Multi-line polarisation analysis of stellar spectra

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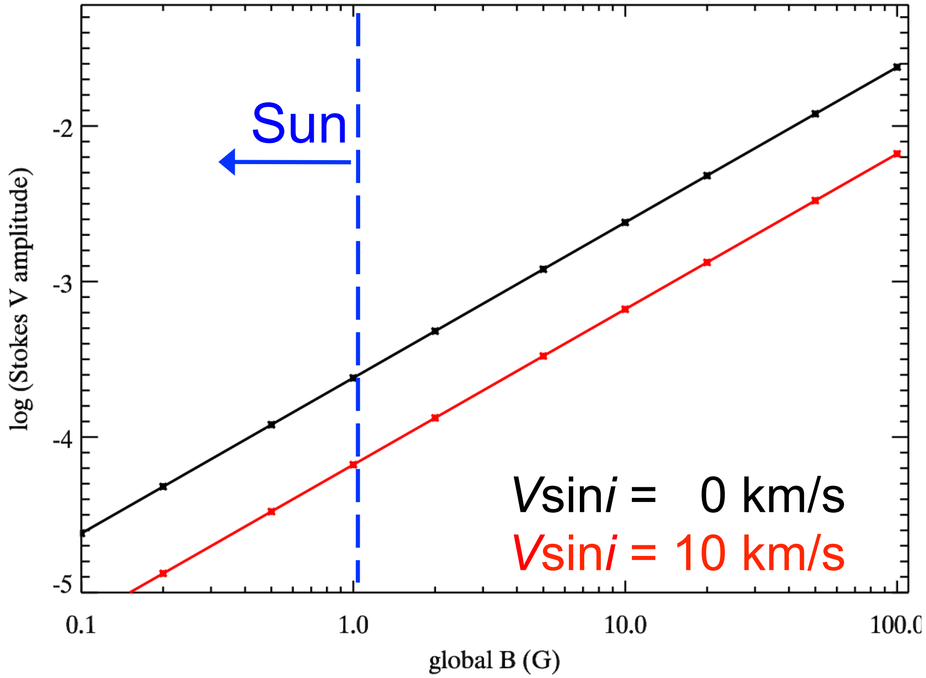
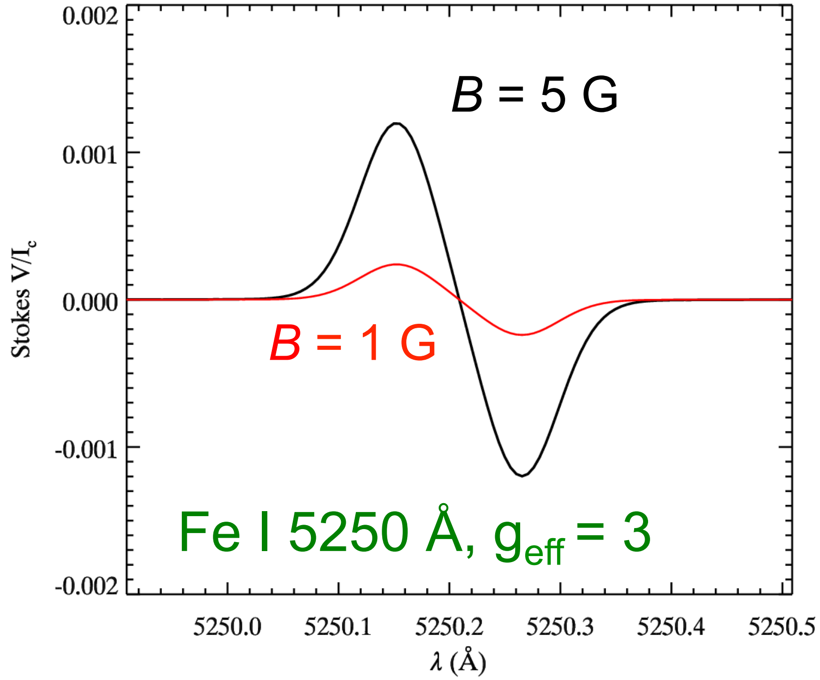
Zeeman effect in stellar spectra

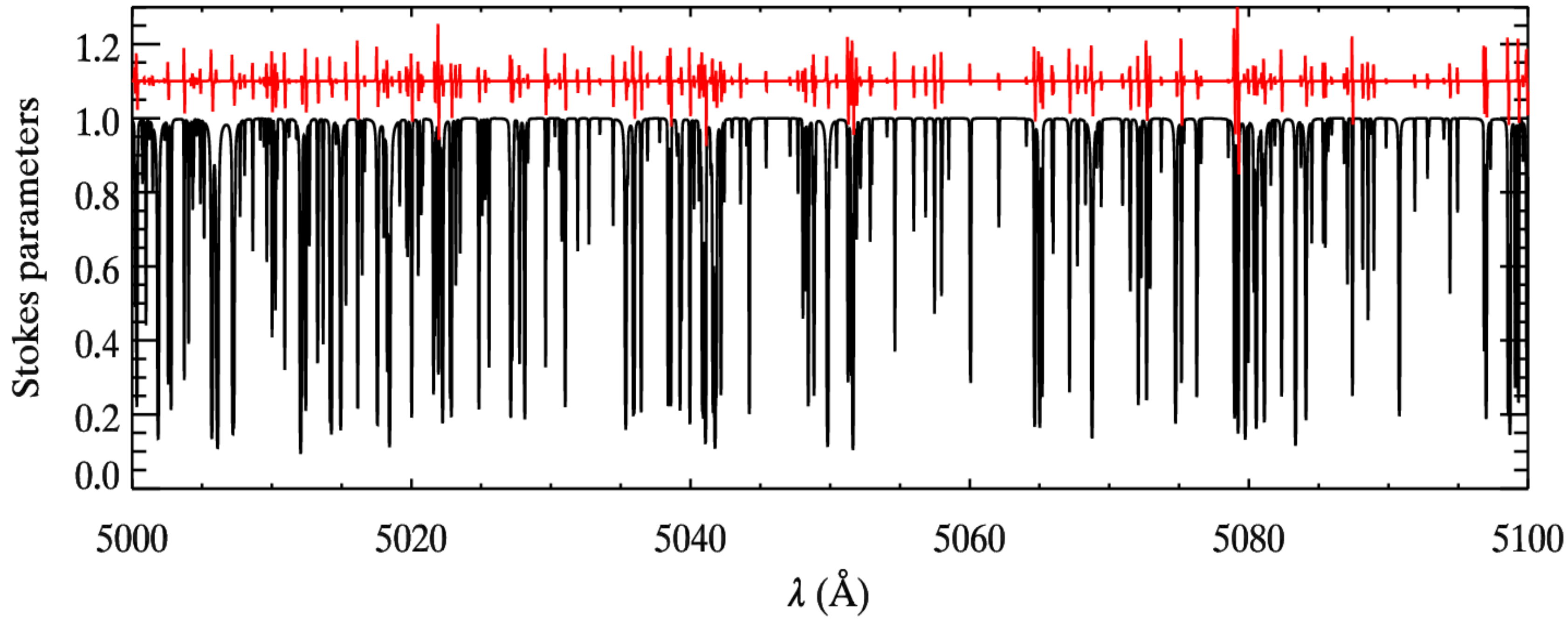
- Polarisation due to Zeeman effect is weak
- Required precision/sensitivity
  - strongly magnetic/active stars: ~10^{-2} to 10^{-3}
  - other, "typical" stars: ~10^{-4} to 10^{-5}





Multi-line analysis?

- Polarisation signatures of different lines are similar
- Modern stellar spectropolarimetric observations have a wide wavelength coverage



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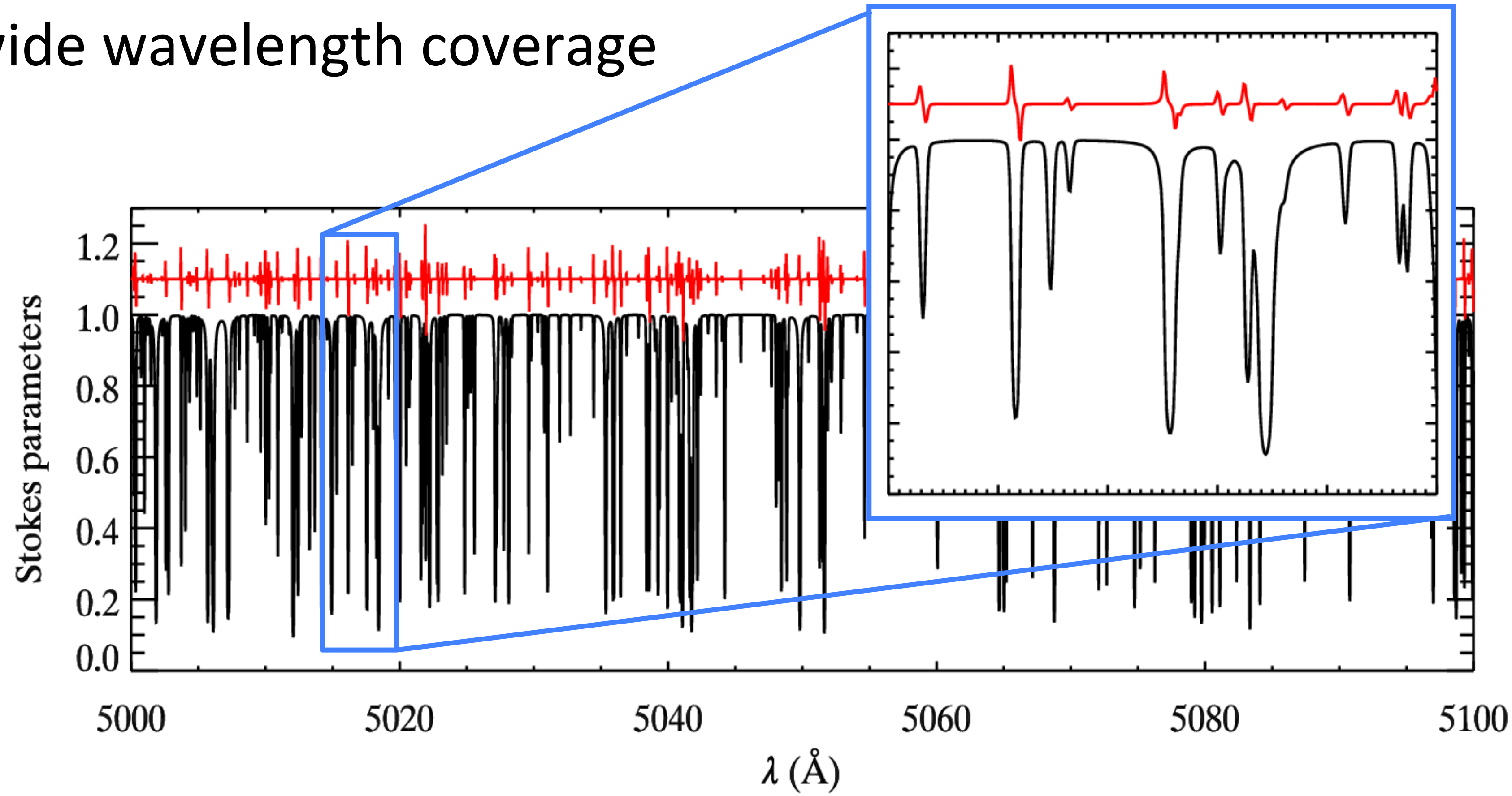


