Backward Monte Carlo modeling of polarization in planet atmospheres

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Motivation (I)

Observationally

- Atmospheric gases and aerosols leave distinct signatures in light scattered/emitted from a planet.
- Most observatories are equipped with polarimetric capabilities.
- Not so extended in space missions (Polder, Glory, …).

Polarimetry remains as an underexploited technique in characterization of planetary atmospheres

Exoplanets are bringing new interest into the technique

Motivation (II)

Theoretically

$$\mathbf{s}\cdot\nabla\mathbf{I}(\mathbf{x},\mathbf{s}) = -\gamma(\mathbf{x})\mathbf{I}(\mathbf{x},\mathbf{s}) + \beta(\mathbf{x})\int_\Omega d\Omega(\mathbf{s}')\mathbb{P}(\mathbf{x},\mathbf{s},\mathbf{s}')\mathbf{I}(\mathbf{x},\mathbf{s}')$$

- **Solution to RTE with multiple scattering. Scalar vs. vector. Plane parallel vs. other geometries, …**

Models: Quick, accurate, flexible

- **Scattering matrix**

$$\mathbb{P}(\mathbf{x},\mathbf{s},\mathbf{s}') = \mathbb{L}(\pi - i)\,\mathbb{M}(\mathbf{x},\mathbf{s},\mathbf{s}')\,\mathbb{L}(-i')$$

$$\mathbb{M}(\mathbf{x},\cos\theta) = \begin{pmatrix} a_1(\mathbf{x},\theta) & b_1(\mathbf{x},\theta) & 0 & 0 \\ b_1(\mathbf{x},\theta) & a_1(\mathbf{x},\theta) & 0 & 0 \\ 0 & 0 & a_3(\mathbf{x},\theta) & b_2(\mathbf{x},\theta) \\ 0 & 0 & -b_2(\mathbf{x},\theta) & a_3(\mathbf{x},\theta) \end{pmatrix}$$

Goal of the presentation

Address the performance of a Backward Monte Carlo algorithm for the vector RTE in planetary atmospheres

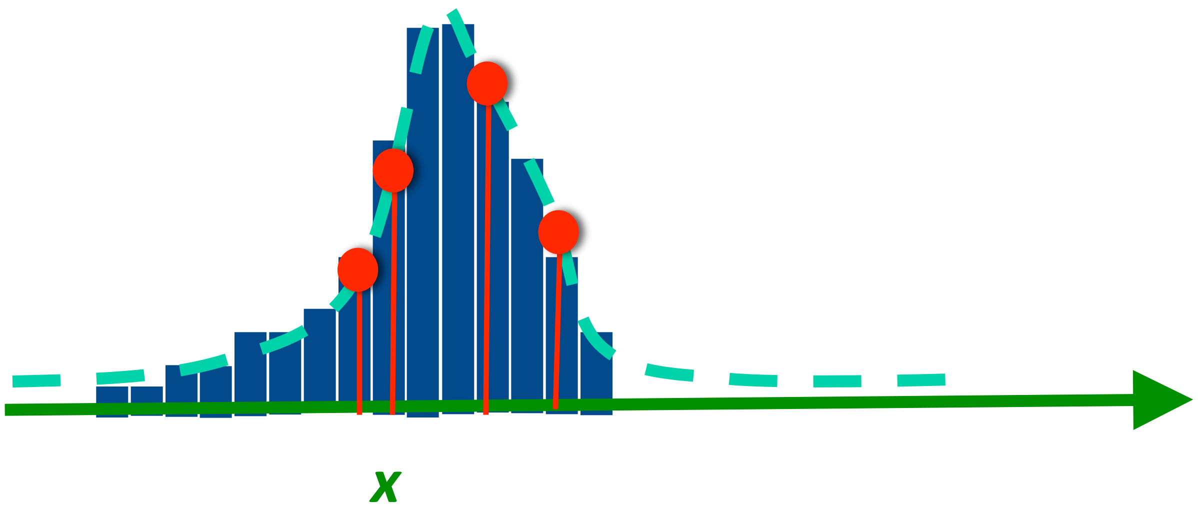
Through examples, demonstrate the capacities of the technique

Besides a demonstration exercise, useful as validation

Monte Carlo solutions to the vector Radiative Transport Equation

- **Flexibility:**
  - Geometry: Limb viewing, nadir, disk-integration…
  - Reflection, but also emission (thermal or not).
  - Easy implementation. Spherical & Non-spherical particles.
- **Accuracy:**
  - MC integration is often a standard. Exact solutions.
  - Sometimes it is the ***only*** option.
  - Adjustable, 0.1%, 1%, 5%, …
- **Swiftness:**
  - Depends on desired accuracy.

How MC integration works



Simpson's rule:

$$\int_0^1 f(x)dx \approx \sum_{i=1}^{N} f(x_i)\Delta x$$

Constant Δx

$$\int_0^1 f(x)dx \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$ **+ $O(N^{-1/2})$**

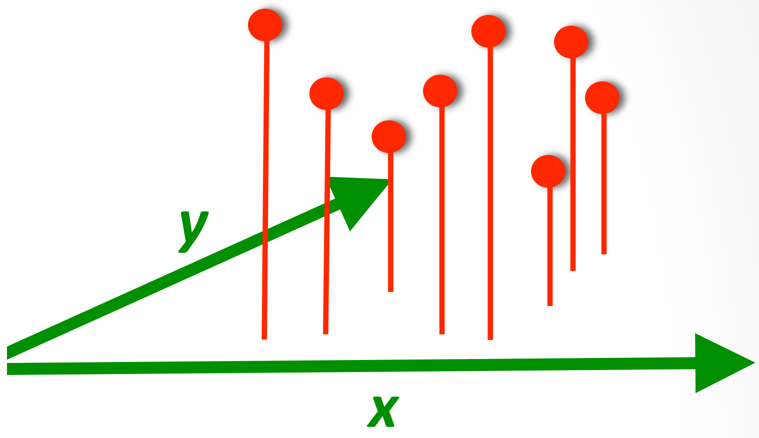
MC algorithm

Draw x_i from uniform [0, 1] distribution

How MC integration works

In 2D:

$$\int_0^1 \int_0^1 f(x,y)\,dx\,dy \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i, y_i)$$



In n-D:

$$\int_0^1 \dots \int_0^1 f(x_1, x_2, \dots, x_n)\,dx_1 dx_2 \dots dx_n \approx \frac{1}{N}\sum_{i=1}^{N} f(x_1^{[i]}, x_2^{[i]}, \dots, x_n^{[i]}) \quad + O(N^{-1/2})$$

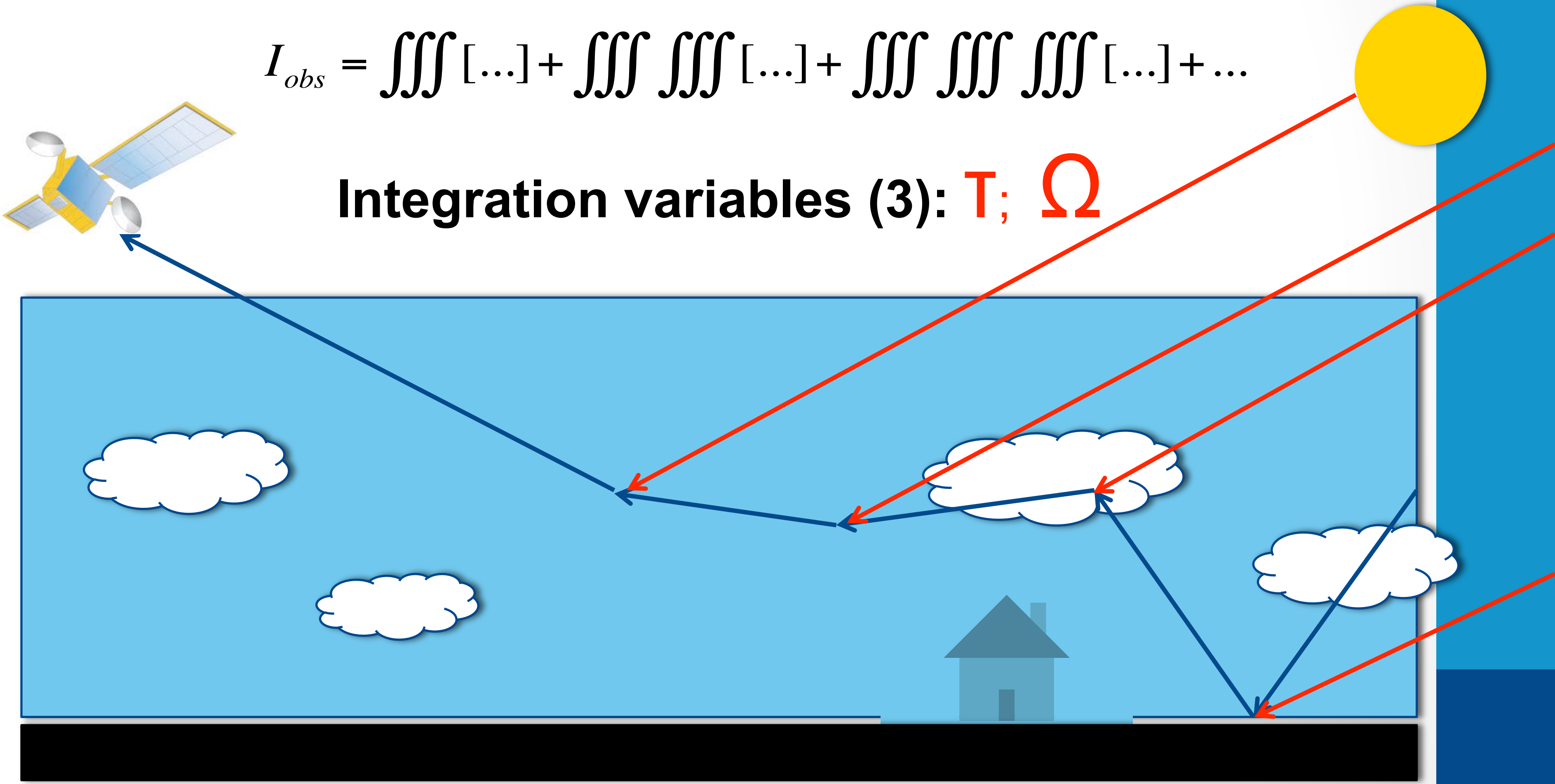
Draw $x^{[i]}_1$, $x^{[i]}_2$,…, $x^{[i]}_n$ from uniform [0, 1] distributions

