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Polarimetric definitions

Presentation given at the conference "The future of photometric, spectrophotometric, and polarimetric standardisation" (2006), and at the workshop on polarimetric calibration in Zurich (2013)

References

Landi Degl' Innocenti & Landolfi

POLARIZATION IN SPECTRAL LINES

Kluwer Academid Pub

The Stokes Parameters

I ("natural" light, or "intensity")

Q & U ("linear polarization")

V ("circular polarization")

Georges Gabriel Stokes (1852)

Definition of the Stokes parameters

- Statistical averages of bilinear products of the components of the electric field along two perpendicular axes, both perpendicular to the direction of the propagation of the radiation

- Operational definition (through ideal filters)

Stokes parameters: the choice of a reference system

Right handed coordinate system (*x*,*y*,*z*), where:

- *z*-axis is directed along the direction of propagation
- x-axis is directed along an "arbitrary" direction

x-axis:

celestial meridian passing
through the object

Stokes parameters: the basic definition

$E_x(t)$, $E_y(t)$

$$E_x(\omega) = \mathrm{F.T.}(\mathrm{E}_x(t))$$
$$E_y(\omega) = \mathrm{F.T.}(\mathrm{E}_y(t))$$

$$I(\omega) = k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle + \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$Q(\omega) = k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle - \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$U(\omega) = k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

$$V(\omega) = \mathrm{i}k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

Stokes parameters: the basic definition

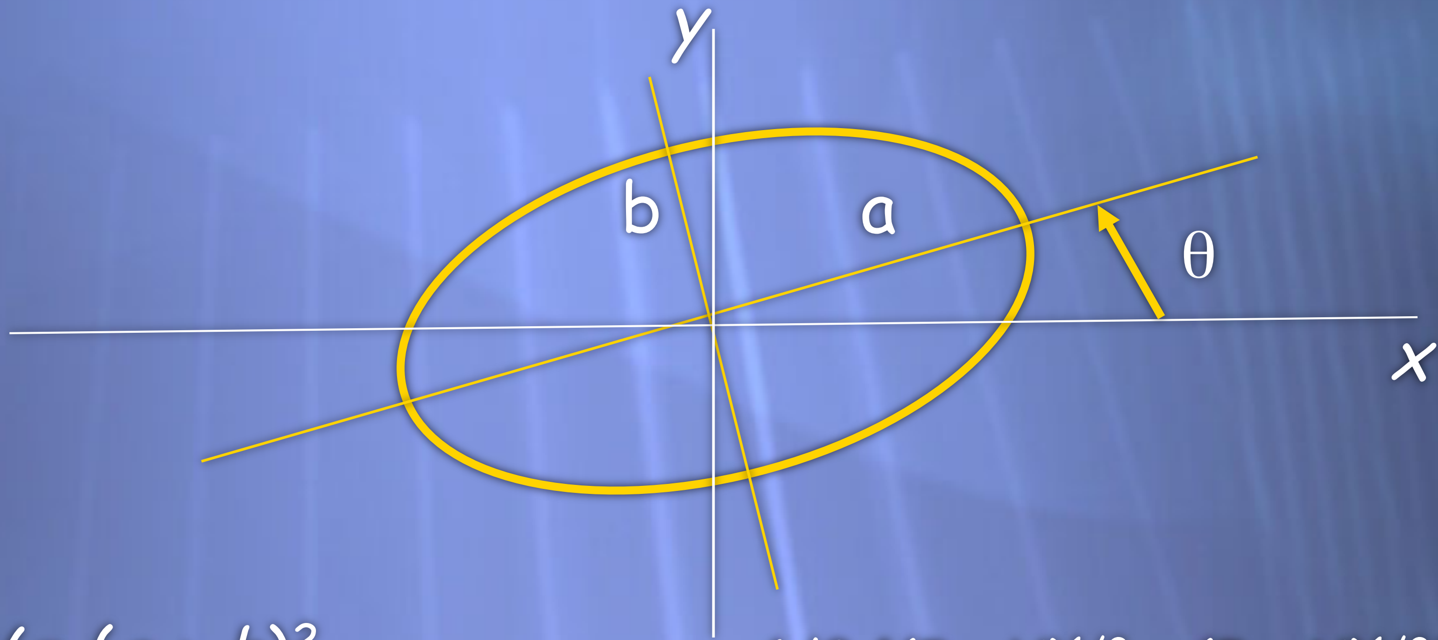
$E_x(t)$, $E_y(t)$

$$E_x(x,y,t) = \Re\left(\mathcal{E}_x(x,y,t)\,\mathrm{e}^{-\mathrm{i}\omega t}\right)$$
$$E_y(x,y,t) = \Re\left(\mathcal{E}_y(x,y,t)\,\mathrm{e}^{-\mathrm{i}\omega t}\right)$$