Simplified spectrum description

- Weak field ($B \leq 1$ kG) and weak line ($d \leq 0.3$) assumptions

$$1 - I_{\rm loc}(v) \propto d P_I(v)$$

$$V_{\rm loc}(v) \propto \lambda \bar{g} B_\parallel \frac{\partial I_{\rm loc}}{\partial v} = d \lambda \bar{g} B_\parallel P_V(v)$$

$$Q_{\rm loc}(v) \propto \lambda^2 \bar{G} B_\perp^2 \frac{\partial^2 I_{\rm loc}}{\partial v^2} = d \lambda^2 \bar{G} B_\perp^2 P_Q(v)$$

- Disk integration

$$\iint V_{\rm loc}(v - v_{\rm Dop}) {\rm d}S \propto d \lambda \bar{g} \iint B_\parallel P_V(v - v_{\rm Dop}) {\rm d}S = d \lambda \bar{g} Z_V(v)$$

Least-squares deconvolution

- Spectrum = superposition of shifted and scaled profiles

$$I(v) = 1 - \sum_i w_I^i Z_I(v - v^i), \quad w_I^i = d_i$$

$$V(v) = \sum_i w_V^i Z_V(v - v^i), \quad w_V^i = \bar{g}\lambda_i d_i$$

$$Q(v) = \sum_i w_Q^i Z_Q(v - v^i), \quad w_Q^i = \bar{G}\lambda_i^2 d_i$$

Donati et al. (1997)
Wade et al. (2000)
Kochukhov et al. (2010)

- Equivalent formulation as a matrix multiplication

$$\boldsymbol{V} = \boldsymbol{M} \cdot \boldsymbol{Z}_V$$

model or observed spectrum

spectral line data

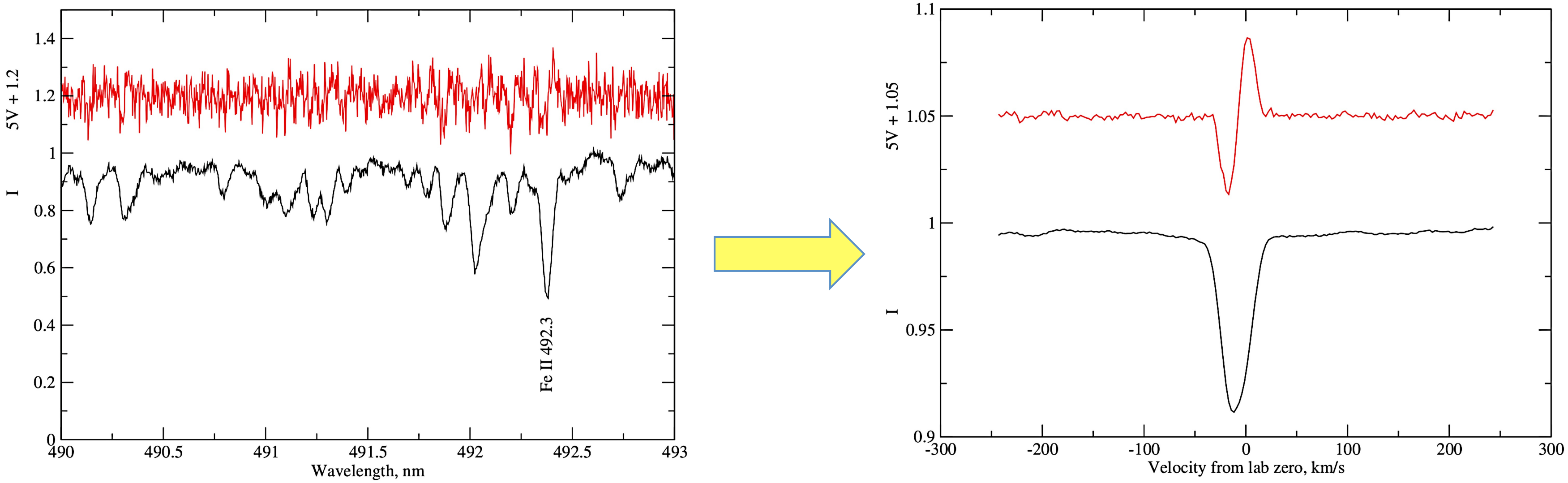
mean profile

Implementation of LSD

- Finding mean profile for a given spectrum and line mask

$$||\boldsymbol{V}_{\mathrm{obs}} - \boldsymbol{M} \cdot \boldsymbol{Z}_V|| \rightarrow \min$$

$$\boldsymbol{Z}_V = \underbrace{(\boldsymbol{M}^{\mathrm{T}} \cdot \boldsymbol{S}^2 \cdot \boldsymbol{M})^{-1}}_{\text{inverse of a.c. matrix}} \cdot \underbrace{\boldsymbol{M}^{\mathrm{T}} \cdot \boldsymbol{S}^2 \cdot \boldsymbol{V}_{\mathrm{obs}}}_{\text{CCF}}$$



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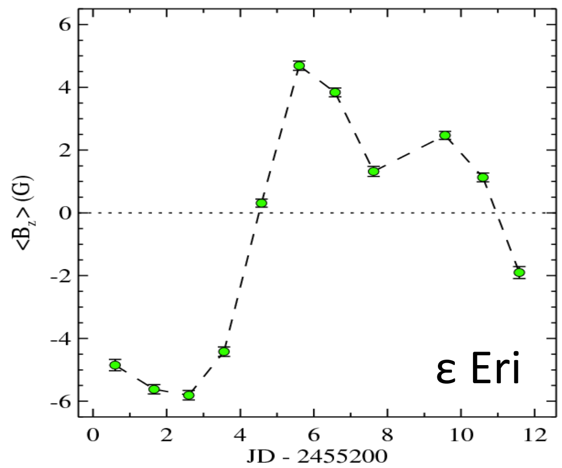
- S/N gain of 10-40, polarimetric sensitivity $<10^{-5}$

- Compression of information (from ~3×10^5 wavelength pixels to ~100 velocity bins)

LSD applications (1)

- Magnetic detection and longitudinal field measurement
  - Mean longitudinal magnetic field

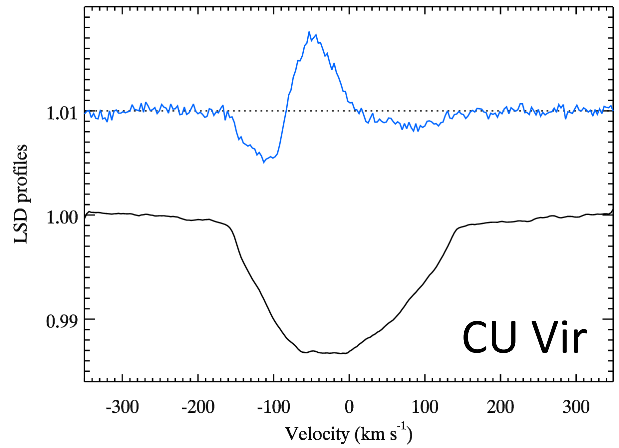
$$\langle B_z \rangle = -\frac{7.145 \times 10^6}{\lambda_0 g_0} \frac{\int V(v - v_0) \mathrm{d}v}{\int (1 - I) \mathrm{d}v}$$