Backward MC integration of the VRTE

Exact solution (O'Brien, 1992): Infinite sum of integrals

$$I_{obs} = \iiint [\ldots] + \iiint \iiint [\ldots] + \iiint \iiint \iiint [\ldots] + \ldots$$

Integration variables (3): τ; Ω



Backward MC integration of the VRTE

Looking at one of those integrals

$$I_n = \iint P(i_1,i_2,\theta)\,d\tau d\Omega \ldots \iint P(i_1,i_2,\theta)\,d\tau d\Omega \ldots \iint P(i_1,i_2,\theta) \ldots d\tau d\Omega$$

Sampling: $d\tau d\Omega$

MC Estimate: Finite summation of products

$$I_n = \sum_k P_k(i_1,i_2,\theta) \ldots P_k(i_1,i_2,\theta) \ldots P_k(i_1,i_2,\theta)$$

How Backward MC integration works

Is the summation convergent? Each term contains:

$$a(\mathbf{x}_0, \mathbf{x}_{0b})\varpi(\mathbf{x}_{0a})\left[\frac{\mathbb{P}(\mathbf{x}_{0a}, \mathbf{s}_0 \cdot \mathbf{s}_{0a})}{\mathbb{M}_{1,1}(\mathbf{x}_{0a}, \mathbf{s}_0 \cdot \mathbf{s}_{0a})}\right] a(\mathbf{x}_{0a}, \mathbf{x}_{0ab})\varpi(\mathbf{x}_{0aa})\left[\frac{\mathbb{P}(\mathbf{x}_{0aa}, \mathbf{s}}{\mathbb{M}_{1,1}(\mathbf{x}_{0aa},}\right.$$

Probabilities

$a(\mathbf{x},\mathbf{x}_b)=1-exp(-\tau) \leq 1$ *That the photon will have a collision* ***within*** *the medium*

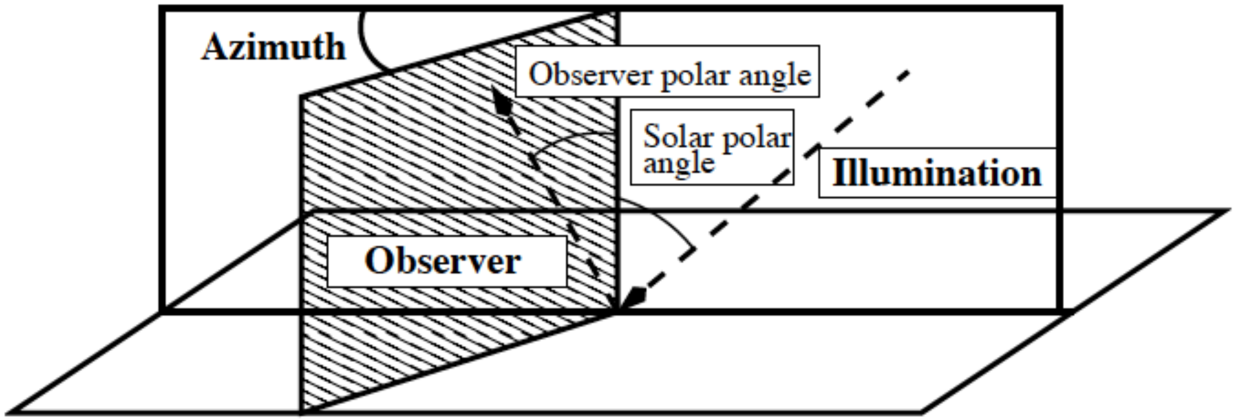
$\omega(\mathbf{x}) \leq 1$ *That the photon will* ***not*** *be absorbed by the medium*

Convergence is faster in thin, absorbing atmospheres

Last term: =1 for scalar problem

My BMC algorithm. Validation

Classical problem for Rayleigh, plane-parallel atmosphere



Optical thickness: *0.02, 0.05, 0.1, 0.15, 0.25, 0.5, 1, 2, 4, 8, 16, 32*
Surface albedo: *0, 0.25, 0.8*
Cos(illumination polar angle): *0.1, 0.2, 0.4, 0.6, 0.8, 0.92, 1*
Cos(Observer polar angle): *0.1, 0.2, 0.4, 0.52, 0.64, 0.84, 0.92, 1*
Azimuth: *0, 30, 60, 90, 120, 150, 180*

