Paschen-Back effect in hyperfine structure states of an atom including the effects of partial frequency redistribution

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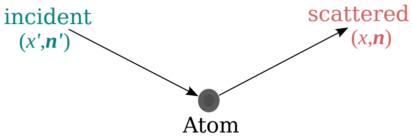
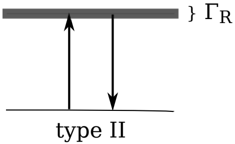
Scattering process





$$\text{scattered} \leftarrow \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \boldsymbol{R} \begin{bmatrix} I' \\ Q' \\ U' \\ V' \end{bmatrix} \leftarrow \text{incident}$$

* Frequency coherent scattering in atom's frame: $\boldsymbol{R}_{\text{II}}$.
* The redistribution matrix $\boldsymbol{R}(x', \boldsymbol{n}', x, \boldsymbol{n}, \boldsymbol{B})$ describes how an incident Stokes vector is transformed into a scattered Stokes vector.

Hyperfine splitting in an atom

* Hyperfine structure splitting (HFS) results from the interaction of the atomic nucleus with the electrons.

* The coupling between the electronic angular momentum J and nuclear spin I_s results in F states satisfying $|J - I_s| \leq F \leq |J + I_s|$.

* F-state interference is the quantum superposition of the hyperfine structure states.

* The transitions among different F states obey the selection rule $\Delta F = 0, \pm 1$.

* Spectral lines like Na I D_1 and D_2, Ba II D_1 and D_2 etc in the Second Solar Spectrum are known to exhibit effects due to HFS.

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Magnetic dipole term

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$$\mathcal{H}_D = \mathcal{A}_J \mathbf{I_s} \cdot \mathbf{J}$$

and

Electric quadrupole term

$$\mathcal{H}_Q = \frac{\mathcal{B}_J}{2I_s(2I_s-1)J(2J-1)}\left[3(\mathbf{I_s}\cdot\mathbf{J})^2 + \frac{3}{2}(\mathbf{I_s}\cdot\mathbf{J}) - I_s(I_s+1)J(J+1)\right]$$

and is diagonal in F. The separation ($\Delta\nu_F$) between the F states can be calculated from its eigenvalues.

Role of magnetic fields

Magnetic Hamiltonian

$$\mathcal{H}_B = \mu_0(\mathbf{J} + \mathbf{S}) \cdot \boldsymbol{B}$$

- The magnetic field produces a shift $\Delta\nu_B$.
- When $\Delta\nu_B \sim \Delta\nu_F$ the magnetic components of different lines start to cross leading to mixing of F states $\Longrightarrow$ Paschen-Back Effect.
- Magnetic Hamiltonian can no longer be treated as a perturbation to the atomic Hamiltonian.

Magnetic field strength regimes

Zeeman effect regime

$\Downarrow$

$\Delta\nu_B \ll \Delta\nu_F$

$\Downarrow$

Magnetic splitting is linear

Magnetic field strength regimes

Zeeman effect regime	Incomplete PBE regime
$\Downarrow$	$\Downarrow$
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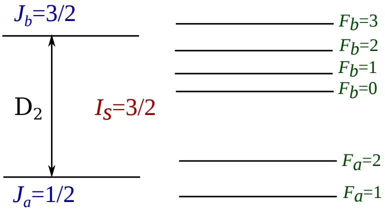
Magnetic field strength regimes

Zeeman effect regime	Incomplete PBE regime	Complete PBE regime
$\Downarrow$	$\Downarrow$	$\Downarrow$
$\Delta\nu_B \ll \Delta\nu_F$	$\Delta\nu_B \sim \Delta\nu_F$	$\Delta\nu_B \gg \Delta\nu_F$
$\Downarrow$	$\Downarrow$	$\Downarrow$
Magnetic splitting is linear	Splitting ceases to be linear	Linearity sets in again

The atomic system

* Na I D_2 line transition between the $^2P_{3/2}$ and $^2S_{1/2}$ levels.
* Wavelength in air for this transition is 5889.95095 Å.

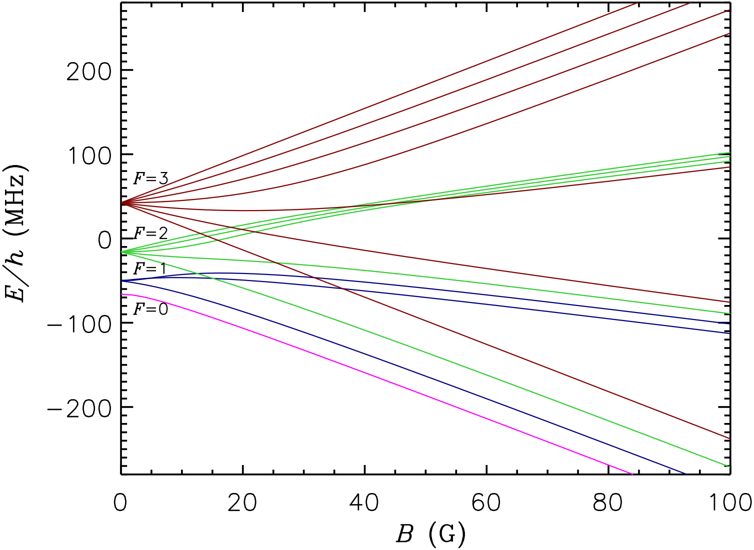


Level	$\mathcal{A}$(MHz)[1]	$\mathcal{B}$(MHz)[1]
$^2S_{1/2}$	885.81	0
$^2P_{3/2}$	18.534	2.724