  - Detection of polarisation signatures in lines profiles

$$FAP = 1 - \Gamma(\nu/2, \chi^2/2),$$

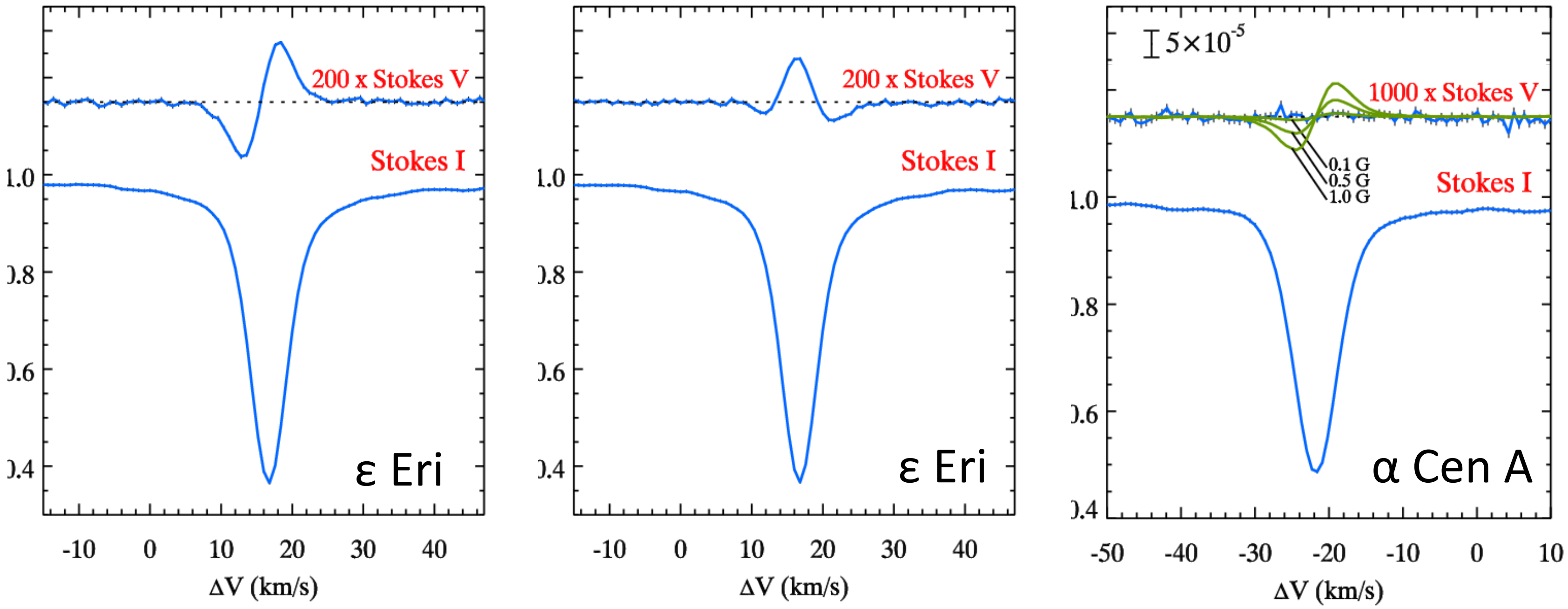
$$\chi^2 = \sum_i V_i^2 / \sigma_i^2$$



LSD applications (2)

- Precise polarisation measurements for solar-type stars

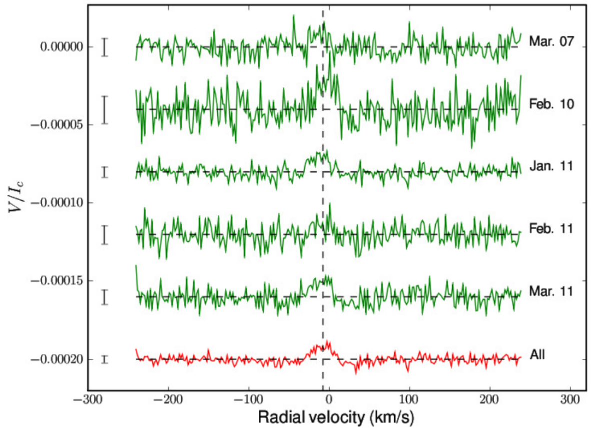
HARPSpol observations of ε Eri and α Cen A (Piskunov et al. 2011)



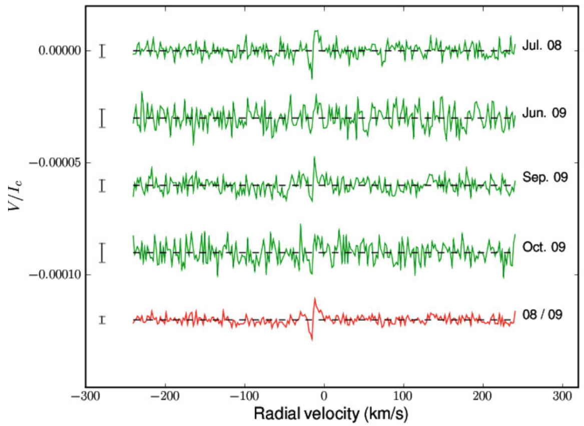
LSD applications (3)

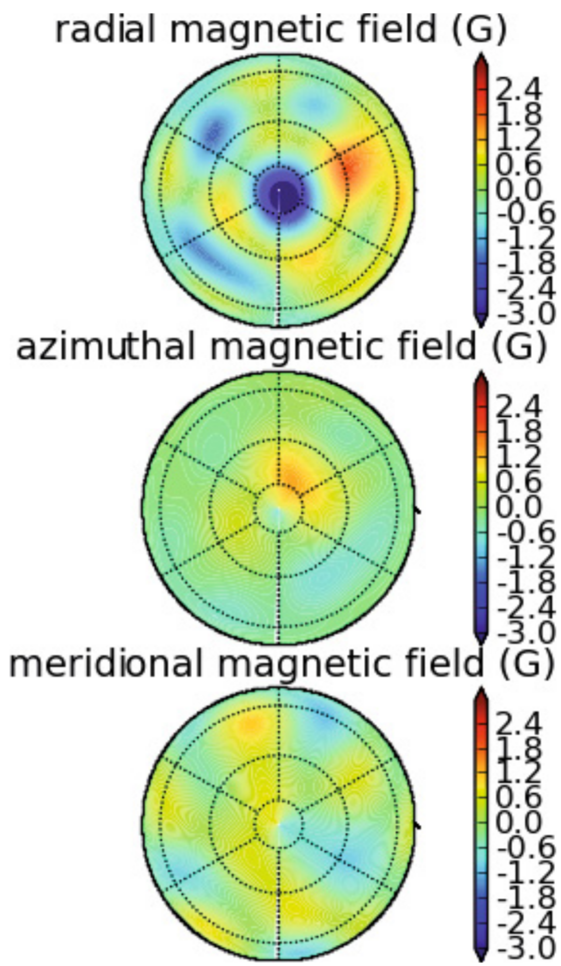
- Detection of weak magnetic fields in normal A-type stars

Sirius



Vega





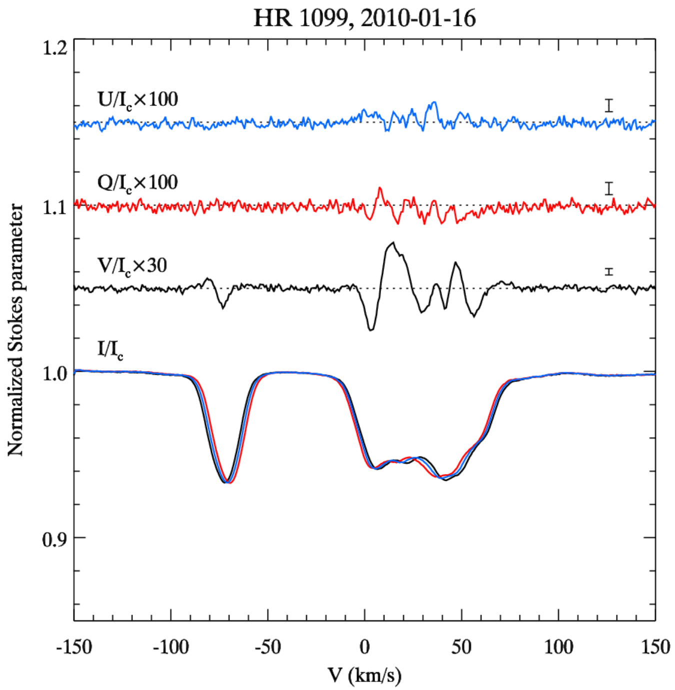
Polarimetric amplitudes $<10^{-5}$

Lignieres et al. (2009); Petit et al. (2010, 2011)

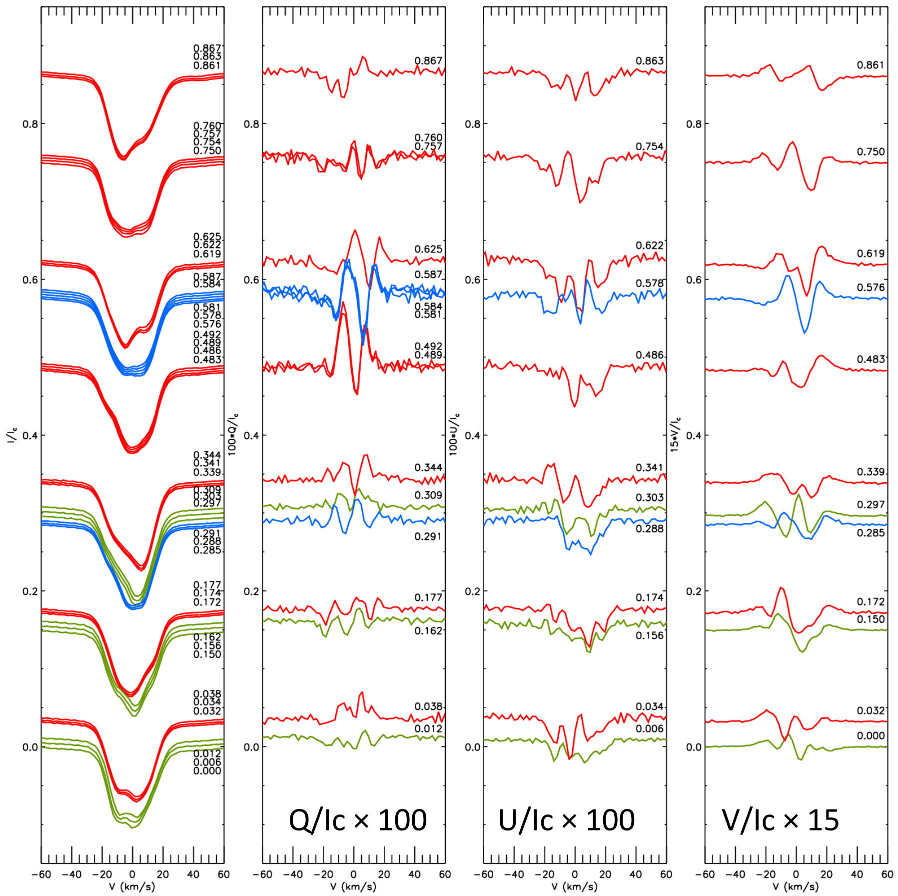
LSD applications (4)