Total # cases*= 12 x 3 x 7 x 8 x 7 =* ***14,112***

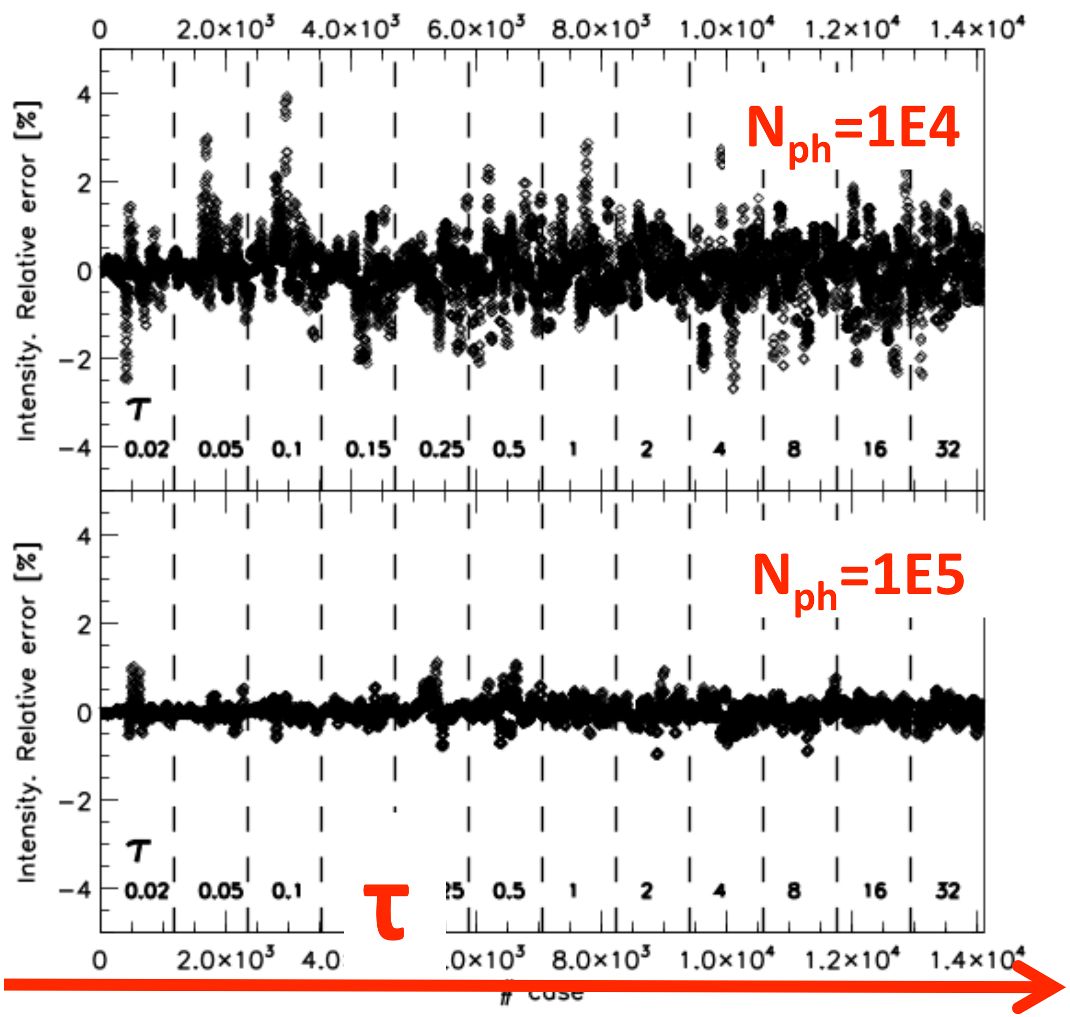
Validation. Plane parallel

Scalar problem: *BMC vs. DISORT. Fully conservative medium.*

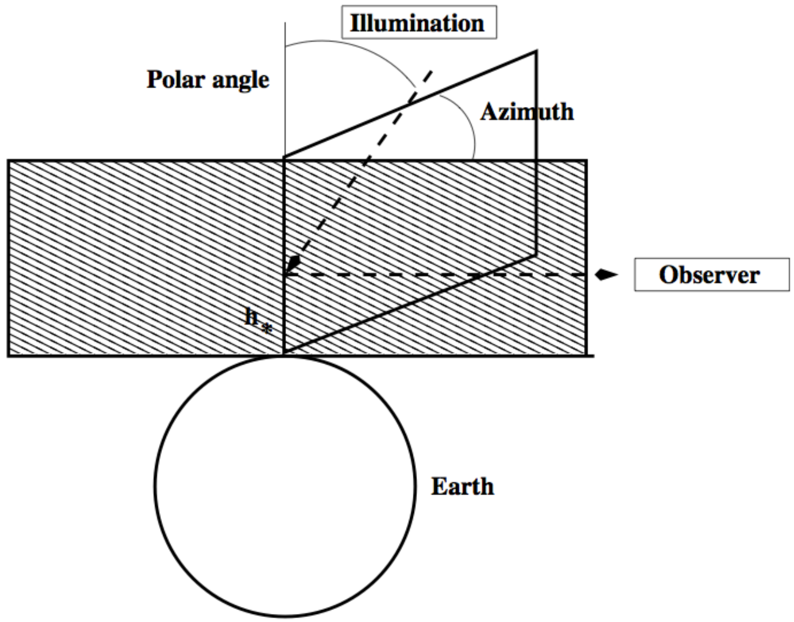
Nph	Mean Deviation [%]	Median Deviation [%]
1E4	0.39	0.28
1E5	0.12	0.08

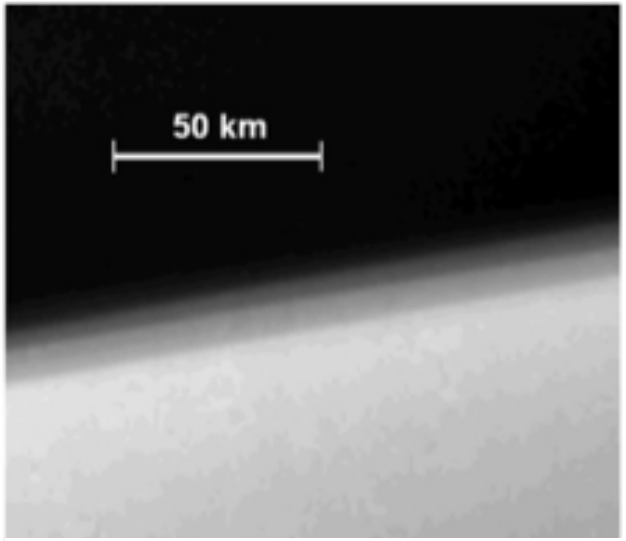
Convergence:

$O(N^{-1/2})$ law



Validation. Limb viewing





VMC image. Titov et al. (2012)

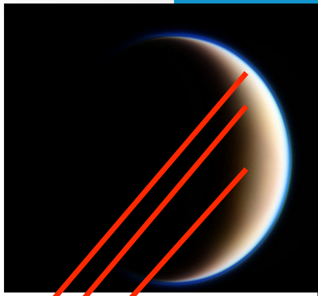
(Postylyakov's) MCC++:

Vertically stratified, shell-symmetric atmosphere, various wavelengths, compositions, with and w/o polarization (Loughman et al., JGR, 2004)

Over about hundred test cases & N_{ph}=1E5, Mean(ΔI)=0.32% & Mean(ΔDoP)=0.48%

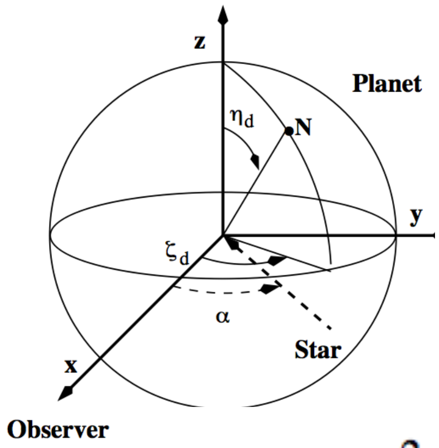
Disk integration

- *Relevant in the context of exoplanet exploration*



Titan by Cassini. Credit: ESA

Equations for disk integration by Horak (1950):



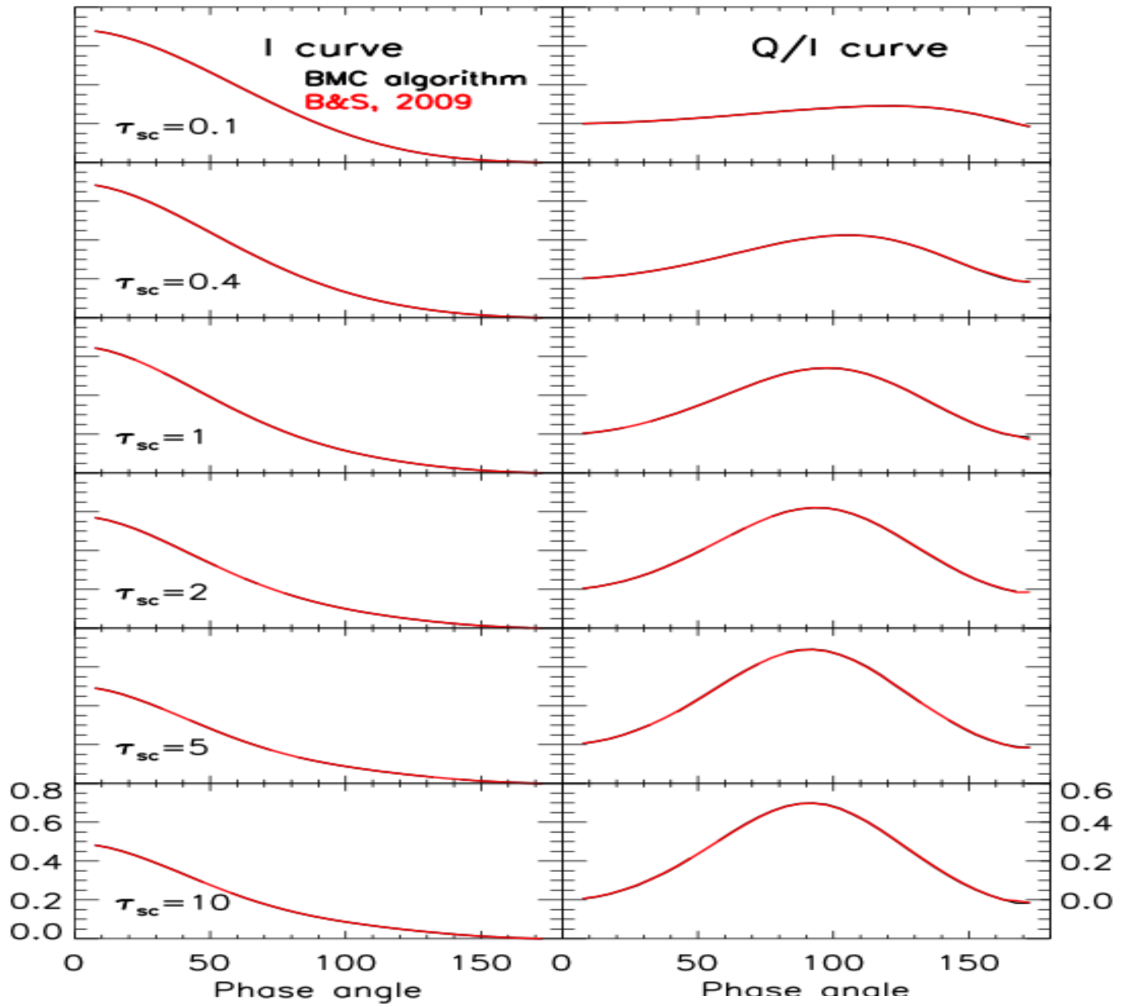
$$\mathbf{F} = \left(\frac{\rho}{\Delta}\right)^2 \int_0^{\pi} d\eta_d \sin^2(\eta_d) \int_{\alpha-\pi/2}^{\pi/2} d\zeta_d \cos(\zeta_d) \mathbf{I}(\zeta_d, \eta_d)$$

$$\mathbf{F} = \left(\frac{\rho}{\Delta}\right)^2 \frac{\pi}{2}(1 + \cos(\alpha)) \frac{1}{n_{\rm ph}} \sum_{i=1}^{n_{\rm ph}} < \mathbf{I}(u_i, v_i) > + O(N^{-1/2})$$