* The selection rules in the PB regime are $\Delta J = 0, \pm 1$ and $\Delta \mu = 0, \pm 1$.

[1] Steck (2003)

Level-crossing



Earlier studies

* Considered a flat-spectrum for the pumping radiation within the separation between the F states[2].

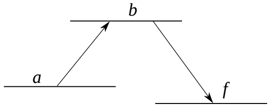
* Landi Degl'Innocenti et al. (1997) used the concept of metalevels and included the effects PRD.

* We follow the Kramers-Heisenberg scattering approach and derive the PRD matrix in the laboratory frame.

[2] Landi Degl'Innocenti (1975, 1978), Landi Degl'Innocenti & Landolfi (2004), Casini & Manso Sainz (2005)

Coherency matrix

* The probability amplitude for scattering from $a \rightarrow b \rightarrow f$ is calculated using the Kramers-Heisenberg formula



$$[w_{\alpha\beta}]_b \sim \sum_b \frac{\langle f|\mathbf{r}\ .\ \mathbf{e}_\alpha|b\rangle \langle b|\mathbf{r}\ .\ \mathbf{e}_\beta|a\rangle}{\omega_{bf} - \omega - \mathrm{i}\gamma/2}$$

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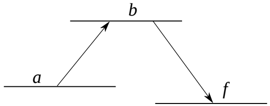

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* $|a\rangle = |J_a I_s i_a \mu_a\rangle$ with $N_{i_a} = 1 + J_a + I_s - \max(|\mu_a|, |J_a - I_s|)$.

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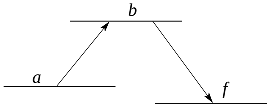

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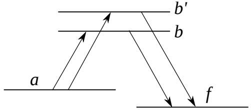
- The scattering cross section is written as coherency matrix $\mathbf{W}$



$$\mathbf{W} = \sum_{a,f} [w_{\alpha\beta}]_b \otimes [w^*_{\alpha\beta}]_{b'}$$

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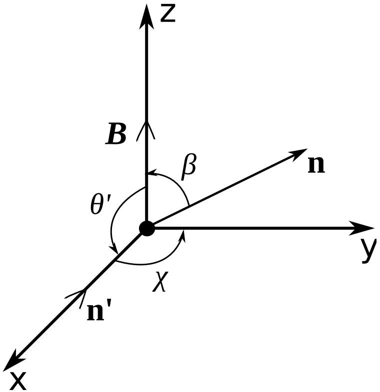
- **M** is analogous to the redistribution matrix $\boldsymbol{R}$.

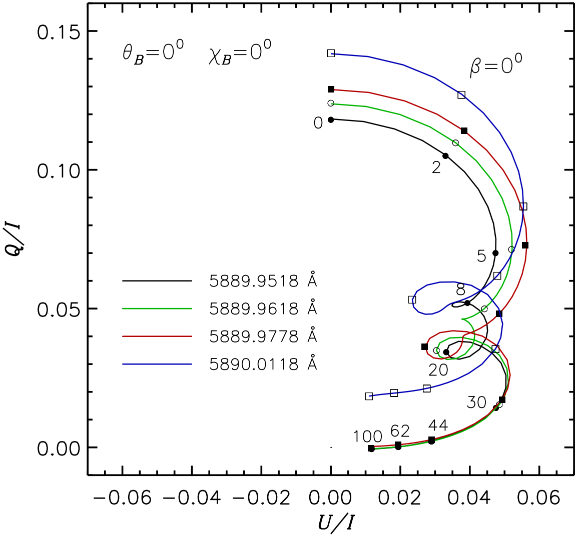
Scattering geometry



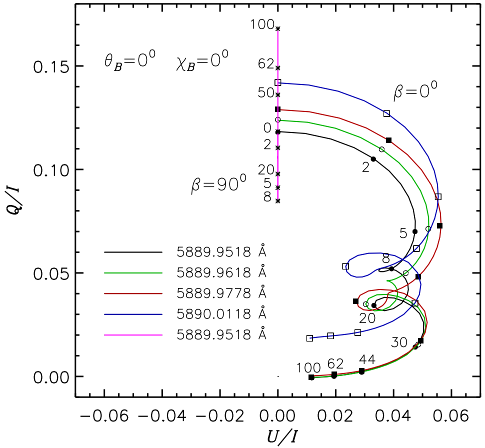
- β is the angle between the magnetic field vector and the scattered ray.
- Incident ray is characterized by $(\theta', \chi') = (90^\circ, 0^\circ)$.
- Scattered ray is characterized by $(\theta, \chi) = (\beta, 90^\circ)$.
- The inclination and azimuth of the magnetic field are $(\theta_B, \chi_B) = (0^\circ, 0^\circ)$.