- Full Stokes vector observations of active stars

Kochukhov et al. (2011)



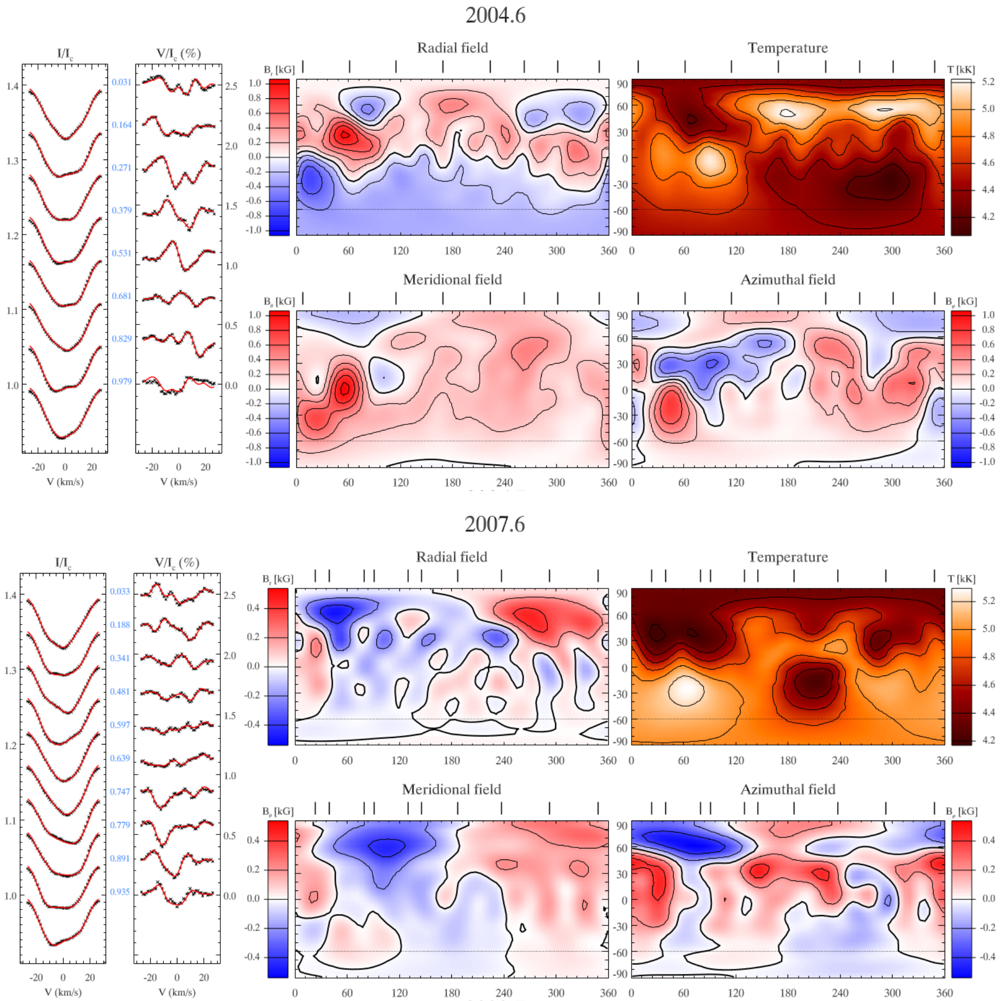
Rosén et al. (2013)



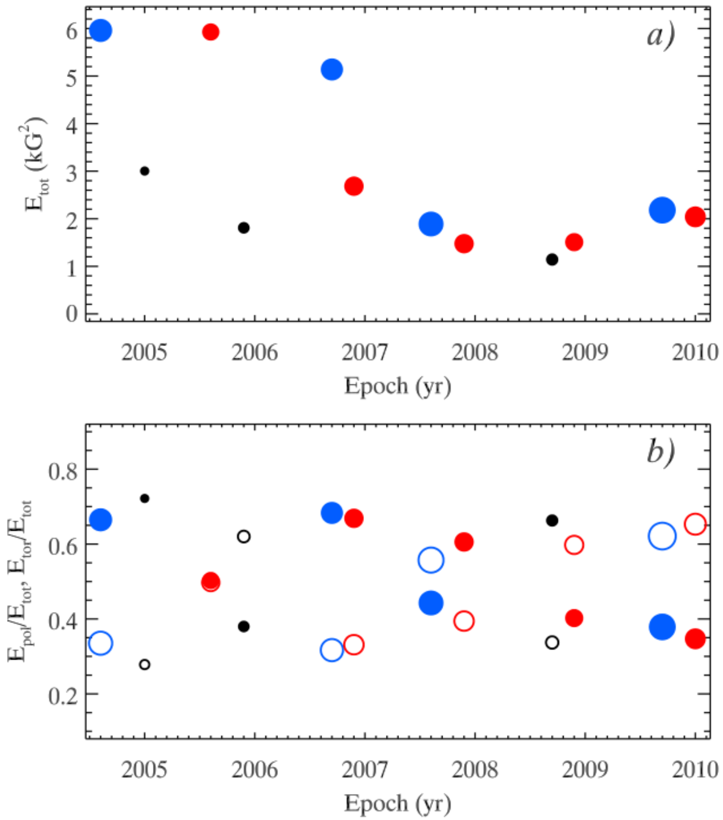
RS CVn stars HR 1099 and II Peg

LSD applications (5)

- ZDI mapping of stellar magnetic fields



Kochukhov et al. (2013)



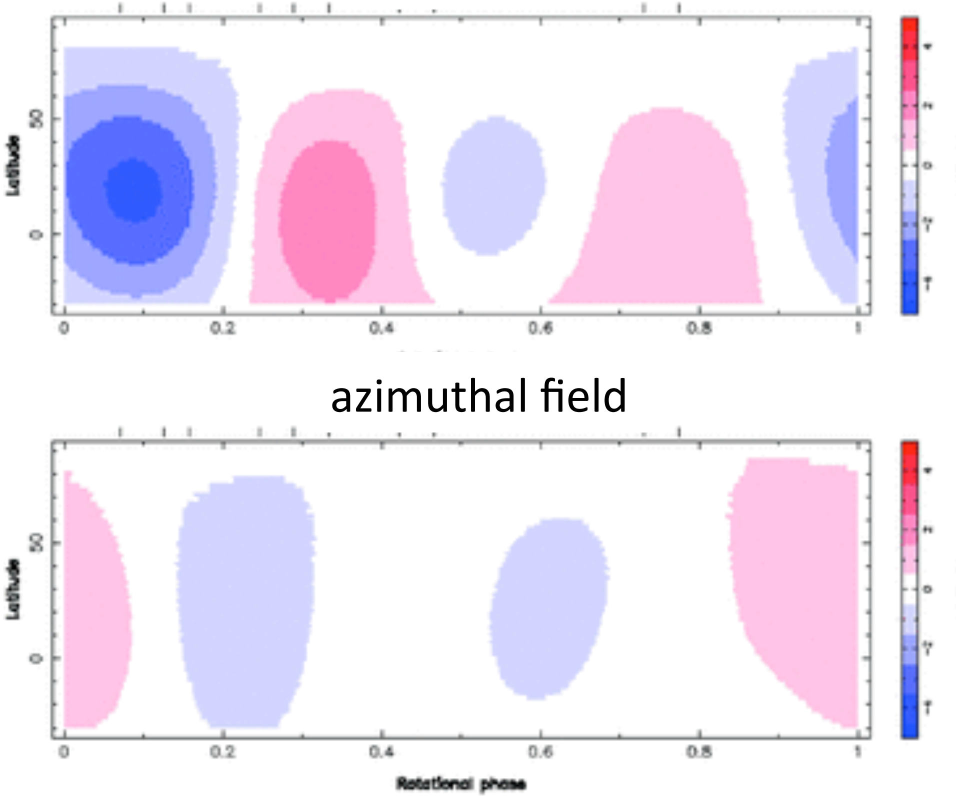
LSD applications (6)