$$I = k\left[\langle\mathcal{E}_x^*\mathcal{E}_x\rangle + \langle\mathcal{E}_y^*\mathcal{E}_y\rangle\right]$$
$$Q = k\left[\langle\mathcal{E}_x^*\mathcal{E}_x\rangle - \langle\mathcal{E}_y^*\mathcal{E}_y\rangle\right]$$
$$U = k\left[\langle\mathcal{E}_x^*\mathcal{E}_y\rangle + \langle\mathcal{E}_y^*\mathcal{E}_x\rangle\right]$$
$$V = \mathrm{i}\,k\left[\langle\mathcal{E}_x^*\mathcal{E}_y\rangle - \langle\mathcal{E}_y^*\mathcal{E}_x\rangle\right]$$

Polarimetric definitions

Geometrical significance of Stokes parameters: the polarization ellipse



$$I - V = (a + b)^2$$

$$I + V = (a - b)^2$$

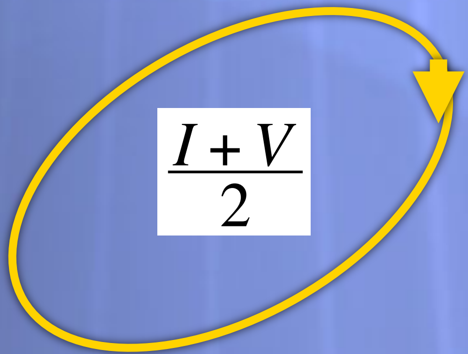
$$U/Q = tg(2\theta)$$

$$a = 1/2\,[(I - V)^{1/2} + (I + V)^{1/2}]$$

$$b = 1/2\,[(I - V)^{1/2} - (I + V)^{1/2}]$$

Geometrical significance of Stokes parameters: the circular polarization

$V =$



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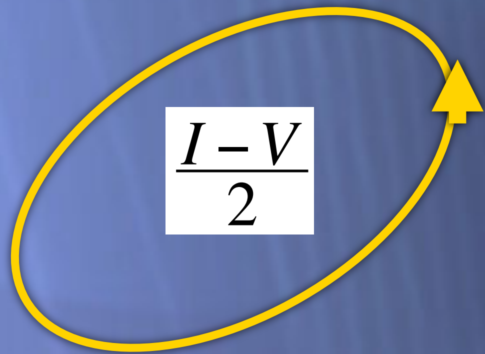
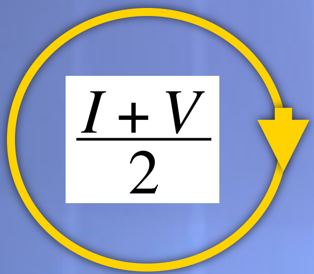


Positive, right handed

Negative, left handed

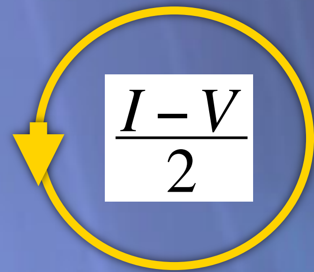
Geometrical significance of Stokes parameters: the circular polarization

If $Q=U=0$ the ellipse degenerate into a circle:
clockwise (as seen from the observer) if $V > 0$,
counterclockwise if $V < 0$