Therefore, same cost for integration over pixel and over disk

Disk integration

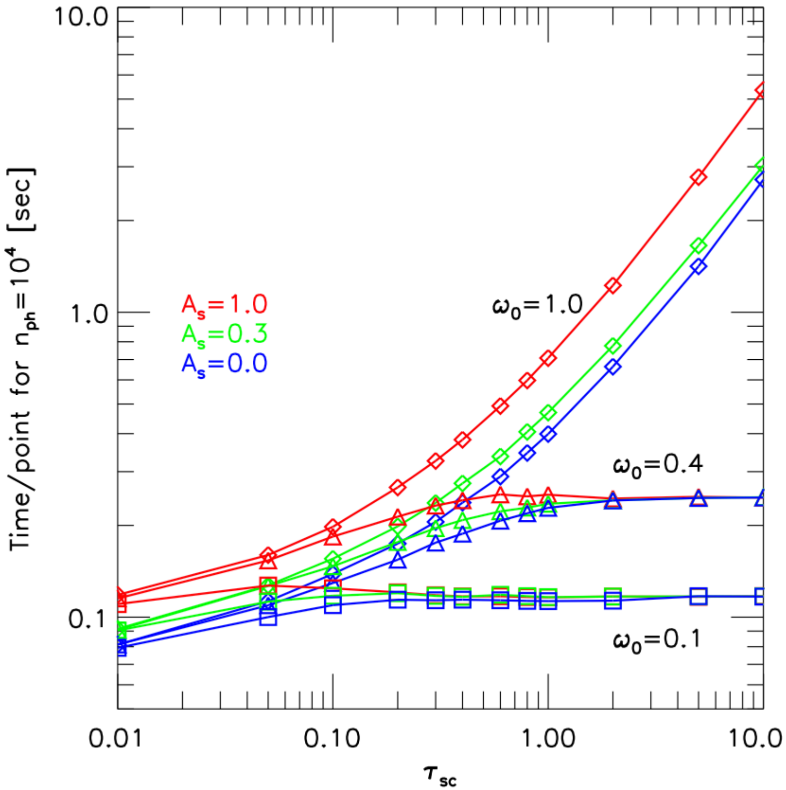
Non-conservative Rayleigh. ϖ=0.95



Comparison vs. Buenzli & Schmid (2009)

Disk integration. Computational time.

(Also for non-disk integration)



Ultimate truth:

Cost depends on # collisions before the photon gets absorbed or escapes the atmosphere

A_s: Surface albedo
ϖ_0: Single scattering albedo
T: Optical thickness

For fully-conservative, semi-infinite solution (As=ϖ_0=1, T=10)

Nph=1E4 → 5 secs / point (<ΔI>=0.55%)
Nph=1E5 → 50 secs / point (<ΔI>=0.19%)

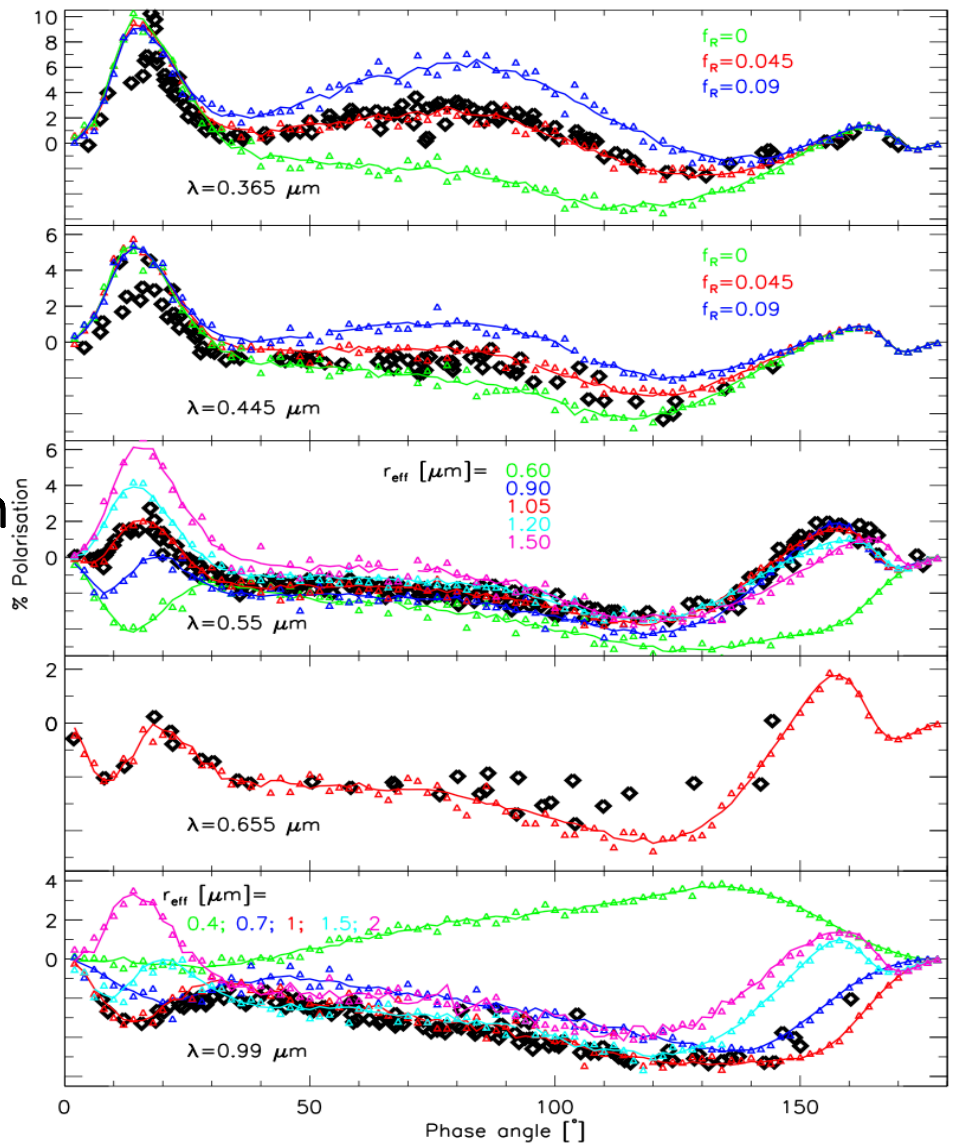
Disk integration. Venus

◆: *Observations*
(Hansen & Hovenier, 1974)

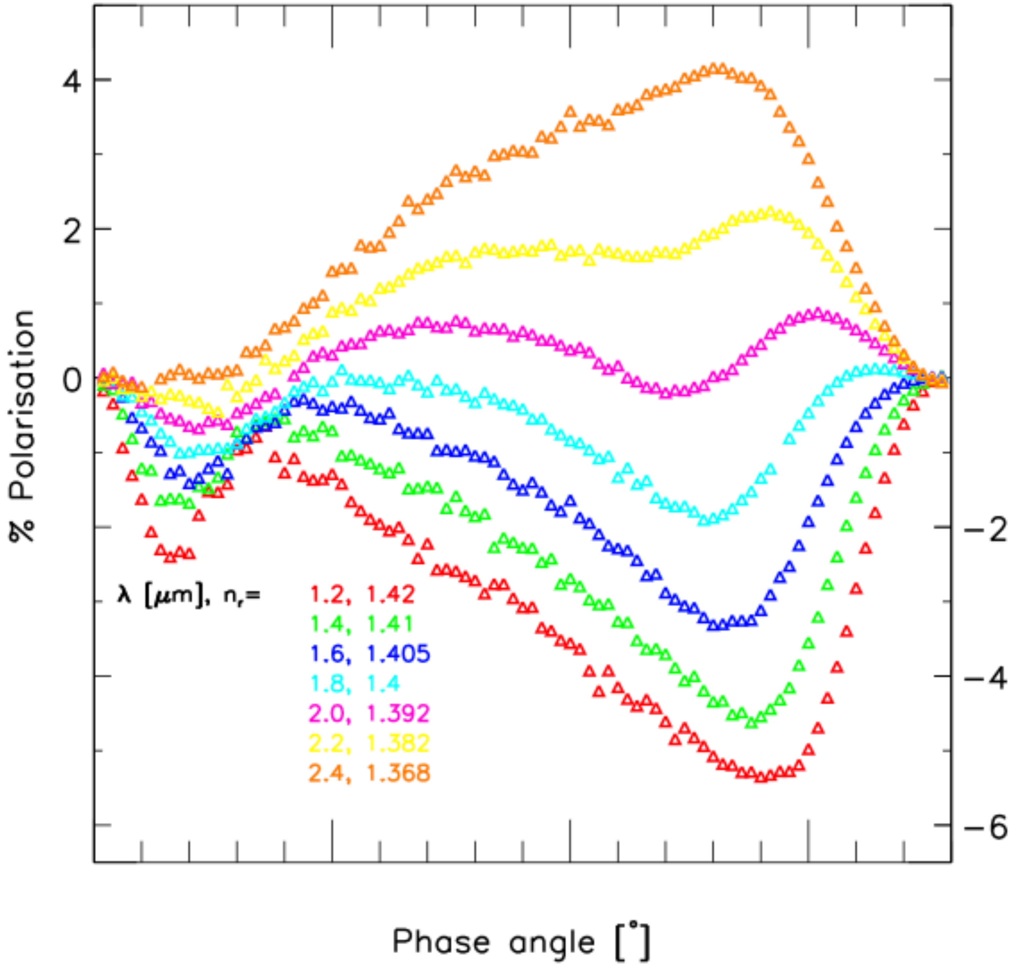
Color ▲ : *Model, N_{ph}=1E4*
(As=ϖ_0=1, T=10)
2 secs/ point x 90 points = 3 min for full light curve