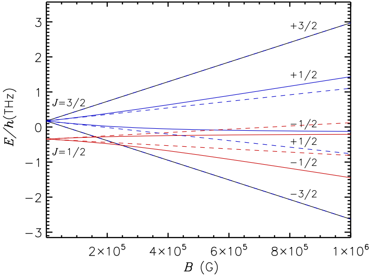
Single 90° scattering of the unpolarized incident radiation.

Polarization diagram



Avoided level crossing[3]



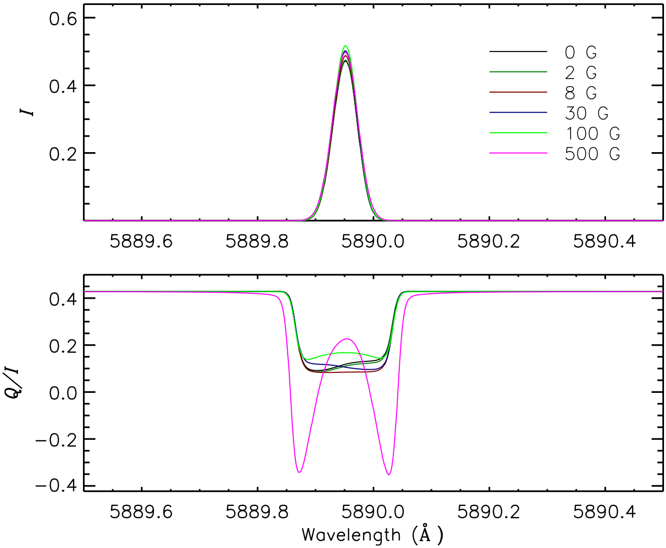


* Because of the non-diagonal coupling terms in the total Hamiltonian, the magnetic substates, instead of crossing, repel each other.
* As a result of avoided crossing we find

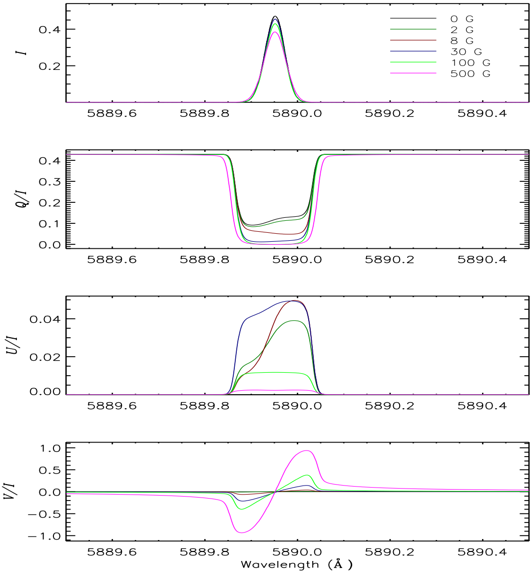
$$\left(\frac{Q}{I}\right)^{I_s\neq0}_{B=0} < \left(\frac{Q}{I}\right)^{I_s\neq0}_{B\to\infty}$$

[3] Bommier (1980) and Landi Degl'Innocenti & Landolfi (2004)

Vertical magnetic field



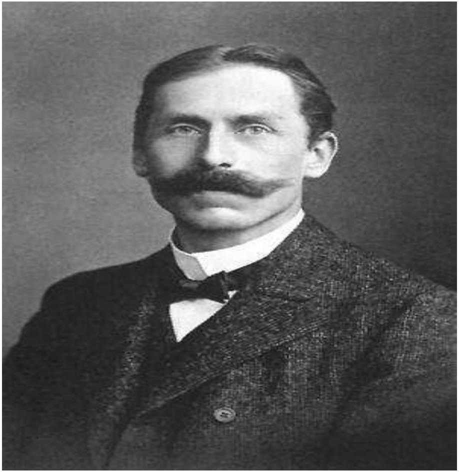
Horizontal field parallel to the LOS



Conclusions

* Using the Kramers-Heisenberg scattering approach we have derived the PRD matrix for PBE in a two-level atom with HFS.
* Incomplete PBE causes non-linear splitting that leads to level-crossings which give rise to loops in the polarization diagrams.
* Avoided level-crossing signatures are prominently seen when the magnetic field is vertical.
* Asymmetric Stokes V profiles are a result of incomplete PBE, because of which net circular polarization remains non-zero.
* These signatures point towards the possibility of using atomic PBE as a tool for the solar magnetic field diagnostics.

Thank You



Louis Karl Heinrich Friedrich Paschen

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