$$\frac{I+V}{2}$$

Positive, right handed



$$\frac{I-V}{2}$$

Negative, left handed

Geometrical significance of Stokes parameters: the linear polarization

Polarimetric definitions

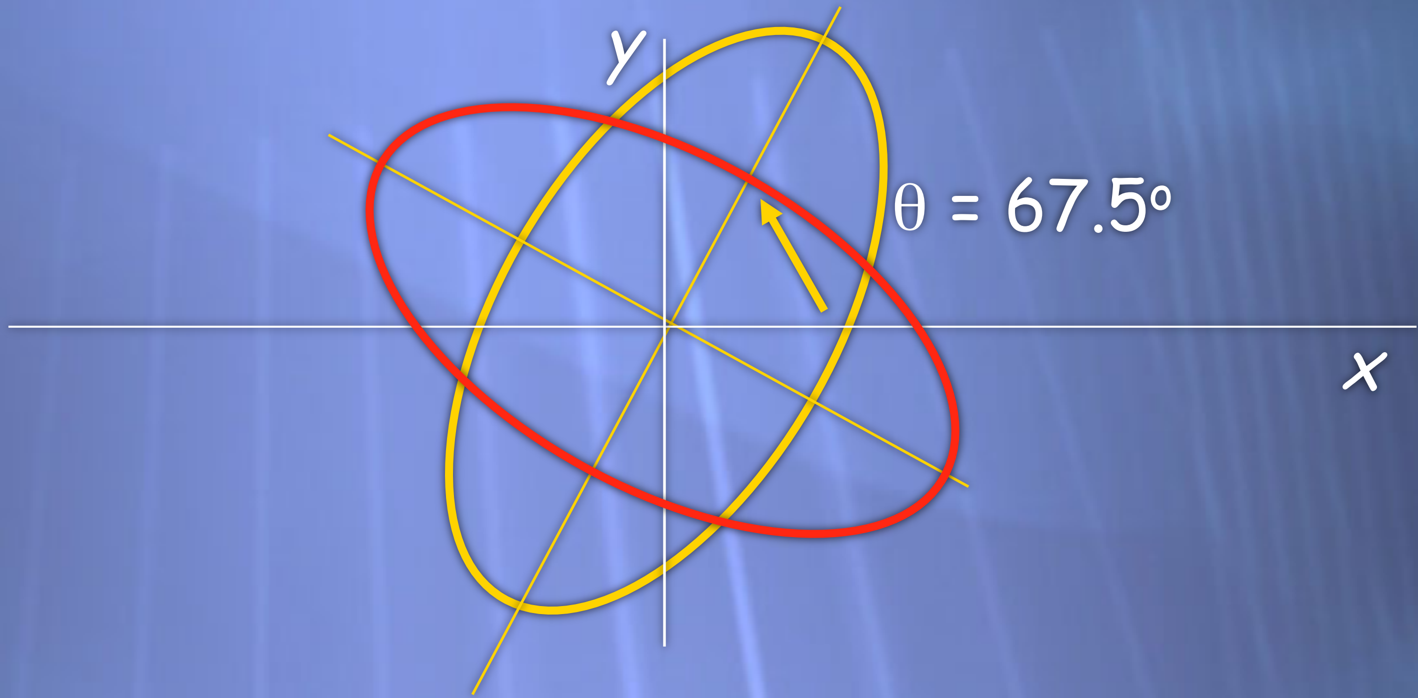
Polarization and its position angle θ

$$P = (Q^2 + U^2)^{1/2}$$

$$Q = P\cos 2\theta \quad U = P\sin 2\theta$$

~~$\theta = 1/2 \arctan (U/Q)$~~

Obtaining the position angle



$Q = P\cos 2\theta \quad U = P\sin 2\theta$

$Q = -P\frac{\sqrt{2}}{2}$ $\quad U = P\frac{\sqrt{2}}{2}$

$$\frac{1}{2}\arctan\left(\frac{U}{Q}\right) = -22.5^{\circ} = 157.5^{\circ}$$

Obtaining the position angle

$$\theta = \frac{1}{2}\arctan\left(\frac{U}{Q}\right) + \theta_0$$

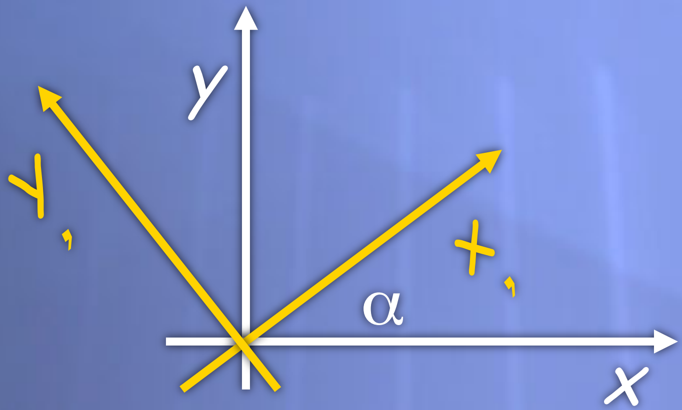
$$\theta_0 = 0 \text{ if } Q > 0$$

$$\theta_0 = \frac{\pi}{2} \text{ if } Q < 0$$

$$\theta = \frac{1}{2}\text{sign}(U)\arccos\left(\frac{Q}{\sqrt{Q^2 + U^2}}\right)$$

V.Bommier, priv. comm.