Color ▬▬ : *Model, N_{ph}=1E5*

30 min for full light curve



Disk integration. Venus



NIR predictions
for N_{ph}=1E5

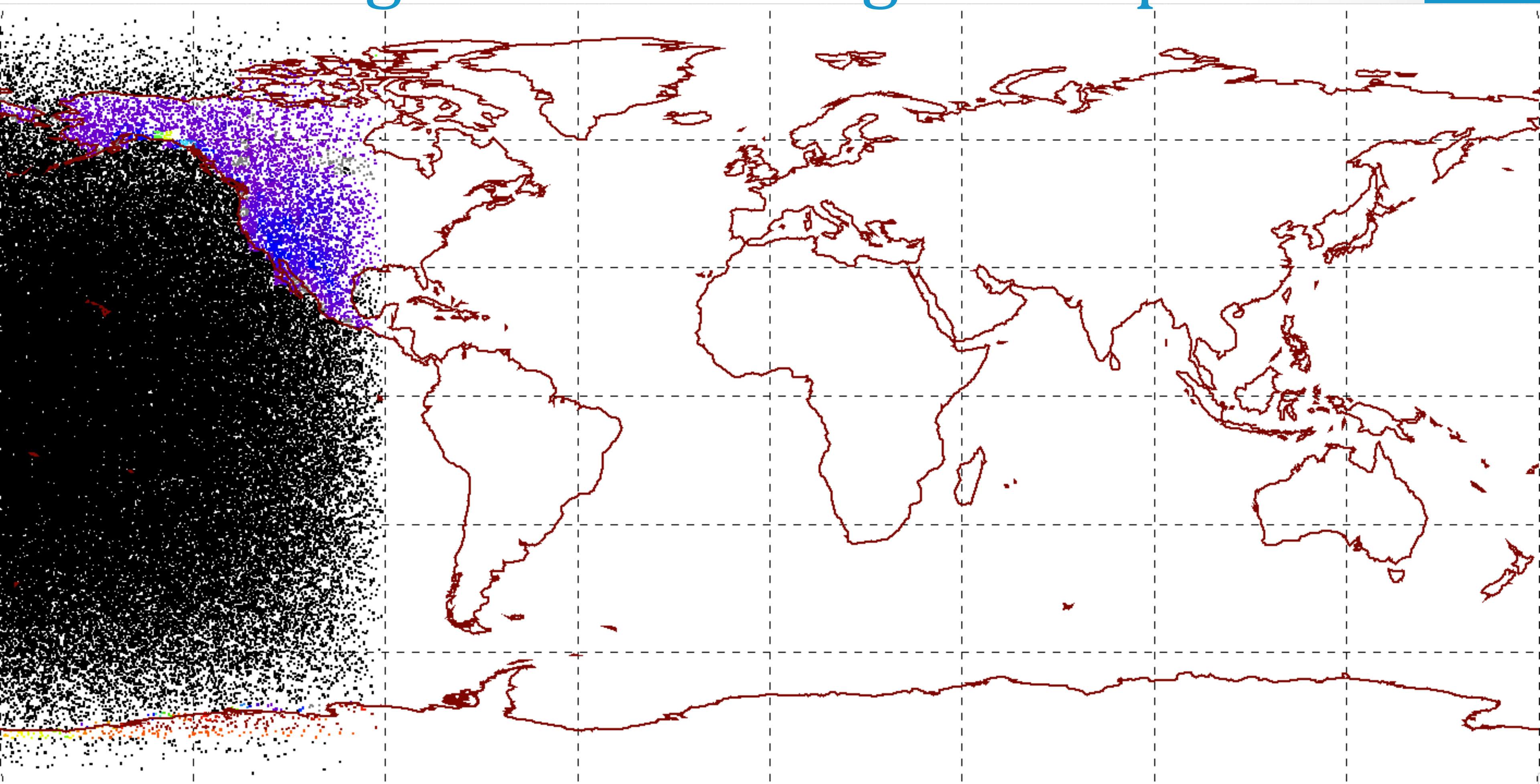
Polarization light curves are rich in details

Disk integration. Inhomogeneous planet

$$\mathbf{F} = \left(\frac{\rho}{\Delta}\right)^2 \frac{\pi}{2}(1+\cos(\alpha))\frac{1}{n_{ph}}\sum_{i=1}^{n_{ph}} < \mathbf{I}(u_i, v_i) > \quad + O(N^{-1/2})$$