- Global magnetic fields of inactive solar-type stars

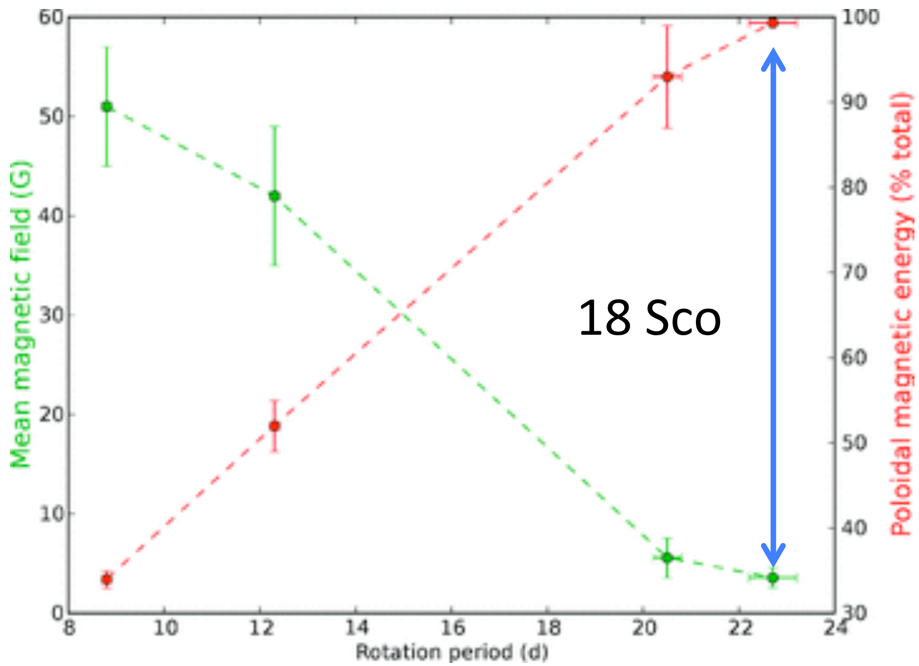
radial field

±5 G

azimuthal field

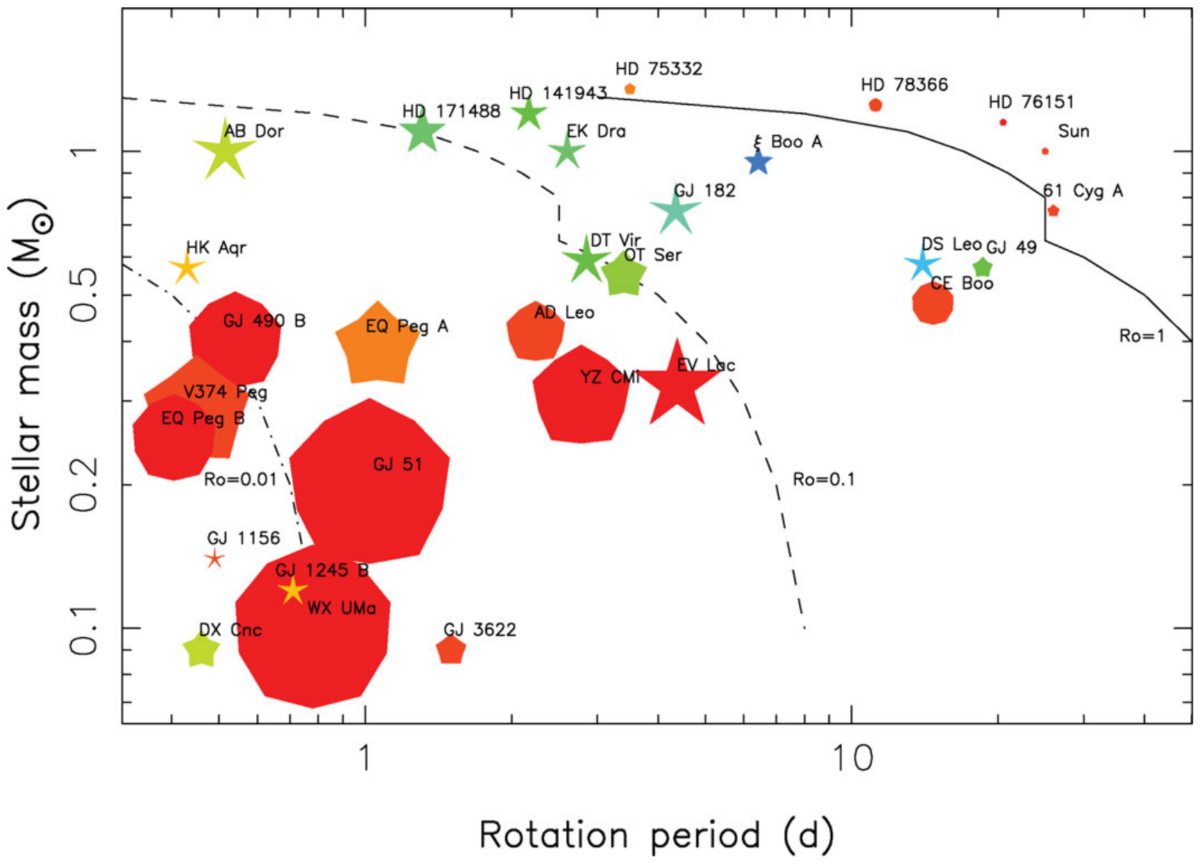


Petit et al. (2008)



LSD applications (7)

◆ Systematic investigation of stellar magnetic field topologies



Size:

field strength

Shape:

● axisymmetric

★ non-axisymmetric

Color:

toroidal

poloidal

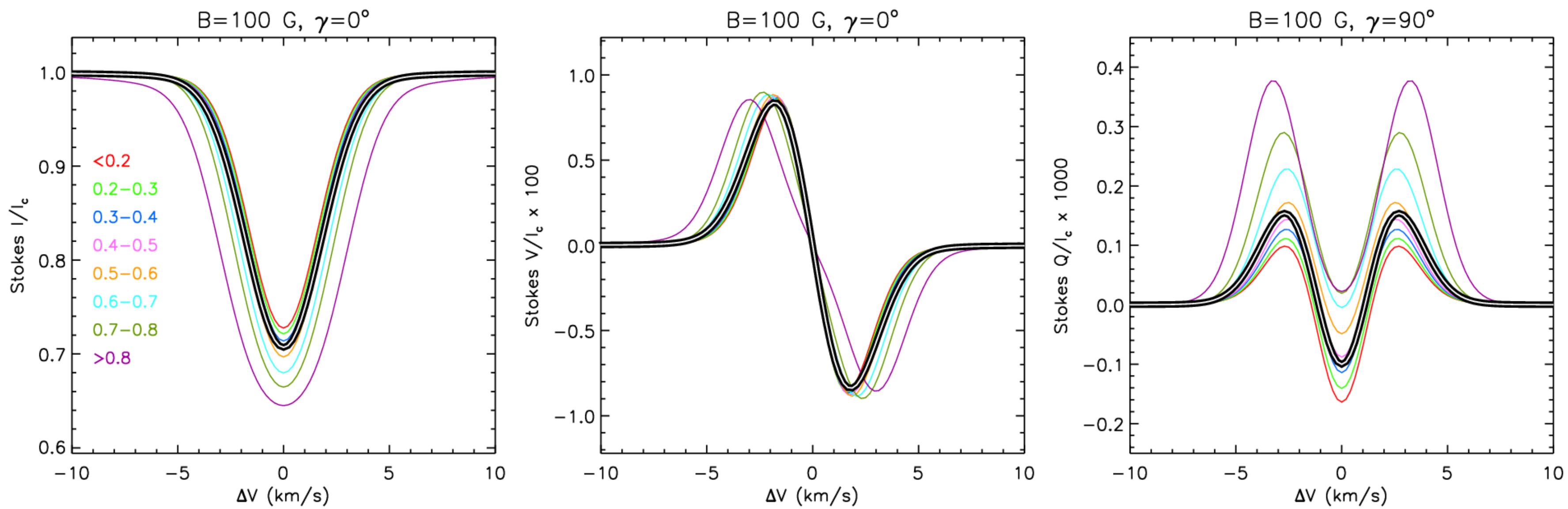
Donati (2011)

Extensions and improvements

- Principal Component Analysis (e.g. Carroll et al. 2012)
- LSD with non-linear profile addition (Sennhauser et al. 2009)
- LSD with multiple mean profiles (Kochukhov et al. 2010)
- LSD de-noising of intensity spectra (Tkachenko et al. 2013)

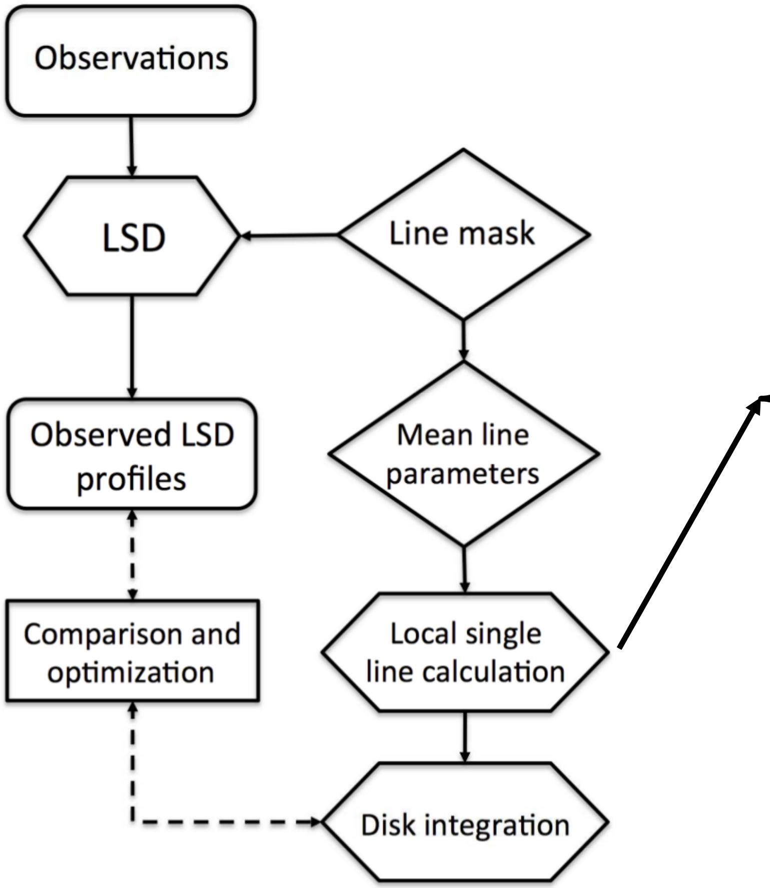
How trustworthy are LSD results?