Changing the reference system



$$P = (Q^2 + U^2)^{1/2} = (Q'^2 + U'^2)^{1/2} = P'$$

is invariant!

$$I' = I$$

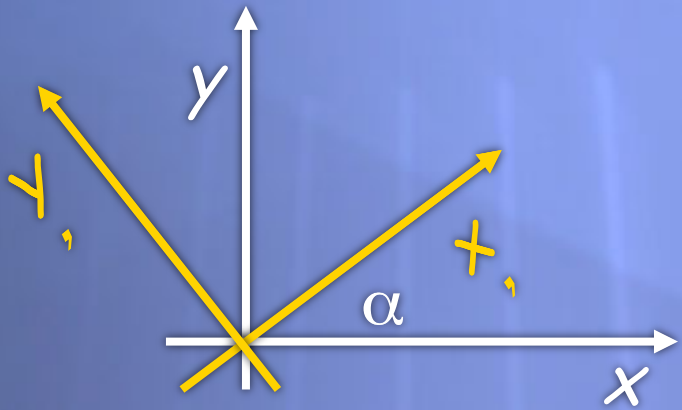
$$Q' = \cos 2\alpha\, Q + \sin 2\alpha\, U$$

$$U' = -\sin 2\alpha\, Q + \cos 2\alpha\, U$$

$$V' = V$$

$$Q = P \cos 2\theta \quad U = P \sin 2\theta$$

Changing the reference system



$$I' = I$$

$$Q' = \cos 2\alpha \; Q + \sin 2\alpha \; U$$

$$U' = -\sin 2\alpha \; Q + \cos 2\alpha \; U$$

$$V' = V$$

$$P = (Q^2 + U^2)^{1/2} = (Q'^2 + U'^2)^{1/2} = P'$$

is invariant!

$$Q = P \cos 2\alpha \quad U = P \sin 2\alpha$$

$$Q' = P \cos^2 2\alpha + P \sin^2 2\alpha = P$$

$$U' = -P \sin 2\alpha \cos 2\alpha + P \sin 2\alpha \cos 2\alpha = 0$$

Three ways to take the opposite sign…

1. Choice of the reference system

Three ways to take the opposite sign...

Right handed coordinate system (x,y,z), where:

- z-axis is directed along the direction of propagation
- x-axis is directed along an "arbitrary" direction

x-axis:

celestial meridian passing through the object

Three ways to take another sign...

Left handed coordinate system (x,y,z):

$$U \longrightarrow -U$$

$$V \longrightarrow -V$$

Three ways to take the opposite sign…

1. Choice of the reference system
2. Basic definitions

Three ways to take the opposite sign…

1. Choice of the reference system
2. Basic definitions

$$I(\omega) = k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle + \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$Q(\omega) = \pm k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle + \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$U(\omega) = \pm k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

$$V(\omega) = \pm i k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

Three ways to take the opposite sign...

1. Choice of the reference system
2. Basic definitions
3. Fourier Transform

Three ways to take the opposite sign...

Fourier transform:

$$E_x(\omega) = \int_0^\infty e^{-i\omega t} E_x(t)\, dt$$

$$E_y(\omega) = \int_0^\infty e^{-i\omega t} E_y(t)\, dt$$

Defining:

$$E_x(\omega) = \int_0^\infty e^{i\omega t} E_x(t)\, dt$$

$$E_y(\omega) = \int_0^\infty e^{i\omega t} E_y(t)\, dt$$

$V \longrightarrow -V$

Using relative units

$$I(\omega) = k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle + \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$Q(\omega) = k\left[\left\langle E_x(\omega)^* E_x(\omega)\right\rangle - \left\langle E_y(\omega)^* E_y(\omega)\right\rangle\right]$$

$$U(\omega) = k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

$$V(\omega) = \mathrm{i}k\left[\left\langle E_x(\omega)^* E_y(\omega)\right\rangle + \left\langle E_y(\omega)^* E_x(\omega)\right\rangle\right]$$

Using relative units

$$P_Q = Q/I$$
$$P_U = U/I$$
$$P_V = V/I$$

Operational definition of Stokes parameters

S = Signal with no filter

S_0, S_{45}, S_{90}, S_{135} = Signals with linear polarizer filters

S_R, S_L = Signals with circular polarizer filters

$I = k' S$; $Q = k' (S_0 - S_{90})$; $U = k' (S_{45} - S_{135})$; $V = k' (S_R - S_L)$

Shurcliff (1962)