Expression valid for inhomogeneous planet

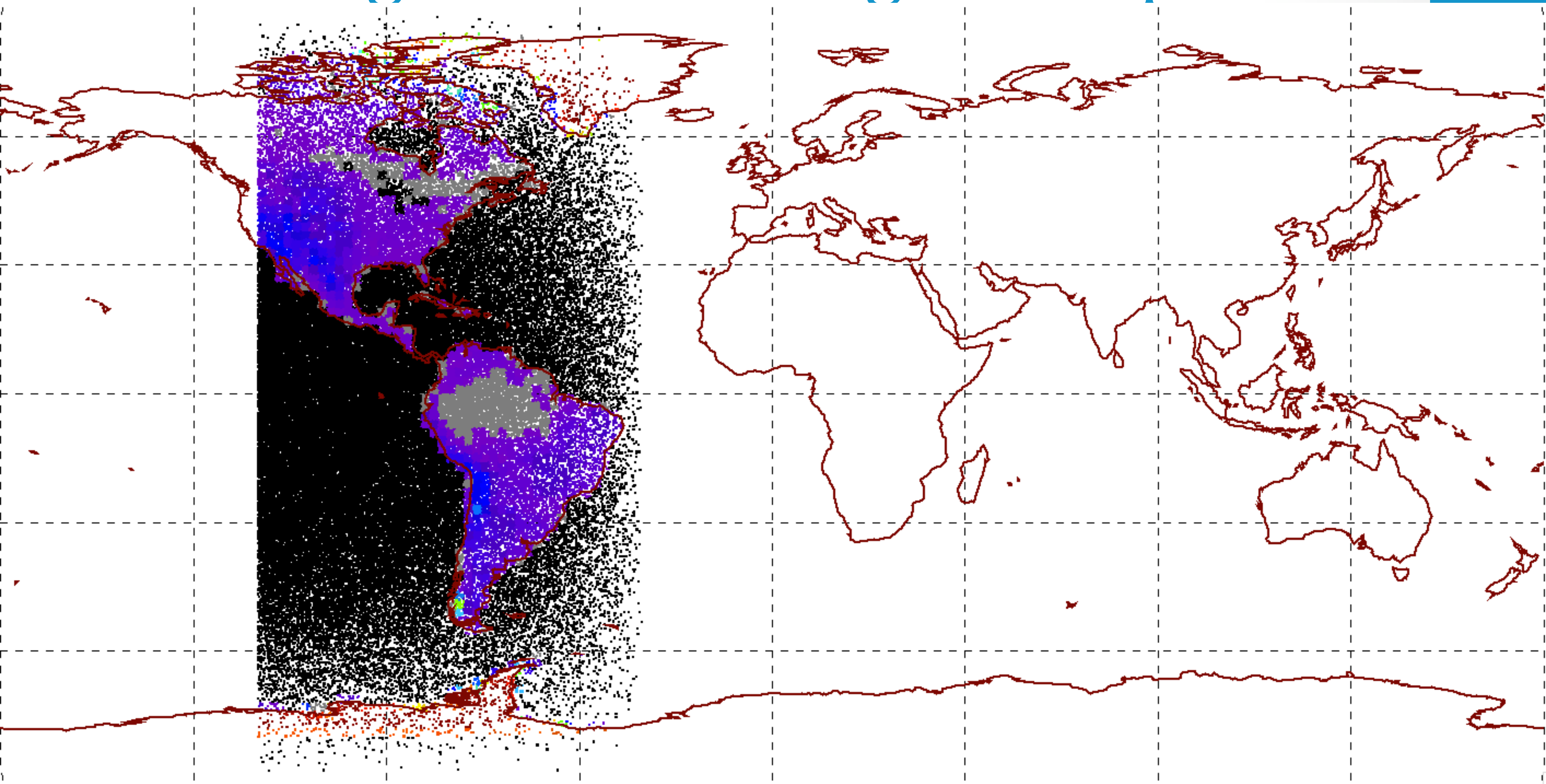
Disk integration. Inhomogeneous planet



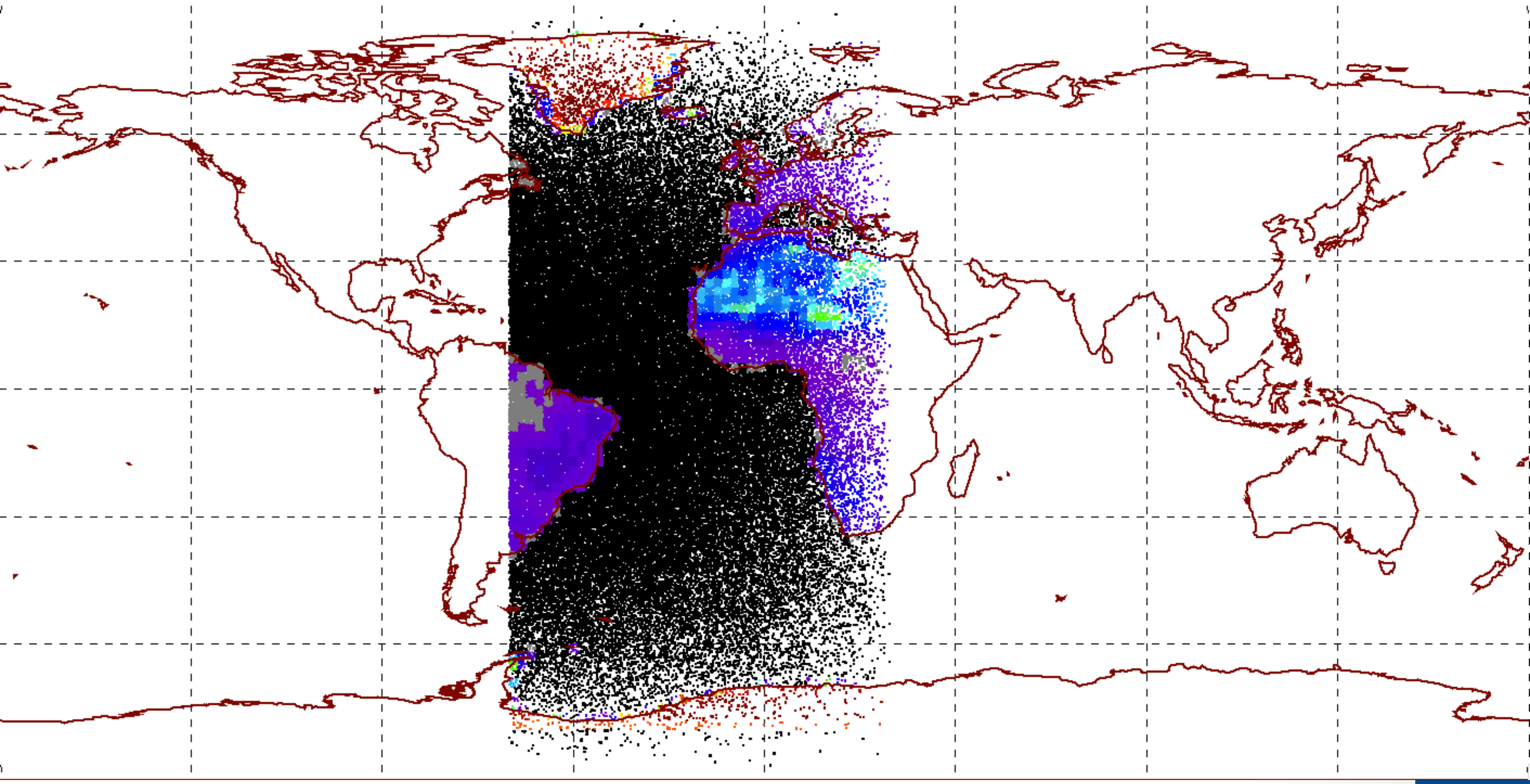
Phase angle = 90°

White-sky albedos from Terra/MODIS
From the UV to the NIR

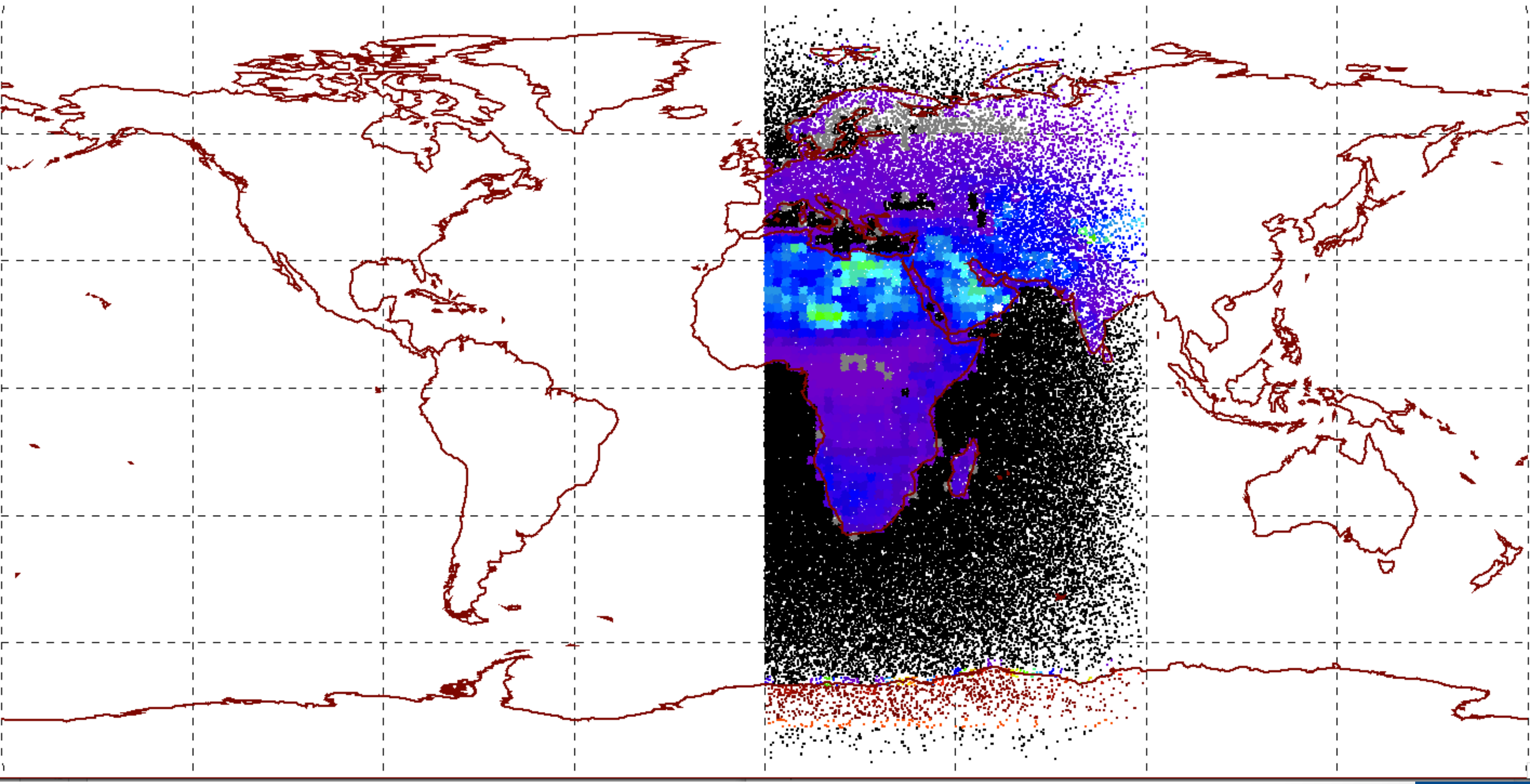
Disk integration. Inhomogeneous planet



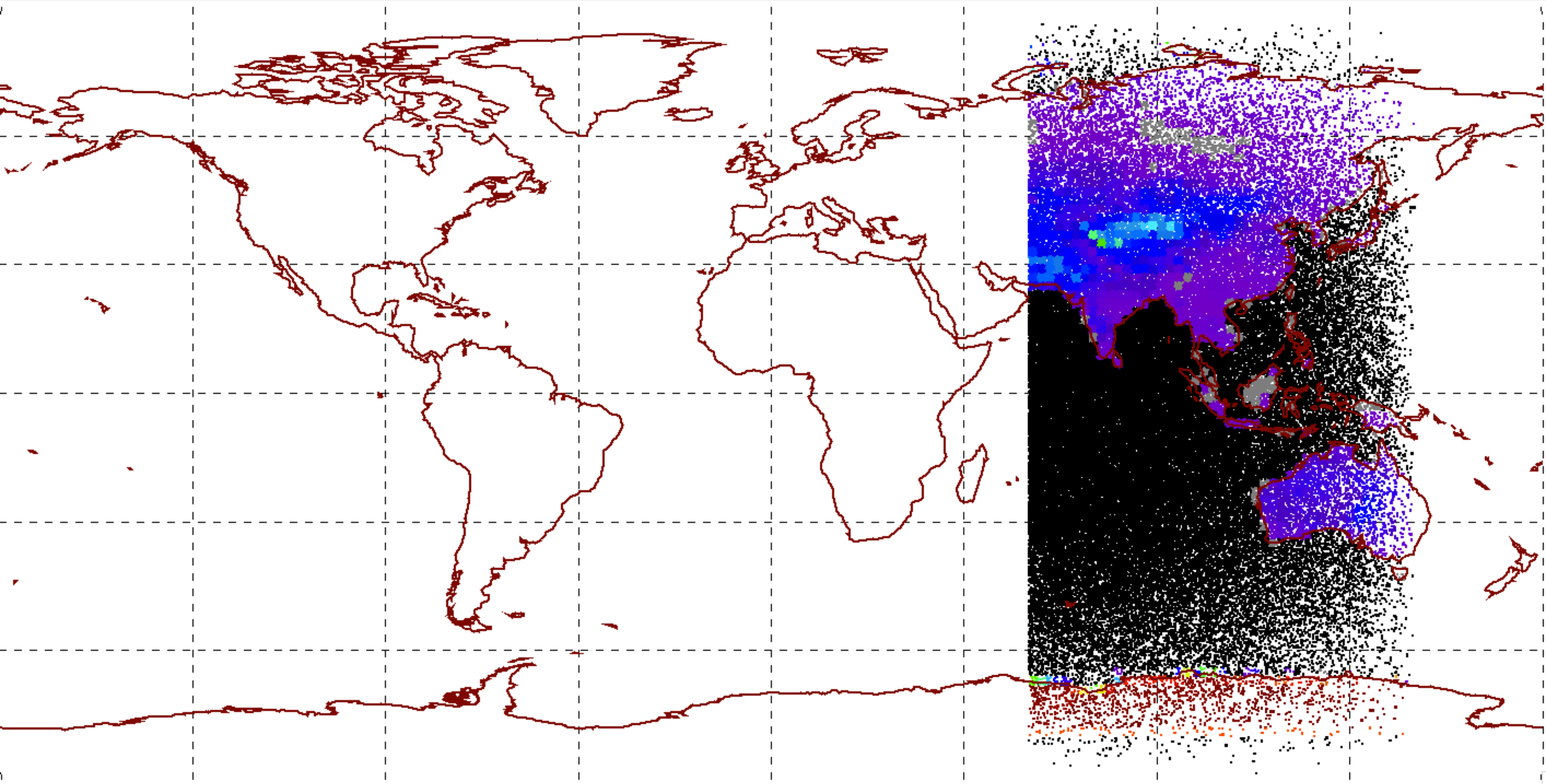
Disk integration. Inhomogeneous planet



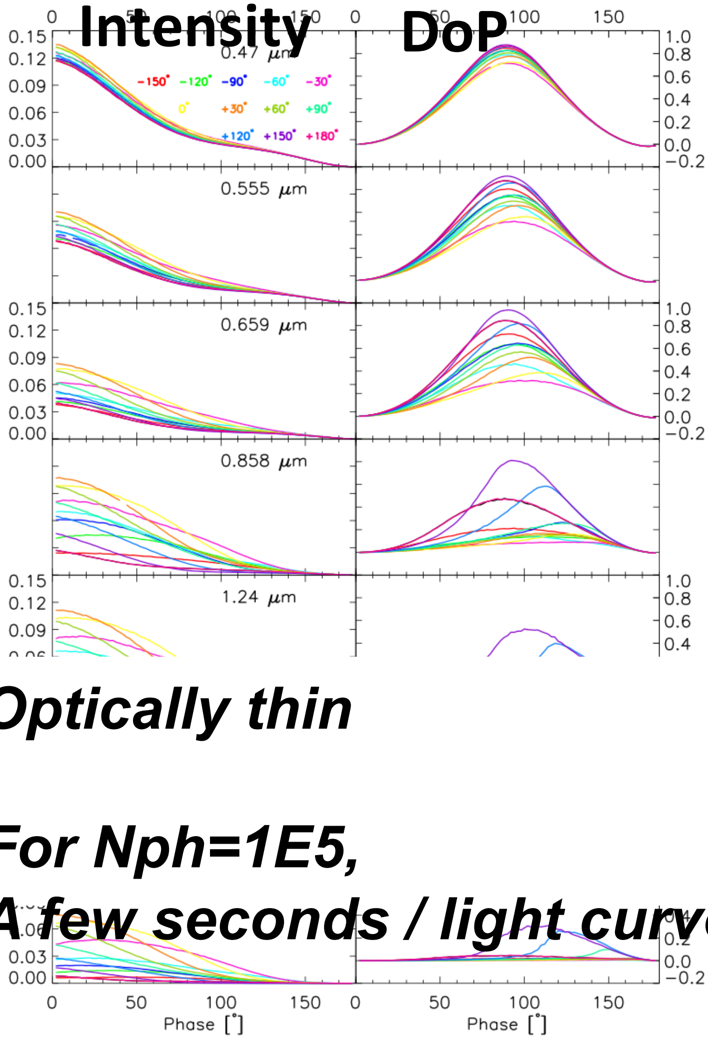