- A very coarse description of line profiles
- Does not capture important physics (star spots)
- Expected to fail for *strong* magnetic fields
- Expected to fail for *strong* lines



Interpretation of LSD profiles

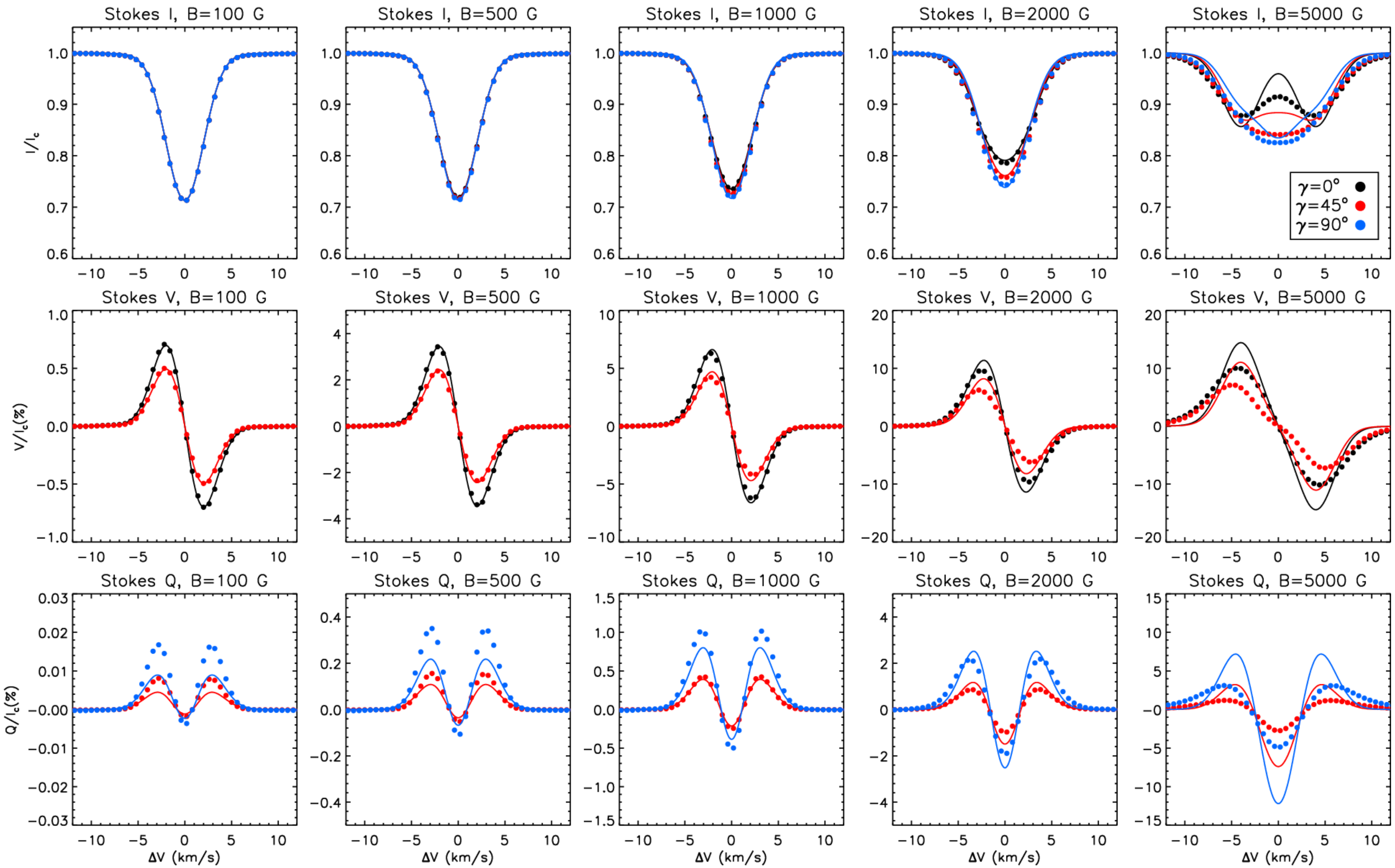
Typically, as a single line with mean parameters



- Gaussian local profile + weak-field approximation
- Unno-Rachkovsky solution
- Spectrum synthesis with mean line parameters

Assessing single-line approximation

Appropriate for Stokes *IV* and *B* < 2 kG (Kochukhov et al. 2010)



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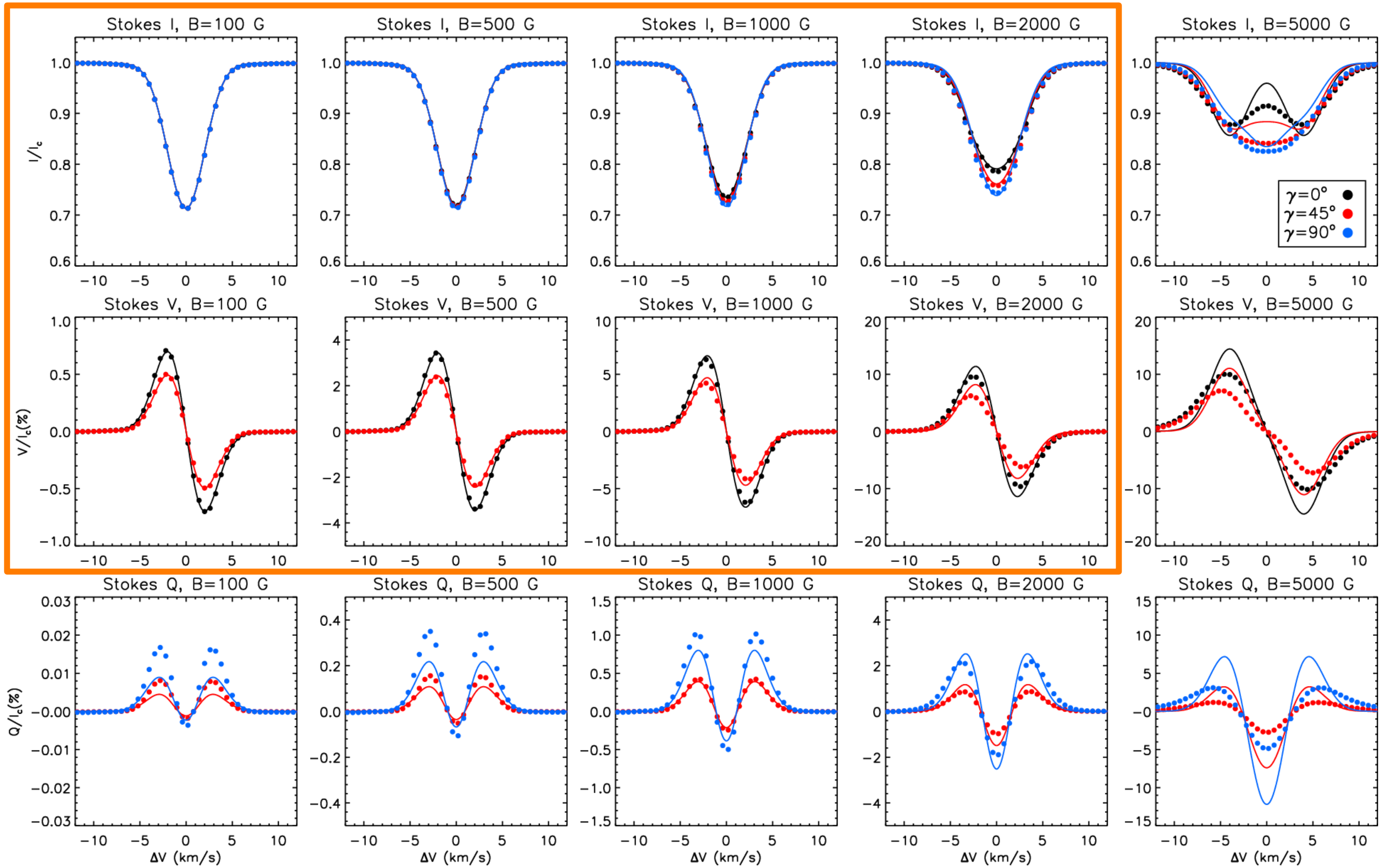
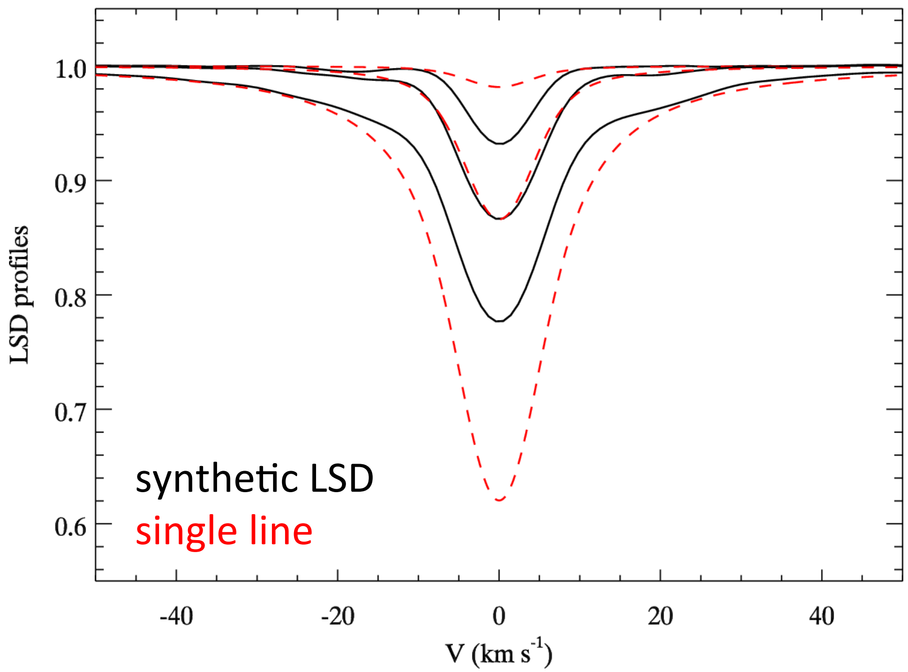


