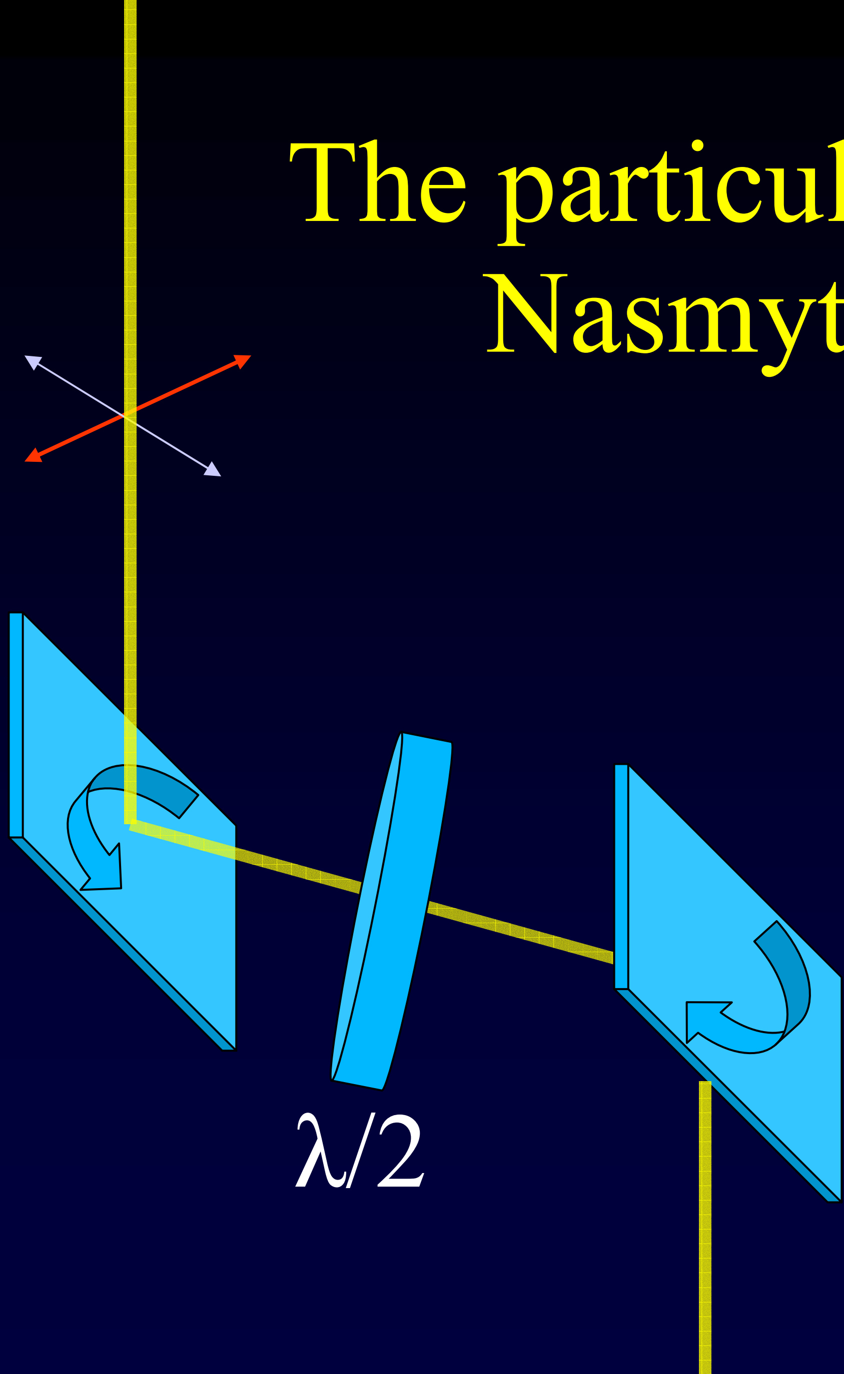


$$M\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix}$$



$$M\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} 1 \\ 0.173 \\ -0.916 \\ 0.352 \end{pmatrix}$$

The particular case of a Nasmyth mirror



$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$