Disk integration. Inhomogeneous planet



Disk integration. Inhomogeneous planet

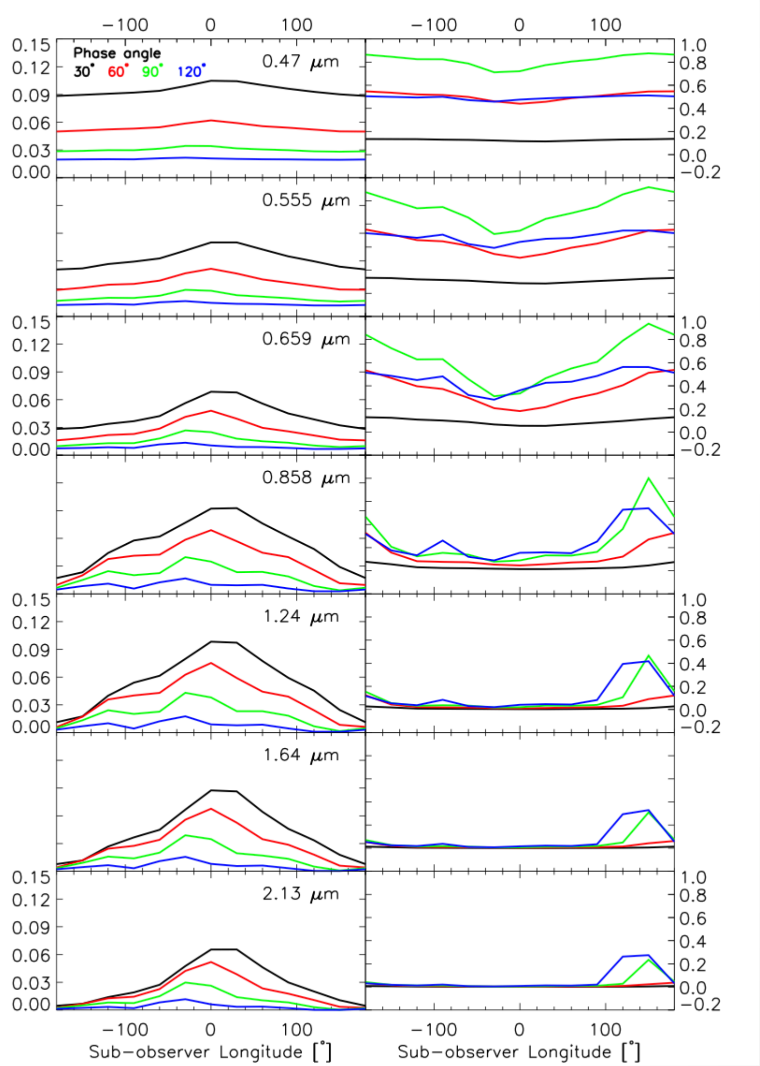


Disk integration. Inhomogeneous planet. Light curves

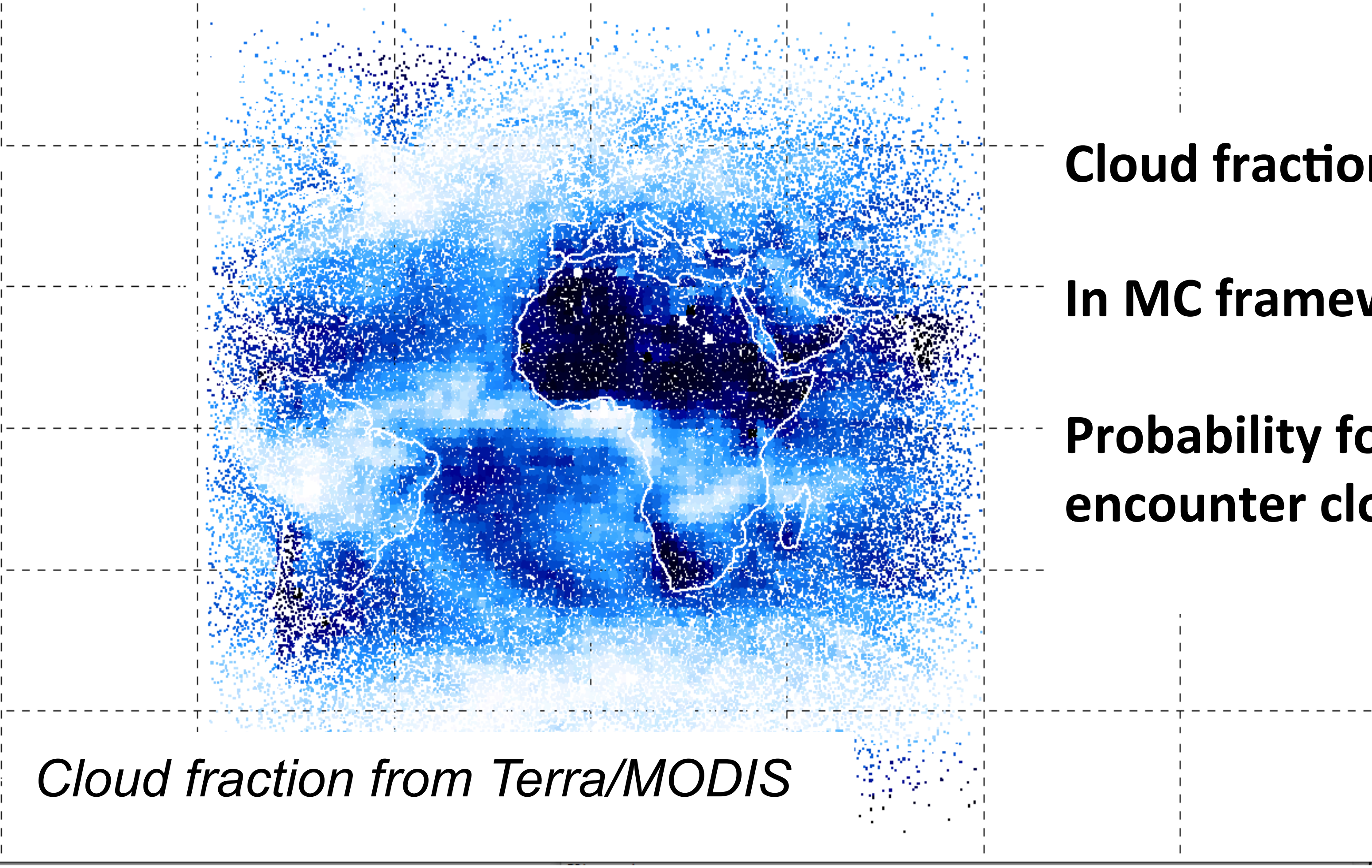


Optically thin

For Nph=1E5,
A few seconds / light curve



Disk integration. Inhomogeneous planet. Light curves



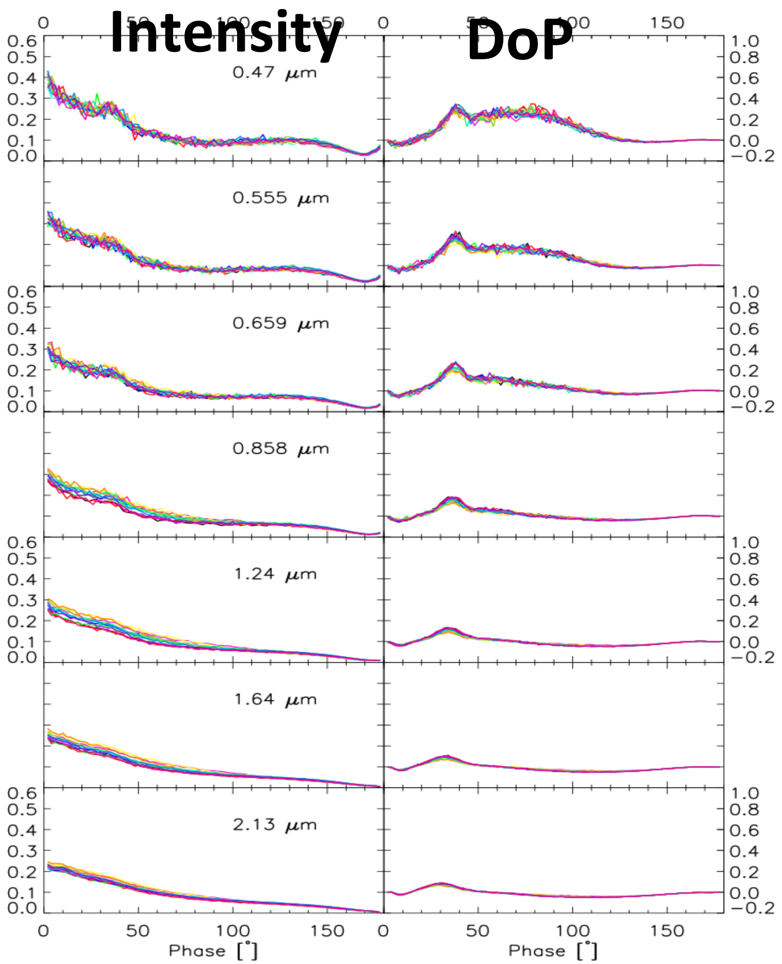
Cloud fraction from Terra/MODIS

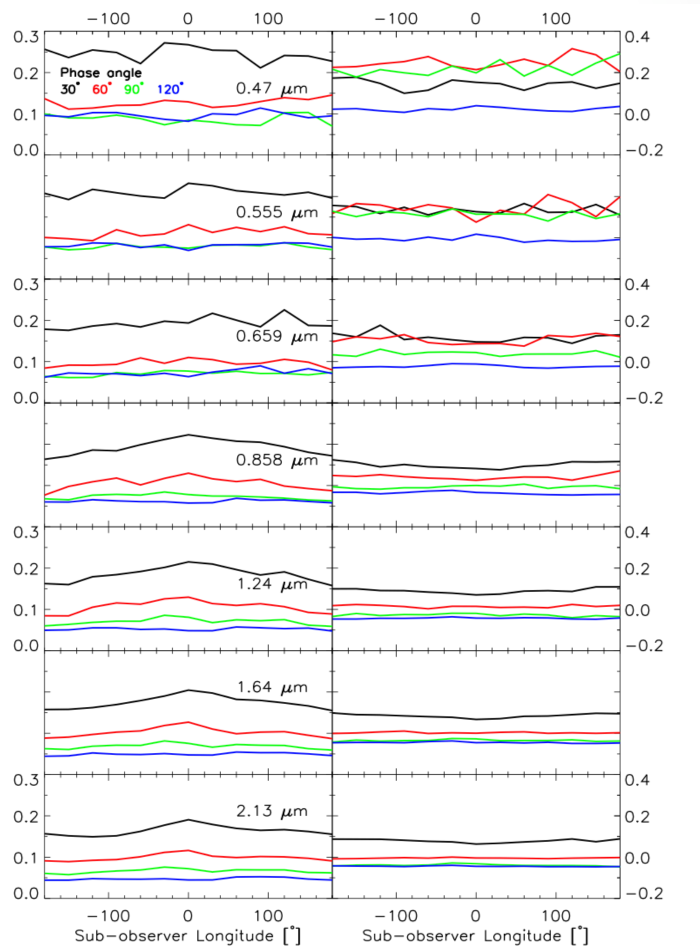
Cloud fraction f

In MC framework

Probability for photon to encounter cloud

Disk integration. Inhomogeneous planet





Summary & future work

- MC integration **is** flexible. It **can** be accurate and quick.
- Ideally suited for problems with disk-integration and moderate accuracy. Spectral integration.
- Explore planet's appearance over shorter timescales with realistic albedo, cloud, composition maps…

Thanks!

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