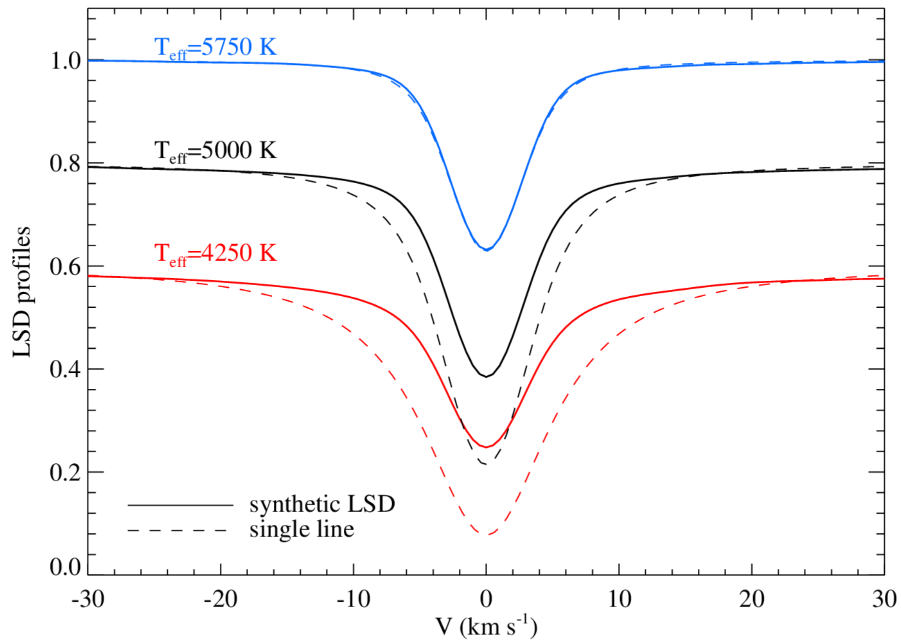
Assessing single-line approximation

Cannot match LSD profile response to the variation of temperature and chemical abundance

T_{eff}=13000 K, [Si]=-1, 0, +1

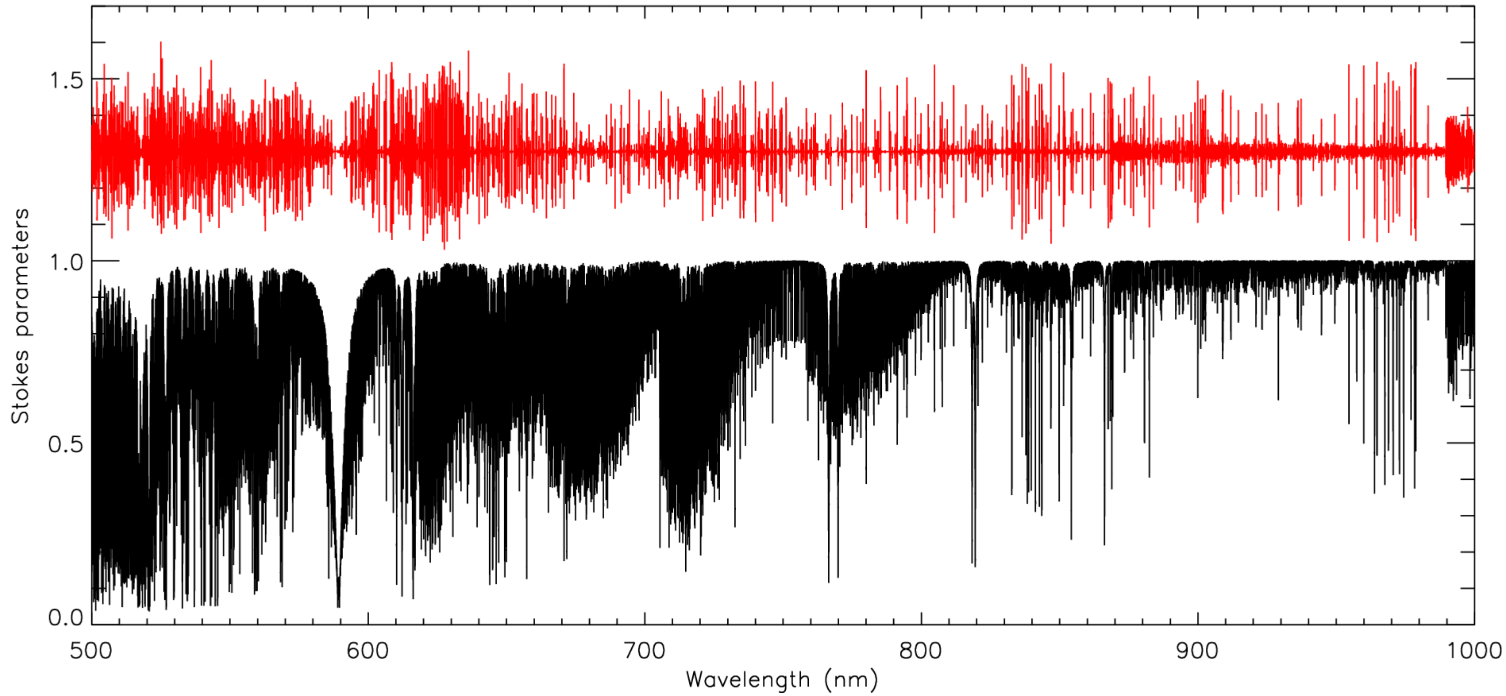


T_{eff}=5750, 5000, 4250 K



Alternative way to interpret LSD profiles

Computing resources allow direct calculation of large chunks of stellar polarisation spectra

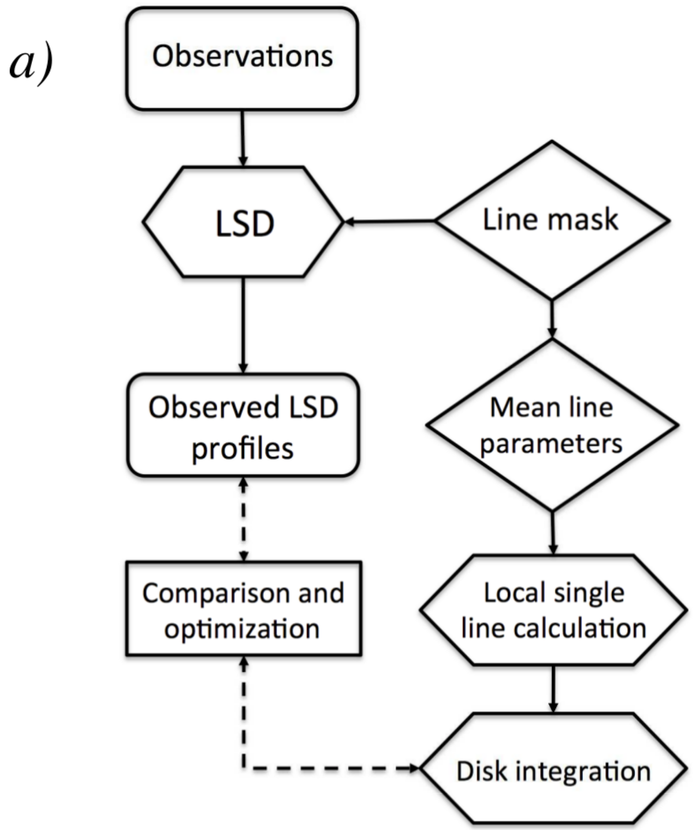


T_{eff} = 3300 K
5000 Å
~10^5 lines

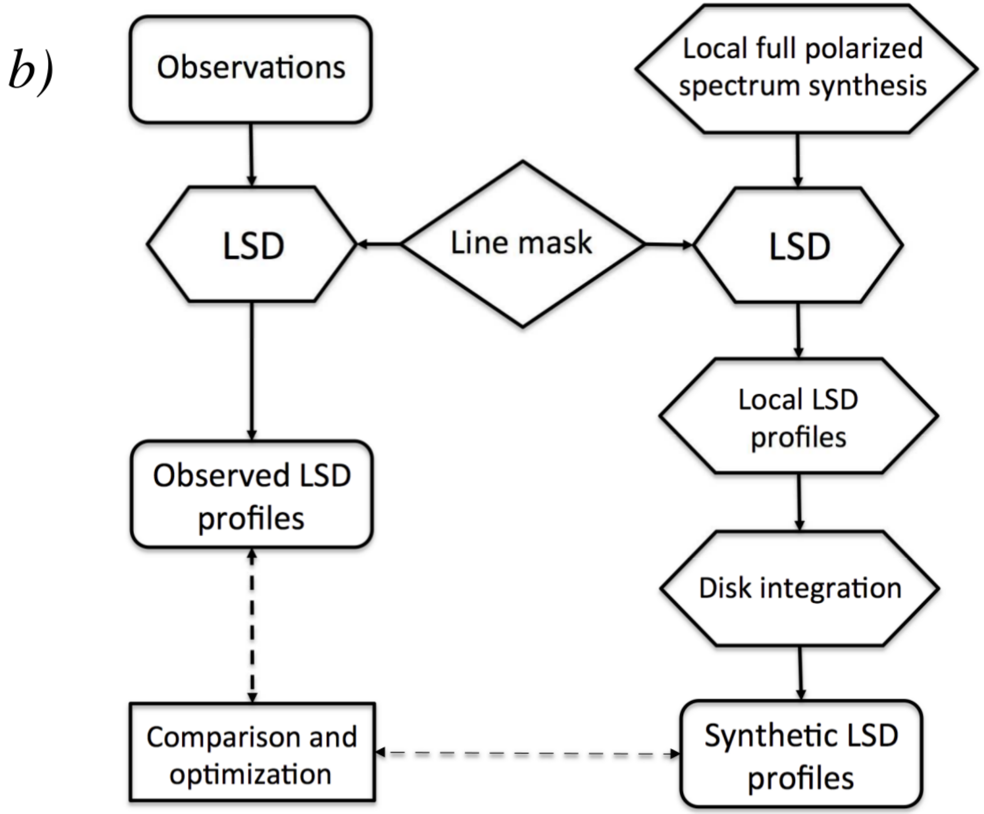
"Brute force" multi-line approach: apply LSD to observations *and* full polarised RT calculations

Alternative way to interpret LSD profiles

Single-line



Brute force multi-line

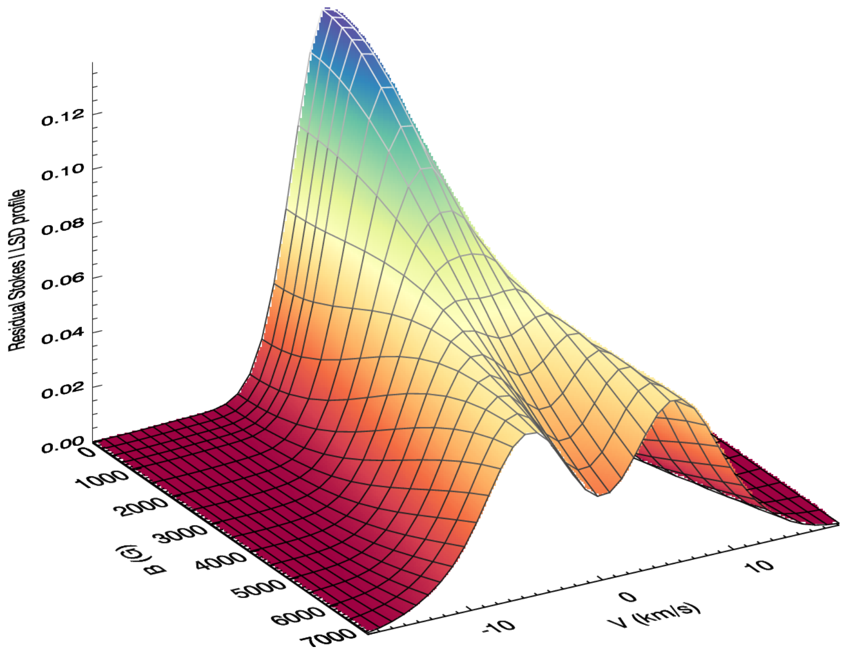


Kochukhov et al. (2014)

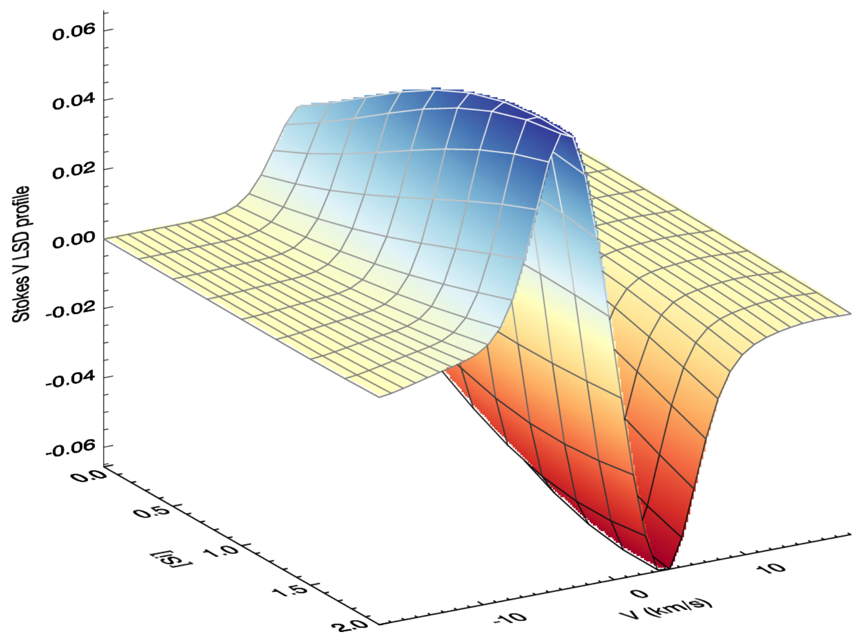
Synthetic LSD profile tables

◆ Detailed line formation physics incorporated in interpretation of LSD profiles

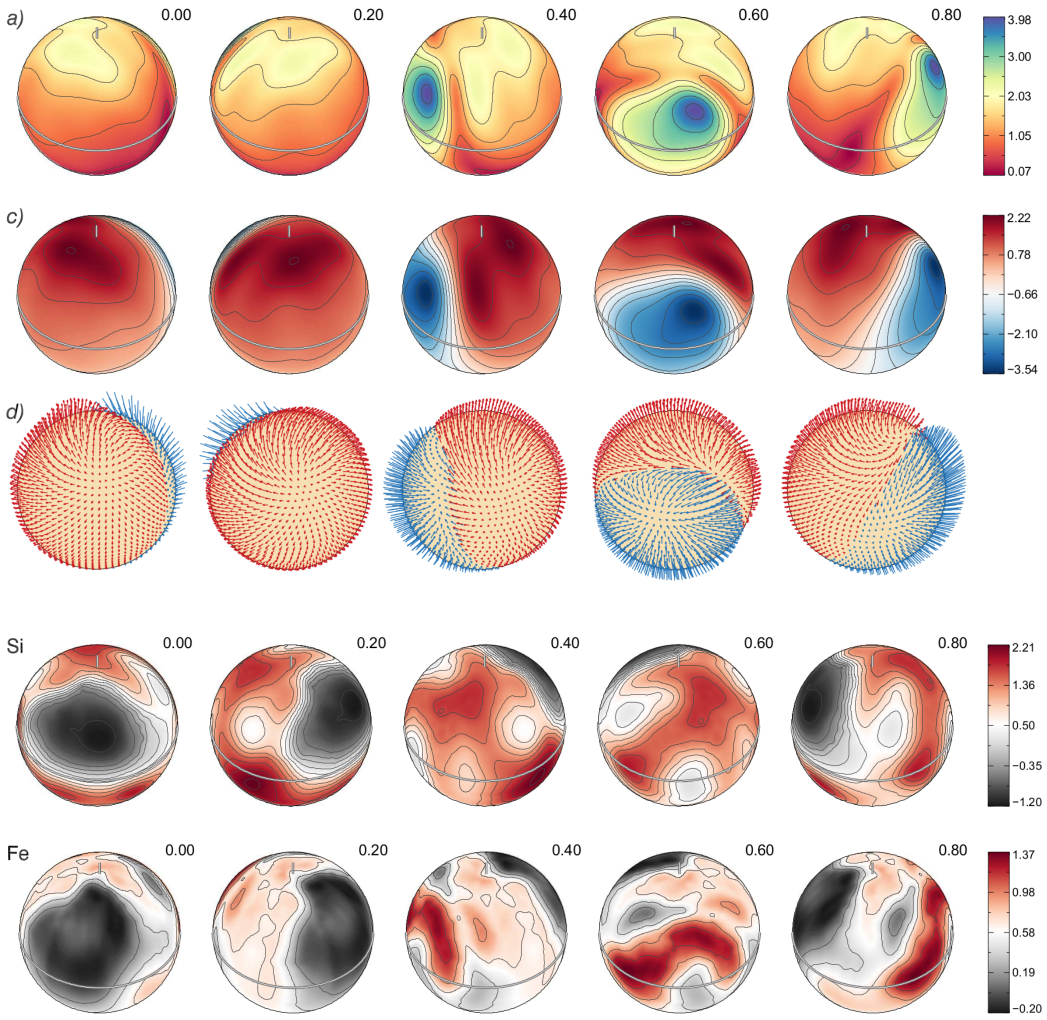
LSD Stokes *I* vs. B

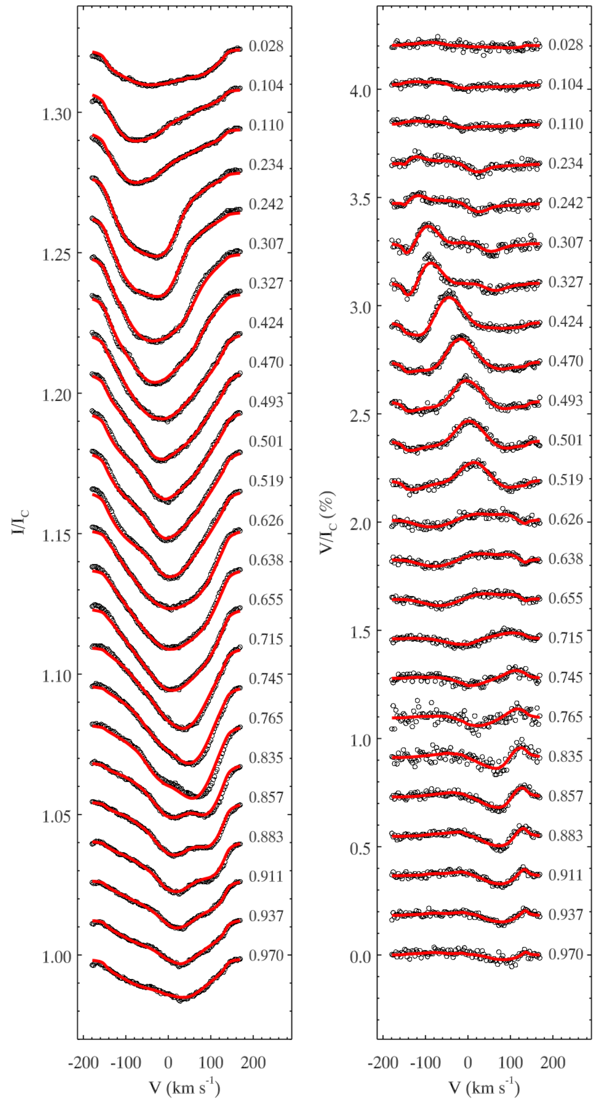


LSD Stokes *V* vs. abundance



Application to weak-field Ap star CU Vir





Conclusions

- Multi-line techniques are essential for stellar spectropolarimetry
- Least-squares deconvolution (LSD) is a widely used and powerful multi-line method
- LSD is robust for magnetic field detections and longitudinal field measurements
- Interpretation of LSD profile shapes is problematic
  - Single-line approach limited to Stokes *IV* and weak fields
  - Multi-line modelling of LSD profiles using detailed polarised spectrum synthesis is feasible